# EE 40

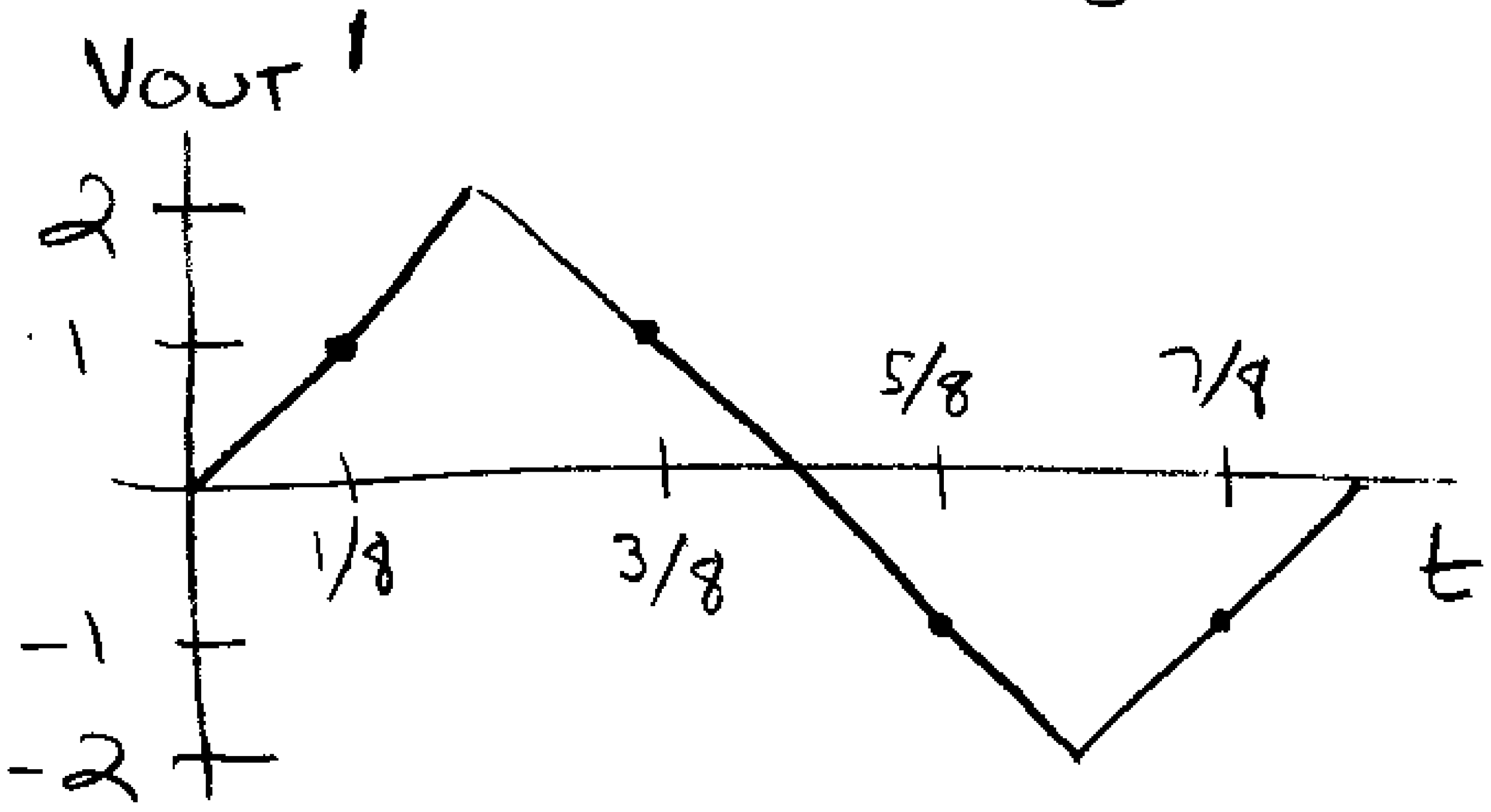
## Final Exam Review Set #1

## Solutions

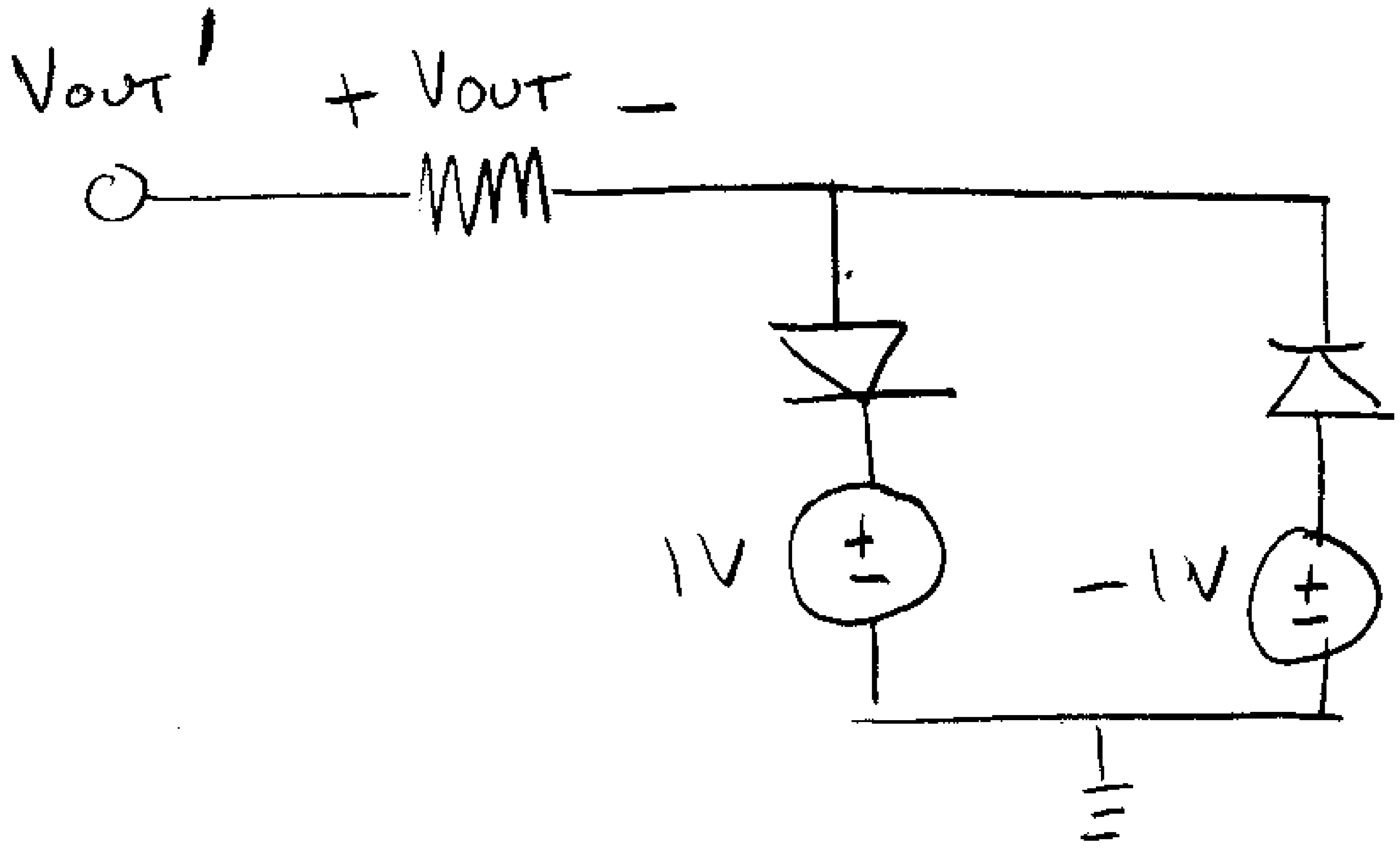
For the design problems, many solutions are possible!

**Problem 1:** Work backwards.

$V_{OUT}$ looks like the "tips" of a triangle wave. Clippers cut off the tips of wave forms. So if I could get a triangle wave input $V_{OUT}'$

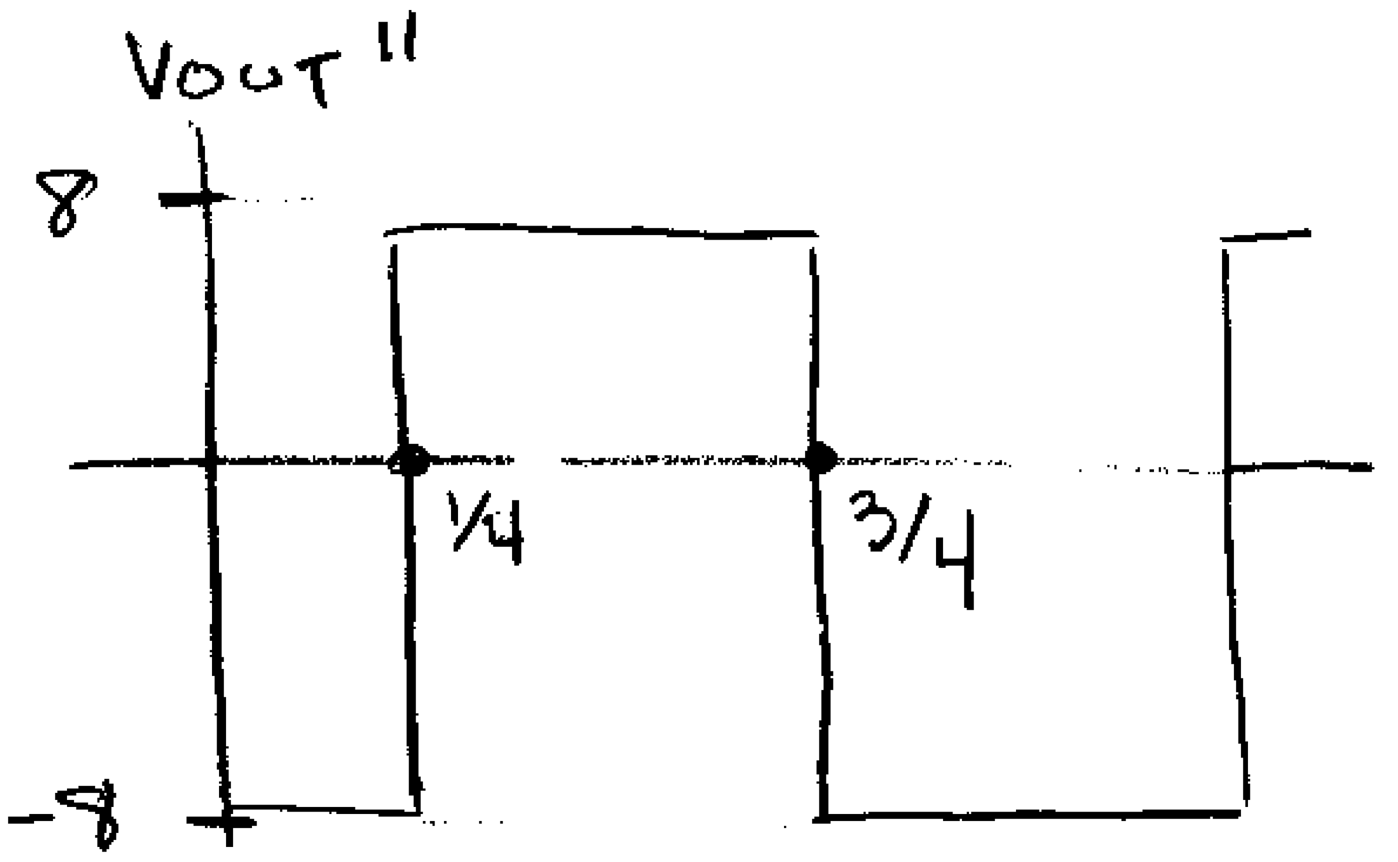



then putting this through a clipper would work.

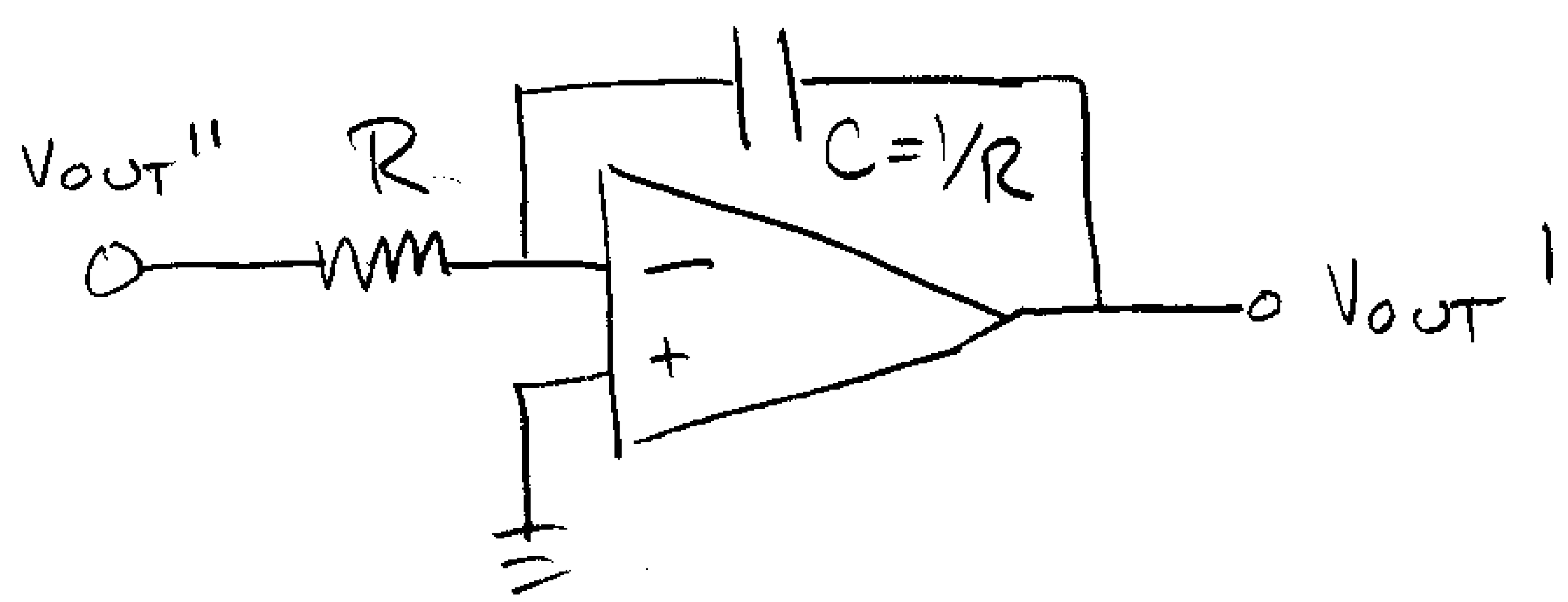



assuming ideal diode model — adjust voltage sources for $V_F \neq 0$.

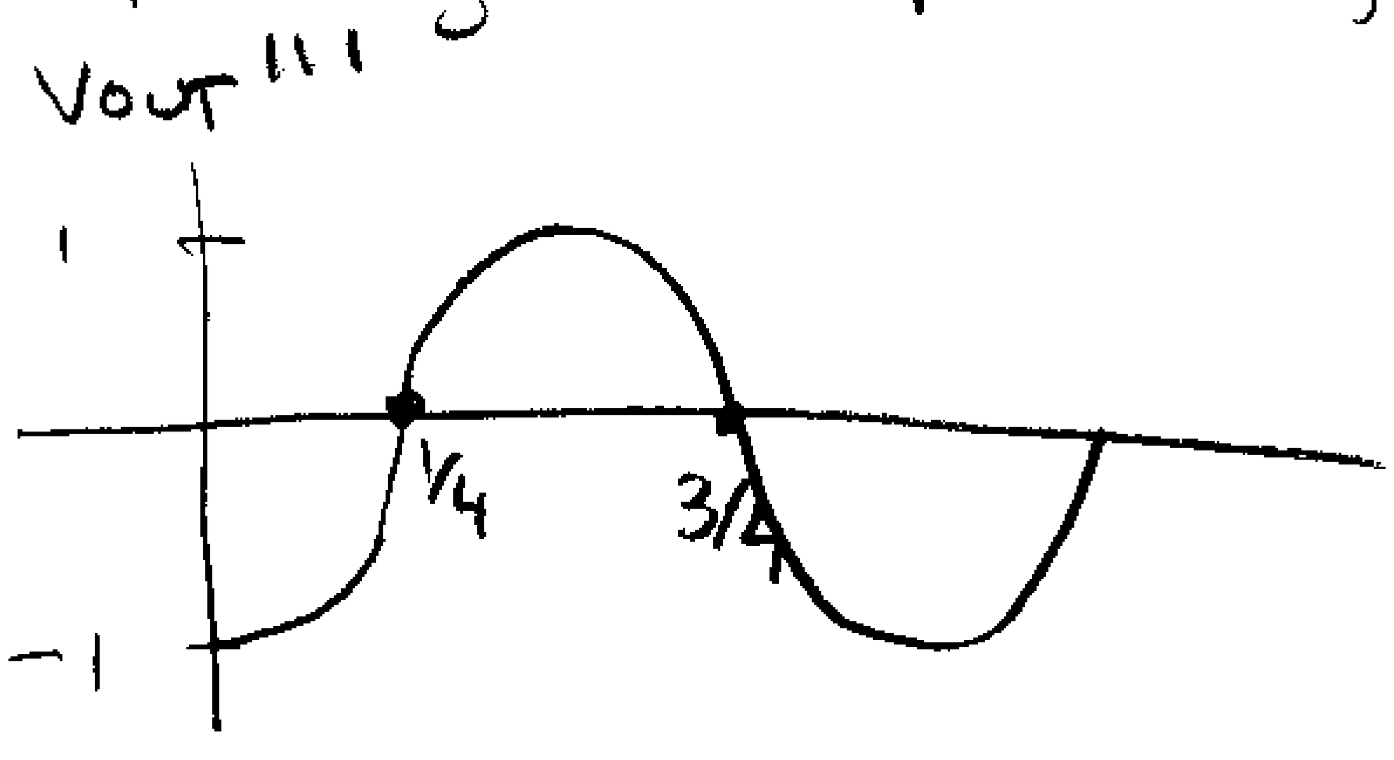
How to get $V_{OUT}'$? Integrate square wave. Suppose I use an integrator with $RC = 1$. Then to get $V_{OUT}'$, send through $V_{OUT}''$:

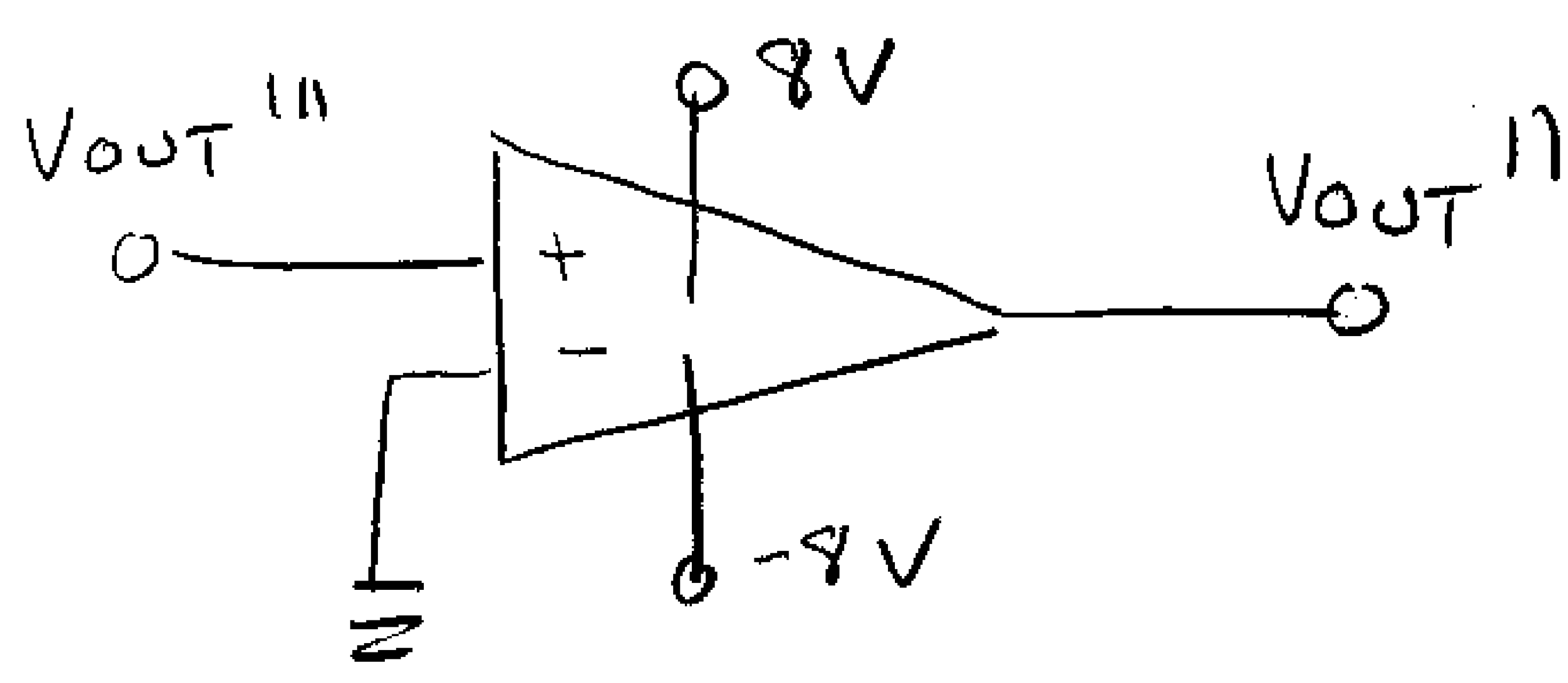



$V_{OUT}'$ increases at a rate of 8 V/s for 1/4 s, decreases at 8 V/s for 1/2 s, increases for 1/2 s, etc. Integrator adds − sign!

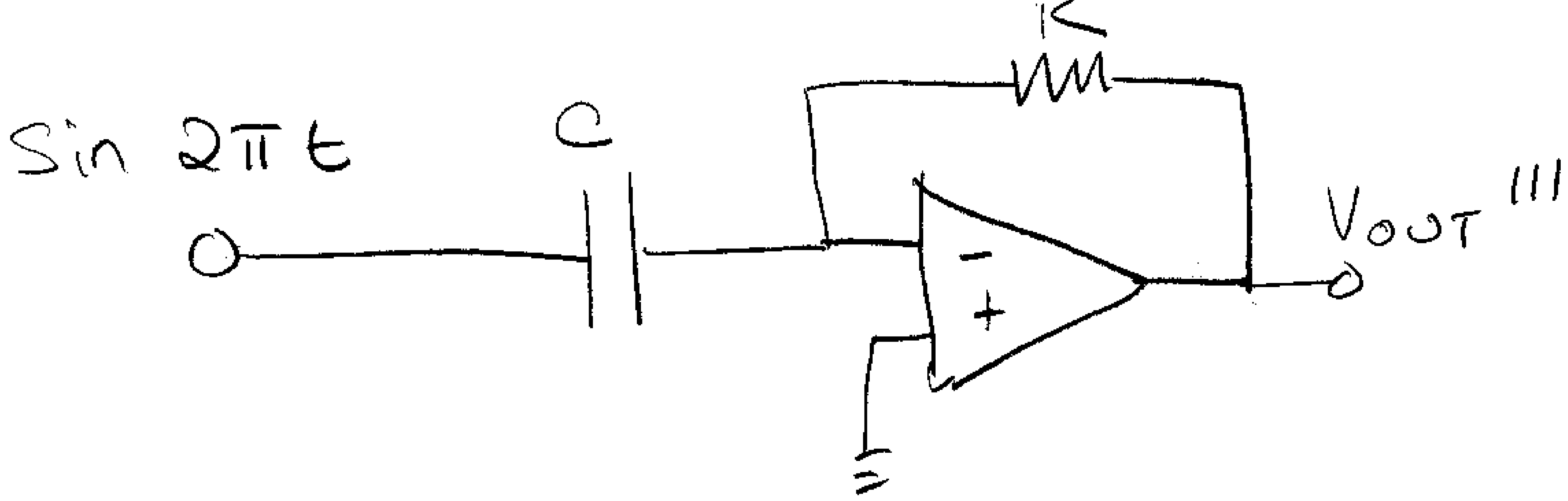



How to get $V_{out}''$? If we put $-\cos(2\pi t)$ ($V_{OUT}'''$) through comparator, that would work.



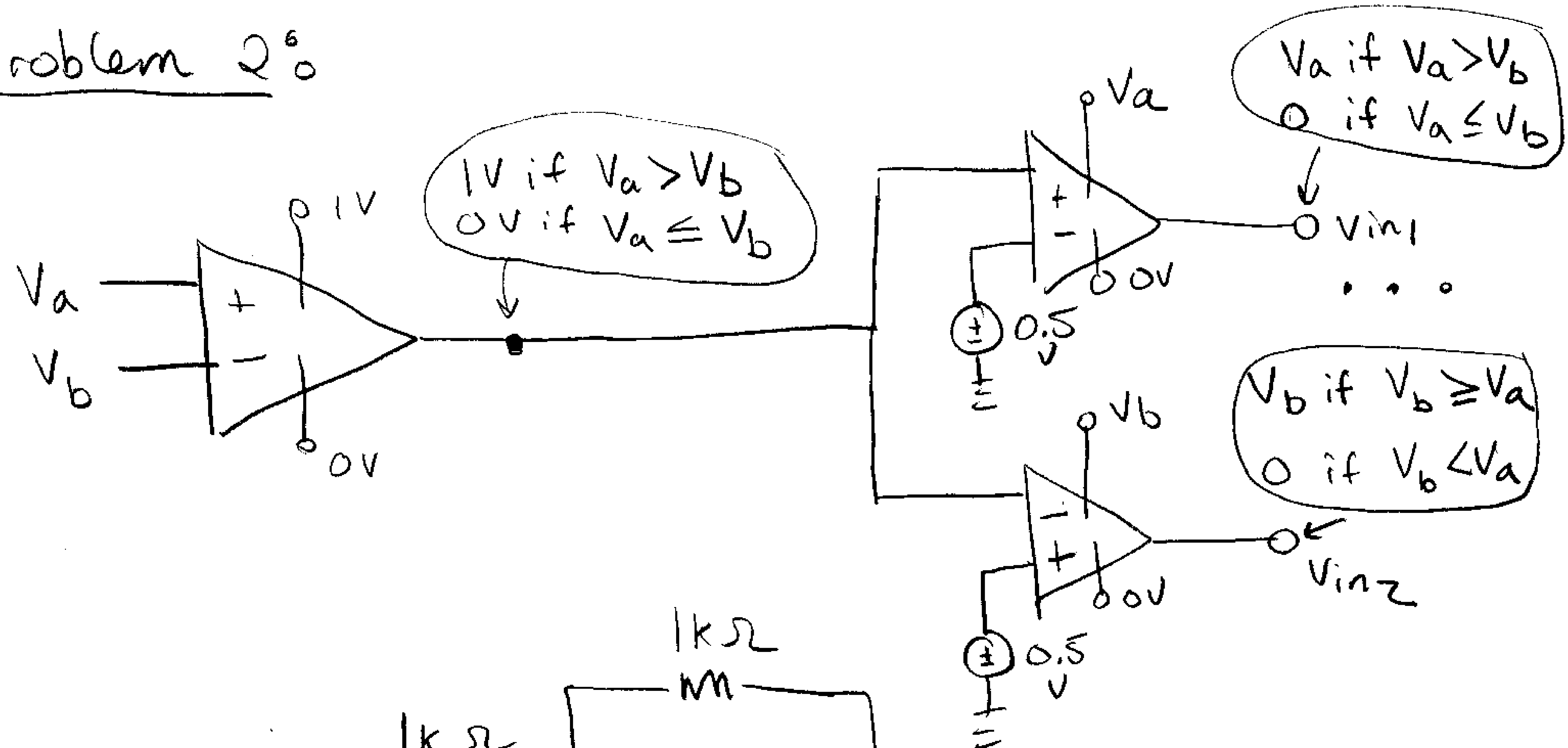



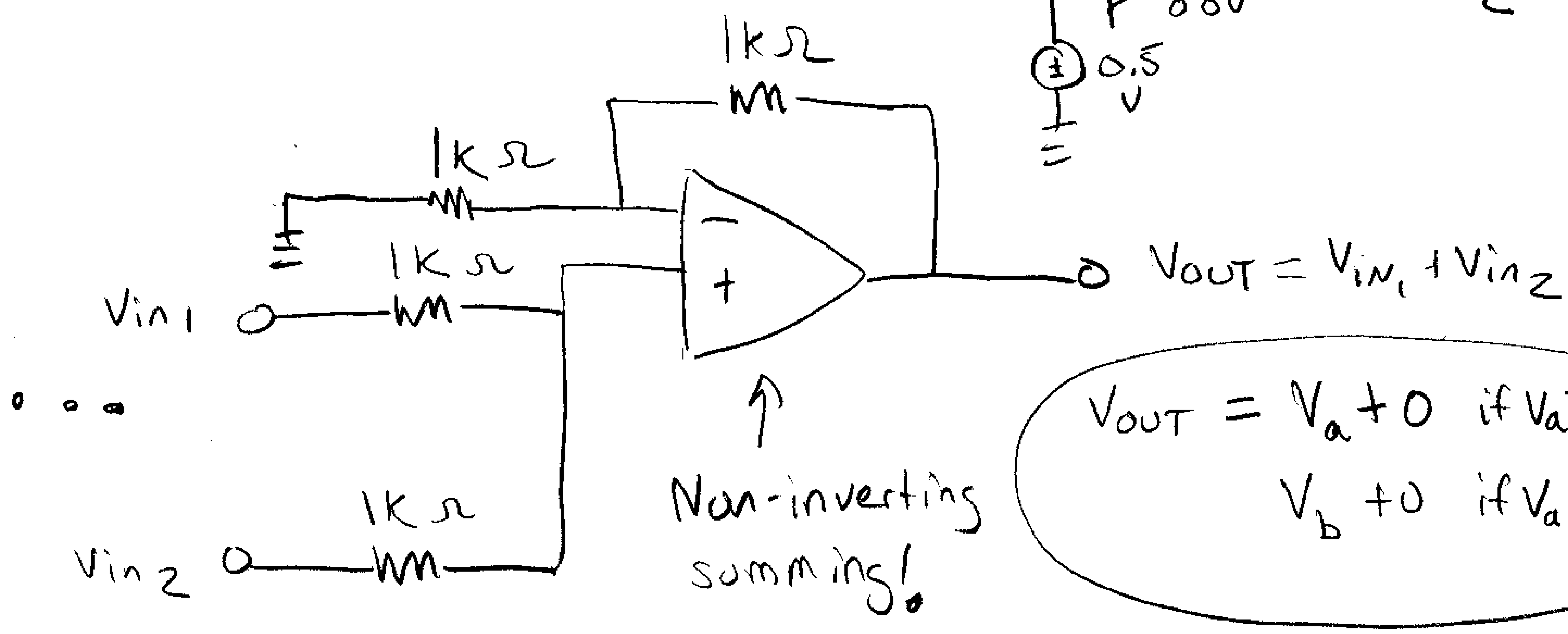

To get $V_{OUT}''' = -\cos 2\pi t$, use the differentiator (which adds − sign). The $2\pi$ scaling factor that comes out can either be cancelled by setting $RC = 1/2\pi$ or ignored since it is lost after the comparator.




Put all the circuits together for final design.

## Problem 2:



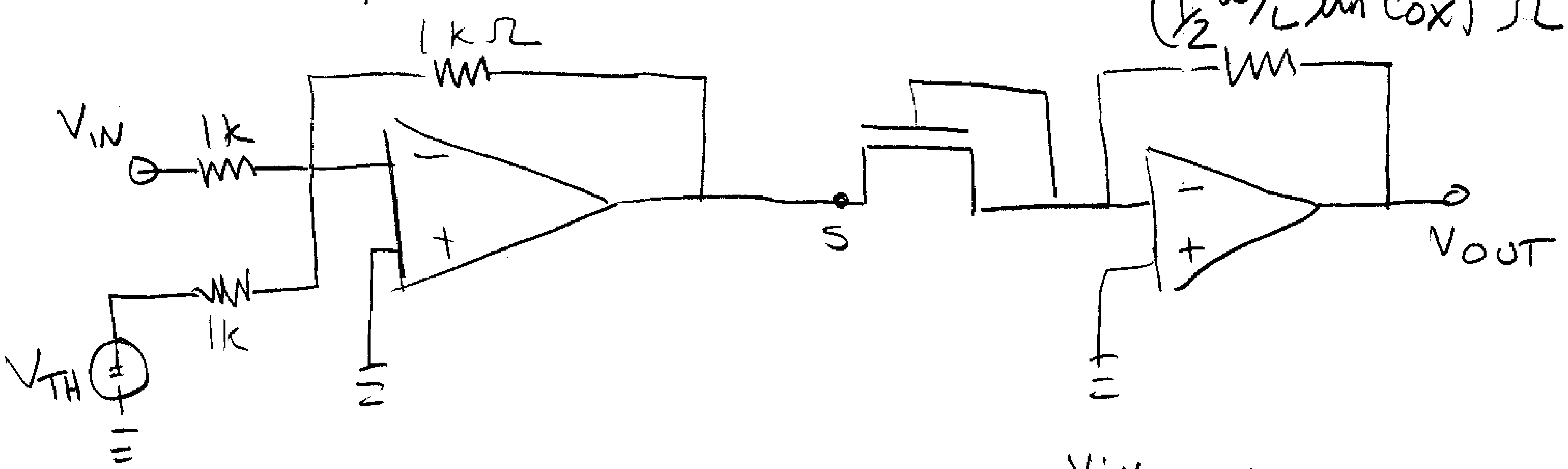

$V_{OUT} = V_a + 0$ if $V_a > V_b$

$\phantom{V_{OUT} = } V_b + 0$ if $V_a \leq V_b$

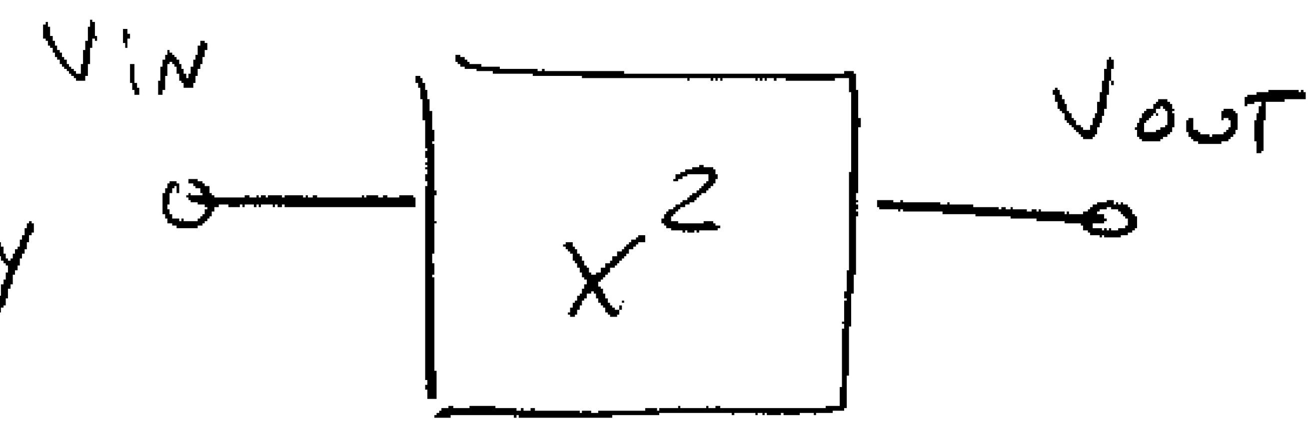
$$V_{OUT} = \max\{V_a, V_b\}$$

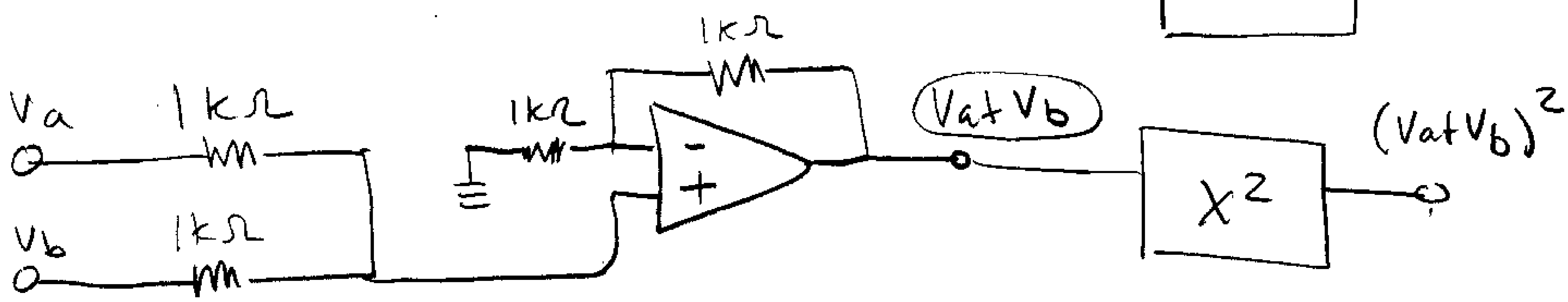
Problem 3: (one of many solutions!)

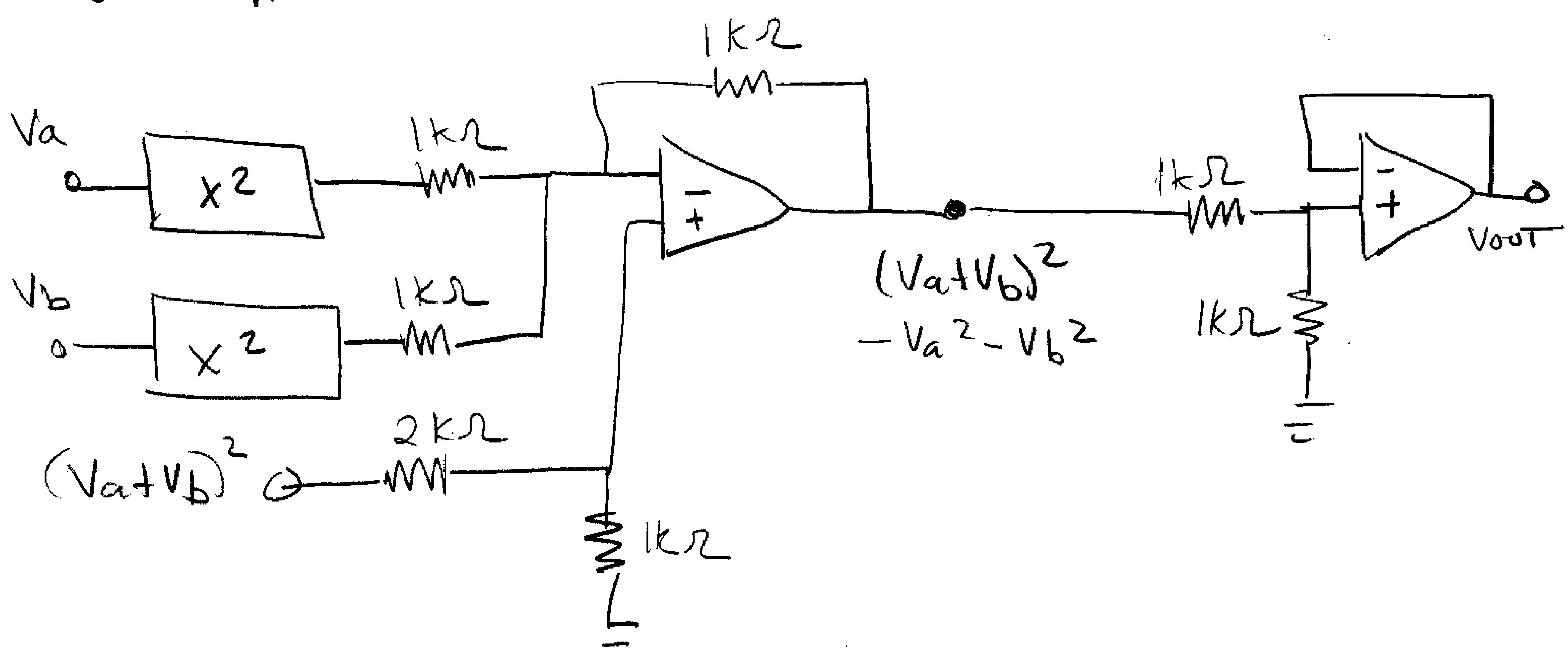
In Homework 7, we found that the following circuit produces $V_{OUT} = V_{IN}^2$:



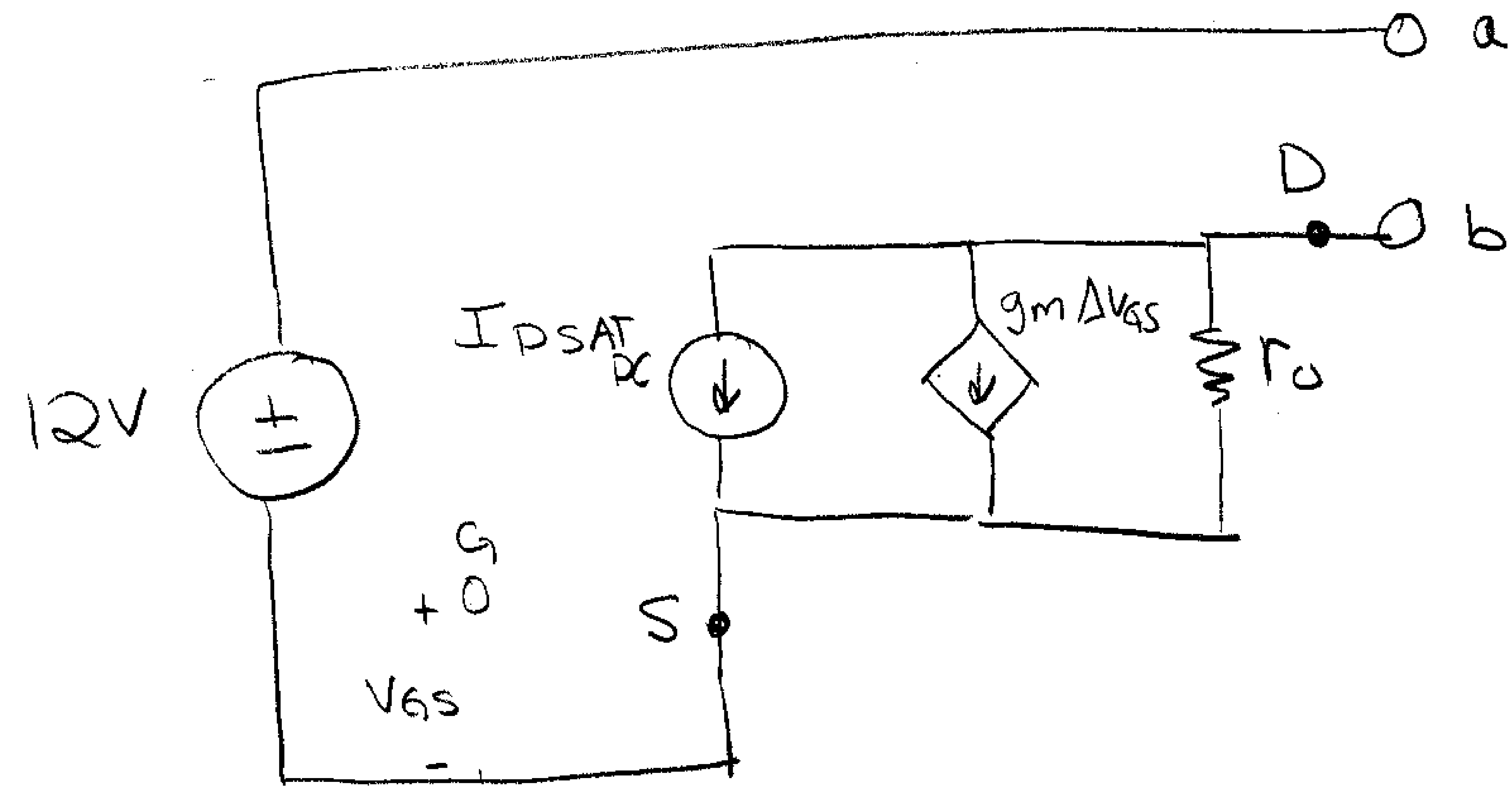

Let's now abbreviate the above by



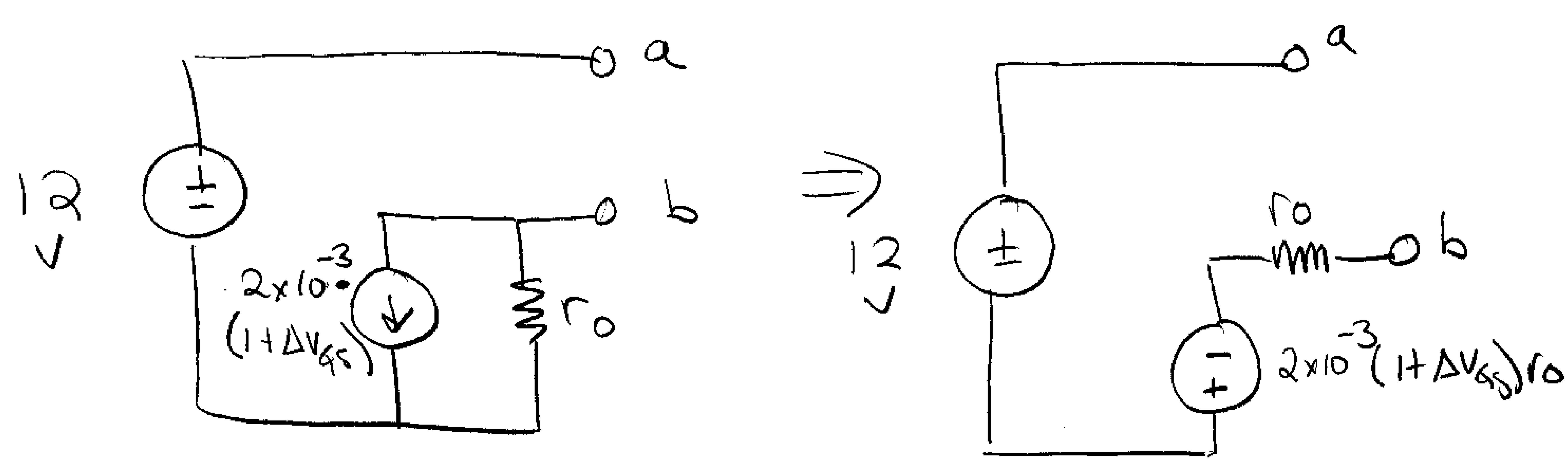
Problem 4: Replace transistor w/ small-signal model:



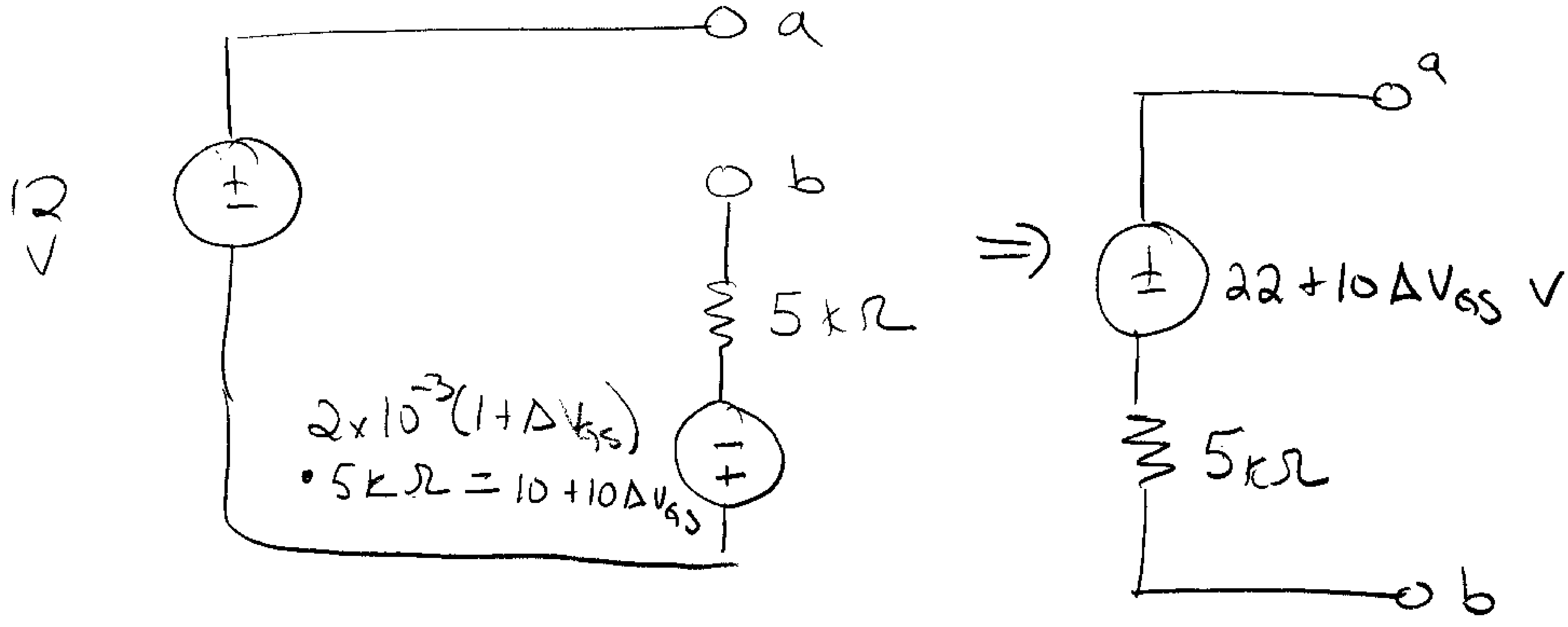

$$I_{DSAT_{DC}} = \tfrac{1}{2}(1mA/V^2)(3V-1V)^2 = 2mA$$

$$g_m = \tfrac{W}{L}\mu_N C_{ox}(V_{GS_{DC}} - V_{TH}) = 1mA/V^2(3V-1V) = 2mA/V$$

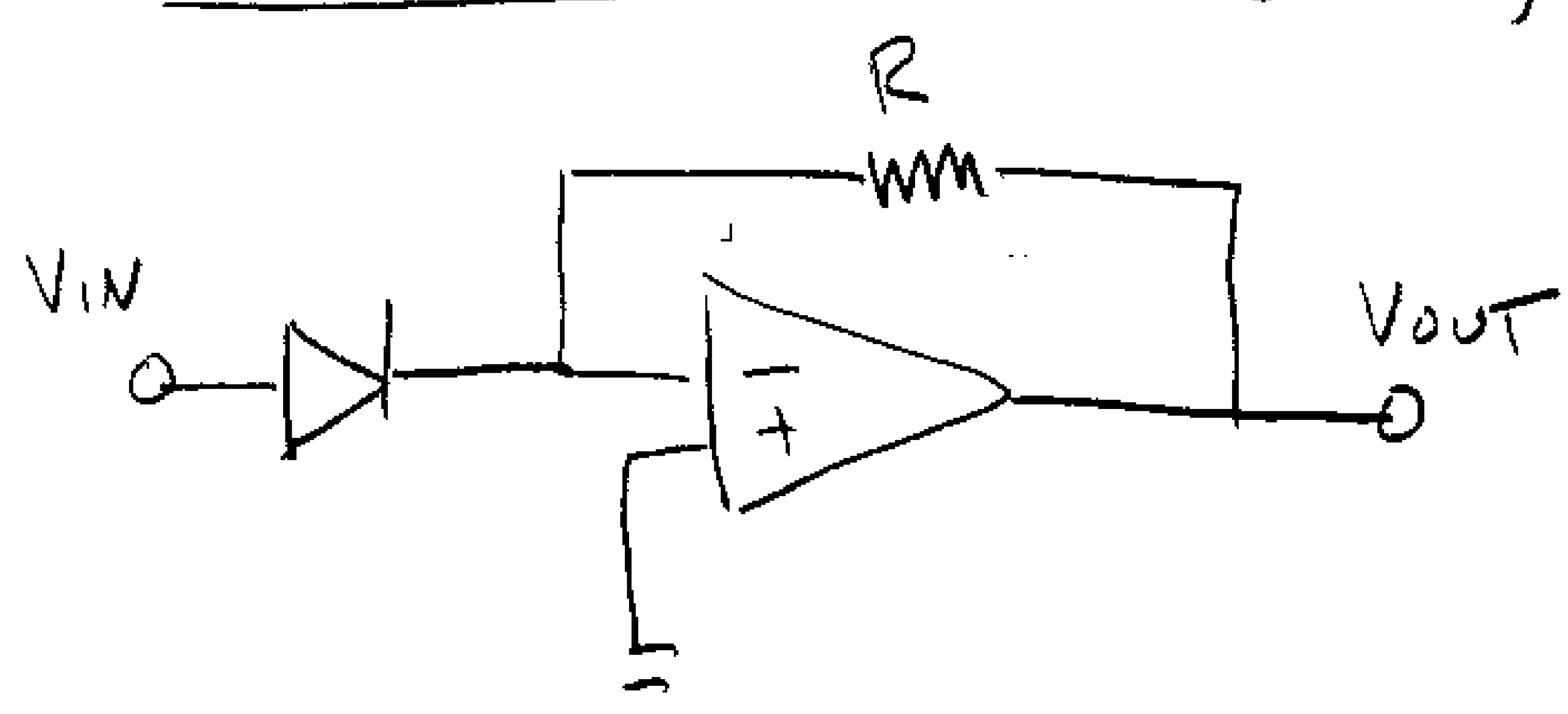
Leaving $\Delta V_{GS}$ in equations so




$$r_o = \frac{1}{\lambda I_{DSAT_{DC}}} = \frac{1}{0.1\cdot 2mA} = 5k\Omega$$



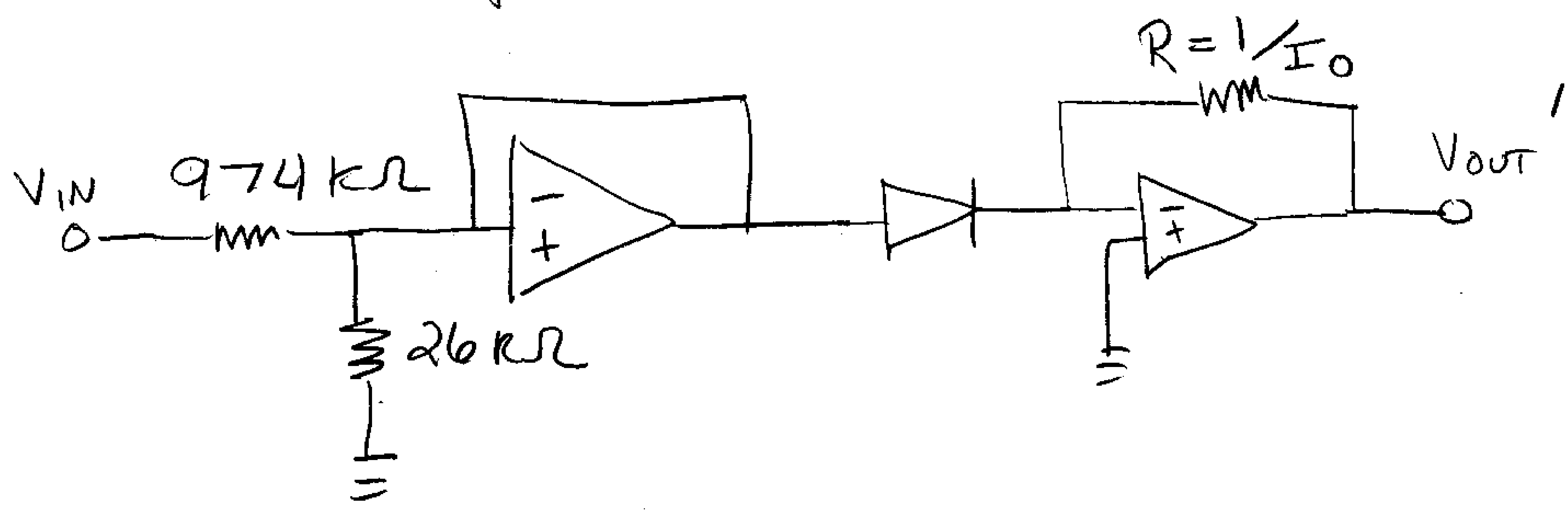

Problem 5: In class, we found that for

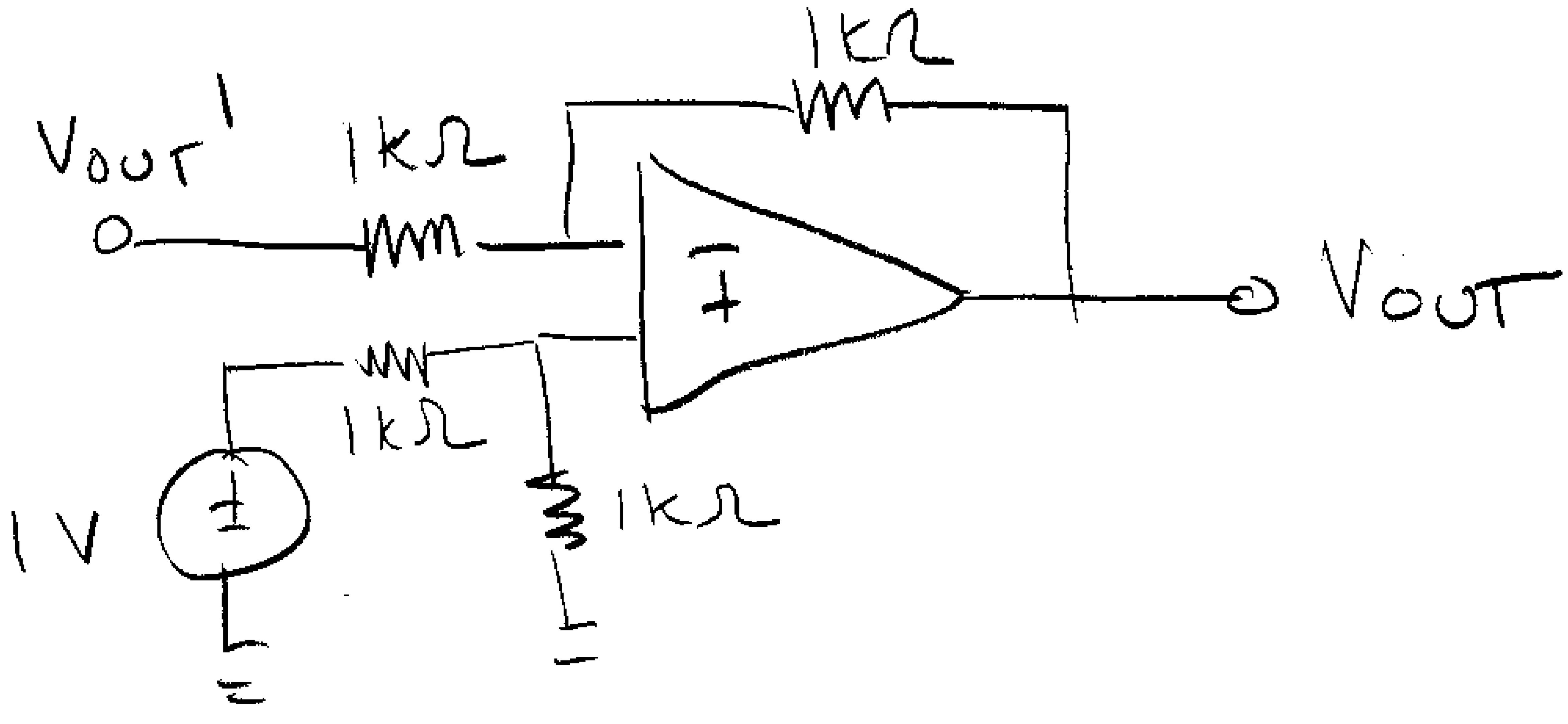



$$V_{OUT} = -R I_0 \left(e^{\frac{V_{IN}}{V_T}} - 1\right)$$

So we can pre-multiply $V_{IN}$ by $V_T$, then scale by $-1/RI_0$, and then add 1 V.

At room temperature, $V_T = 0.026$ V.





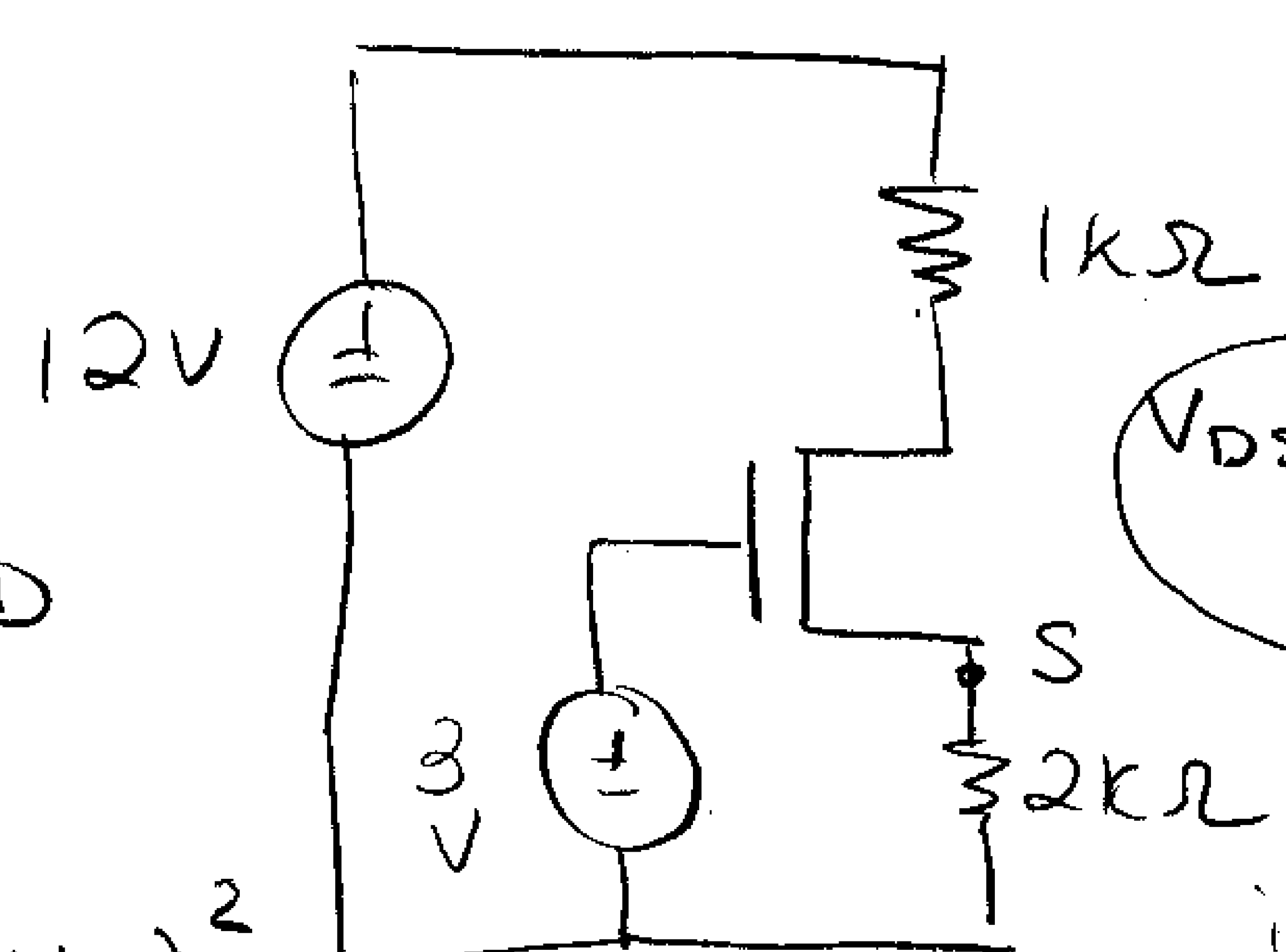

## Problem 6:

- $I_o$ can be extremely small ($10^{-15}$ A). To make $R = 1/I_o$, need "exa-ohm" resistance - not possible. Other gain elements with $10^{15}$ gain also impractical.

- $V_T$ changes with temperature, proportionally. Changes in temperature that normally occur in circuits would throw off results.

Other issues may also exist!

## Problem 7:




$V_{GS} = 3V - 2k I_D$

$12V = 1k I_D + V_{DS} + 2k I_D$

Hope for saturation!

$V_{DS} = 12V - 3k\Omega I_D$
$= 11.66 V$

$I_D = \frac{1}{2} \frac{W}{L} \mu_n C_{ox} (V_{GS} - V_{TH})^2$

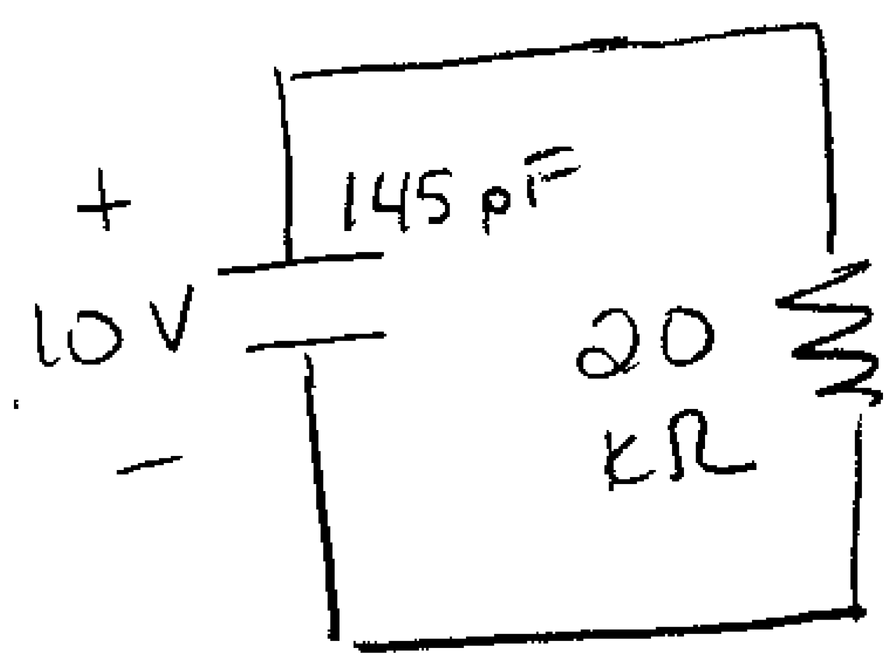
$= 100 \mu A/V^2 (3V - 2k I_D - 1V)^2 \overset{\text{Solve}}{\Longrightarrow} I_D = \{ \sout{4.4 mA}, \underline{114 \mu A} \}$

impossible, results in $V_{DS} < 0$

Problem 8: If $\mu_N$ increases due to heat, then $I_D$ will increase. Then the voltage over the $2k\Omega$ resistor will increase, lowering $V_{GS}$. This counteracts the increase in $I_D$ and results in a current supply which varies less due to temp changes.

Problem 9:

$t = 0$: $V_{OUT} = 1V$



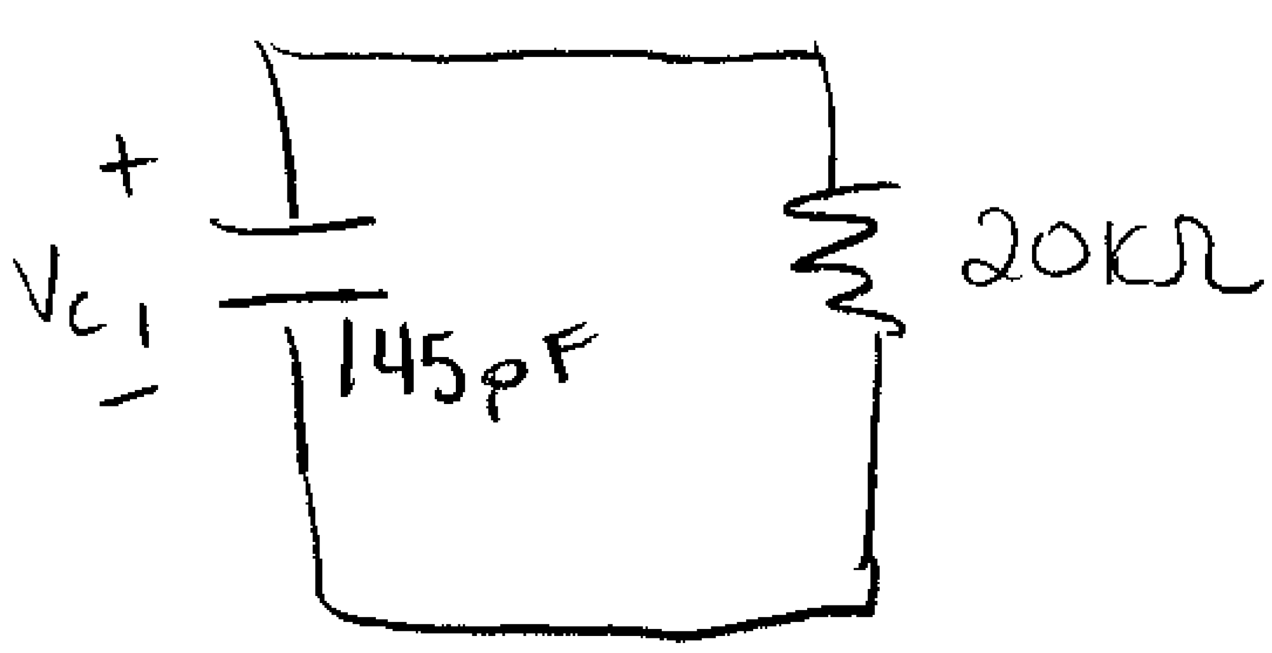

Stays until $V_{C_2} = 5V$.

It takes $0.69\,T$ to discharge halfway.

At $0.69T = 0.69(145pF)(20k\Omega)$
$= 2\mu s$

switch flips.

$t = 2\mu s$ $V_{OUT} = -1V$




$C_1$ has been charging for $2\mu s$ in circuit with 10V & 10kΩ.

$V_{C_1}(t = 2\mu s) = 10V\left(1 - e^{\frac{-2\mu s}{10k \cdot 145p}}\right)$
$= 7.5V$

How long to discharge to 5V?

$5V = 7.5e^{\frac{-\Delta t}{20k \cdot 145p}}$

$\Delta t = 1.176\mu s$

switch flips at $t = 3.176\mu s$

$t = 3.176\mu s$ $V_{OUT} = 1V$

$C_2$ has been charging, starting at 5V, for $1.176\mu s$.

$V_{C_2}(t = 1.176\mu s) =$

$10\left(1 - e^{\frac{-(t - 2\mu s)}{10k \cdot 145p}}\right) + 5e^{\frac{-(t - 2\mu s)}{10k \cdot 145p}}$

$= 7.77V$

How long to discharge to 5V?

$\Delta t = (t - 3.176\mu s) = 1.28\mu s$

switch flips at $4.45\mu s$ $V_{OUT} = -1V$

Ok we get the idea.
How about a general formula?

Suppose we know $\Delta t_K$, the time between the last two switches. Let's find $\Delta t_{k+1}$, the time to the next switch.

The capacitor of interest charged for $\Delta t_K$ in the charging circuit. It charged to:

$$V_{co} = 10\left(1 - e^{-\frac{\Delta t_K}{T_c}}\right) + 5e^{-\frac{\Delta t_K}{T_c}}$$

where $T_c = 10000\Omega \cdot 145pF = 1.45\mu s$

Rewrite more simply as

$$V_{c_o} = 10 - 5e^{-\frac{\Delta t_K}{1.45\mu s}}$$

How long to discharge capacitor down to 5V? $\Delta t_{K+1}$:

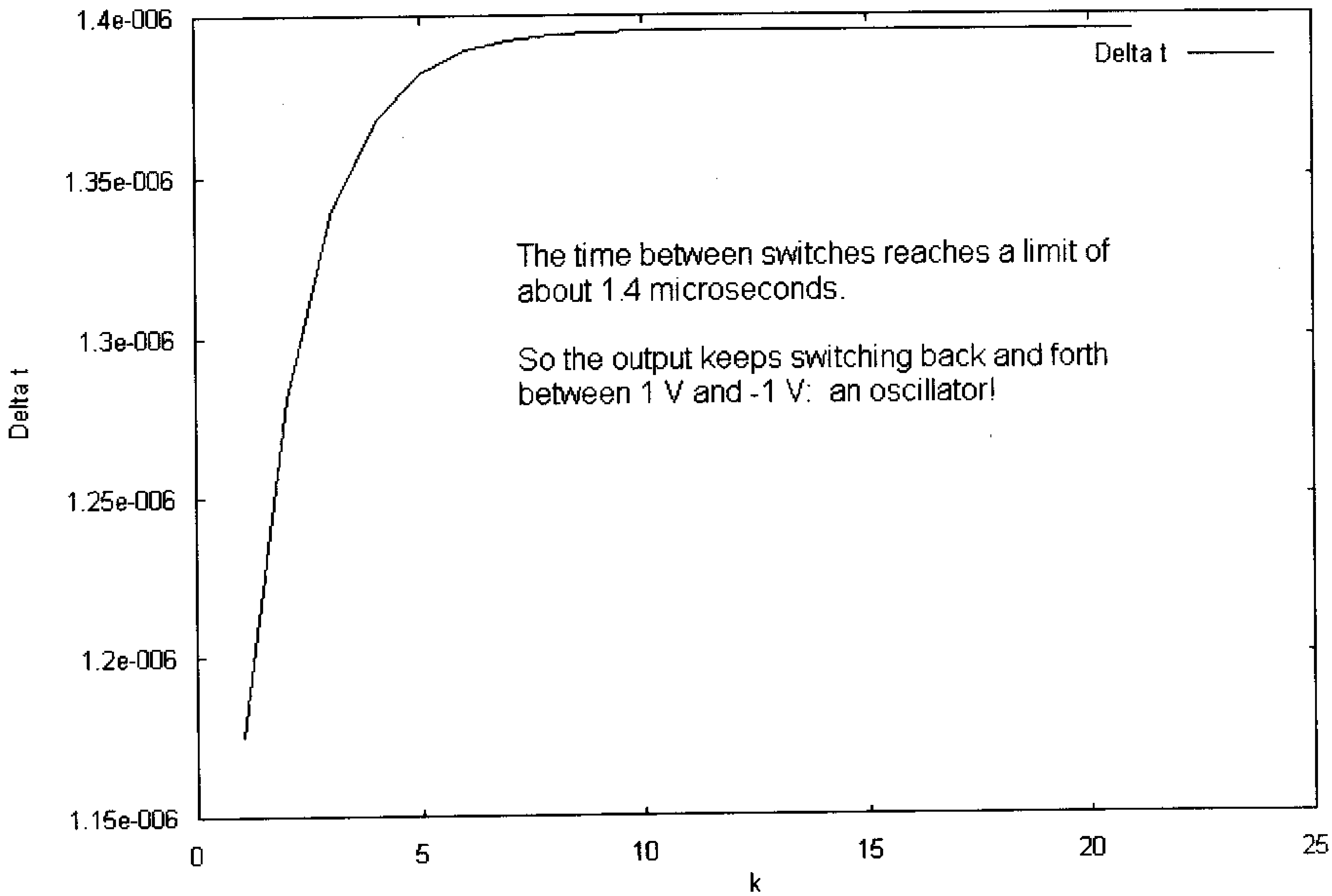
$$5V = V_{co}\, e^{-\frac{\Delta t_{K+1}}{T_d}}$$ where $T_d = 20k\Omega \cdot 145pF = 2.9\mu s$

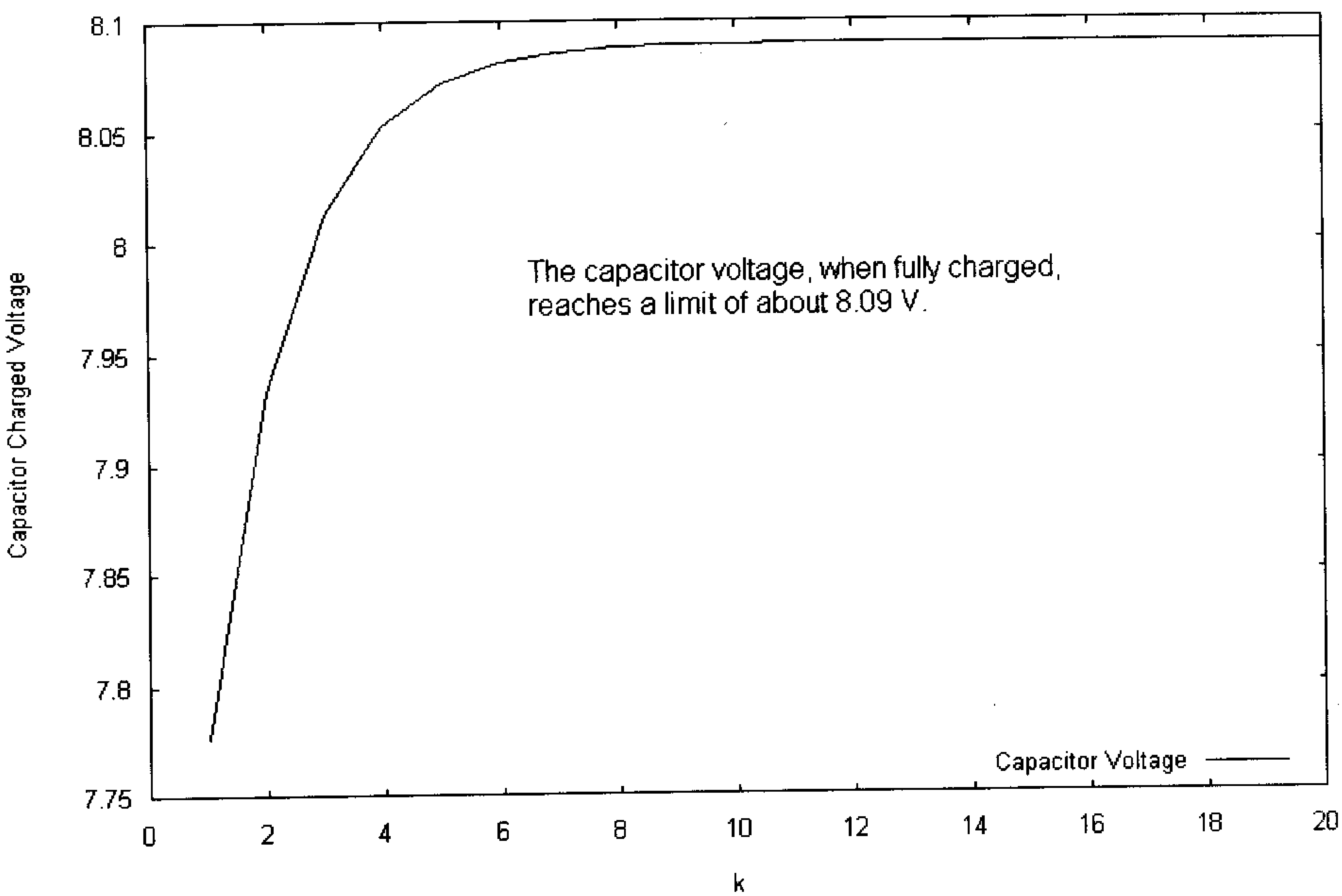
Solve:

$$\Delta t_{K+1} = -2.9\mu s \cdot \ln\left(\frac{5V}{V_{co}}\right)$$

$$= -2.9\mu s \cdot \ln \frac{5V}{10 - 5e^{-\frac{\Delta t_K}{1.45\mu s}}}$$

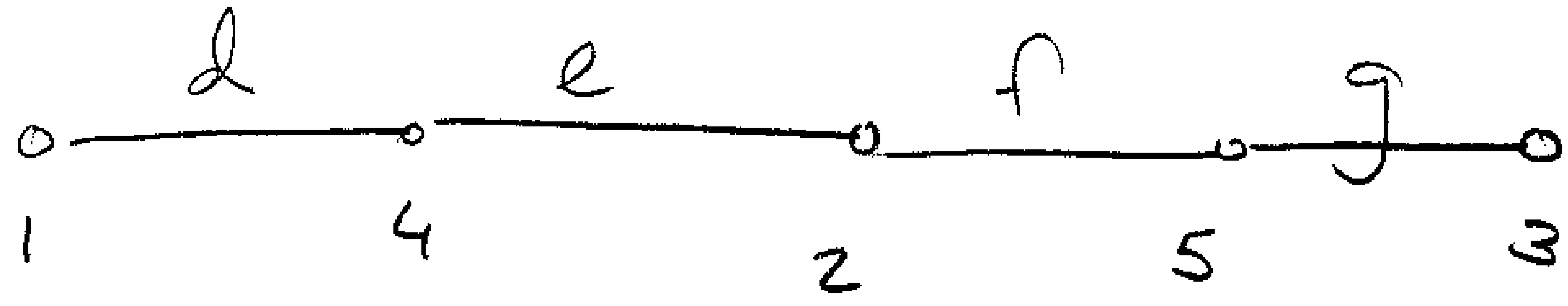
This is our update formula.


1.4e-006
1.35e-006
1.3e-006
1.25e-006
1.2e-006
1.15e-006
Delta t
0
5
10
15
20
25
k
Delta t
The time between switches reaches a limit of about 1.4 microseconds.
So the output keeps switching back and forth between 1 V and -1 V: an oscillator!


8.1
8.05
8
7.95
7.9
7.85
7.8
7.75
Capacitor Charged Voltage
0
2
4
6
8
10
12
14
16
18
20
k
Capacitor Voltage
The capacitor voltage, when fully charged, reaches a limit of about 8.09 V.

# Problem 10:

a)




Fund. cut sets:

Cutting d separates 1 from {4, 2, 5, 3},
To fully separate: cut {d, b, a}

Cutting e separates {1, 4} from {2, 5, 3}.
To fully separate: cut {e, a, b, h}.
Cutting f separates {1, 4, 2} from {5, 3}.
To fully separate: cut {f, h, c, a}.
Cutting g separates {1, 4, 2, 5} from {3}.
To fully separate: cut {a, c, g}.

b) Your answers will vary depending on the reference node: replace the appropriate node voltage with zero.

Set {d, b, a}: Current through d unknown (voltage source) but it tells us $V_1 - V_4 = 10$.

Set {e, a, b, h}: $2A + \frac{V_1 - V_2}{2} + \frac{V_4 - V_2}{2} + \frac{V_4 - V_5}{10} = 0$
"a" "b" "e" "h"

Set $\{f, h, c, a\}$: $\frac{V_2 - V_5}{20} + \frac{V_4 - V_5}{10} + \frac{V_2 - V_3}{4} + 2A = 0$

"f" "h" "c" "a"

Set $\{a, c, g\}$: $2A + \frac{V_2 - V_3}{4} + \frac{V_5 - V_3}{5} = 0$

"a "c" "g"