

Appendix A – Number Systems

1. Introduction

In kindergarten, we are taught to count using base-10 notation, because our hands have 10 fingers. What about another number base? Let us take base-16. This number base is not that much different. Instead of using 10 digits as the base of our number system, we are going to be using 16 (hence the term base-16). Ok, that sounds kind of straightforward. We use 0 – 9 for the first 10 digits, just like decimal. But, what about the next 6? We can use the letters of the alphabet! Table 1 shows the first 20 numbers in base 10, base 2 (also called binary), base 5 and base 16.

Base-10	Base-2	Base-5	Base-16
0	0	0	0
1	1	1	1
2	10	2	2
3	11	3	3
4	100	4	4
5	101	10	5
6	110	11	6
7	111	12	7
8	1000	13	8
9	1001	14	9
10	1010	20	A
11	1011	21	B
12	1100	22	C
13	1101	23	D
14	1110	24	E
15	1111	30	F
16	10000	31	10
17	10001	32	11
18	10010	33	12
19	10011	34	13

Table 1. Counting in different bases.

Let us understand each base separately:

1. Base 10: The one you are most familiar with. We have 10 digits – 0 through 9. After you reach 9, we have exhausted all possibilities for single digits. So, we carry a 1 into the next digit position and use a 0 at the rightmost digit position. We start again 10 through 19. Now, we again have reached 9 at the rightmost digit. So, we increment the 1 in the next digit position. After 1 in base 10 is 2. So, we start from 20 and keep going.

2. Base 2: If you intuitively understand what is going on with base-10, the other bases are a piece of cake! In base-2 we have only two digits – 0 and a 1. So, once we reach 1, just like when we count until 9 in base-10, we have exhausted all the digits. So, we need to carry into the next digit position – 10. We count – 10 becomes 11. Again, we have exhausted the all the digits, so we carry again – 100 and so on.

If you have noticed, smaller the base, the sooner you have to carry. Incidentally base-2 is the smallest base possible. Fermat, the great French mathematician said about base-2, “0 is the devil and 1 is God, and their interplay creates the world”.

3. Base-5: We count the first 5 digits – 0 through 4. We have used all 5 digits, so we carry over – 10. This process keeps repeating...

4. Base-16: The only caveat to base-16 is using the letters of the alphabet. Of course, you need not use the letters of the alphabet. You can use other symbols to represent decimal 10 in base 16. For instance, you could use Greek letters, letters in your own language etc. But, the world standard is to use the letters of the alphabet. So, please stick to it!

Converting between different bases is easy. For instance, converting from hexadecimal to binary:

F	F	F	F
1111	1111	1111	1111

Looking at table 1, we see hexadecimal F is equal to binary 1111. Now you can see the reason why people use hex. It is easier to write:

FFFF

than

1111111111111111!

The reason for using binary numbers is simple: 1 represents the “on” state and 0 the “off” state. Each field in a binary number is called a bit, for instance, the binary representation of FFFF has 16-bits.