Projective 3D geometry
class 4

Multiple View Geometry

Comp 290-089

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Content

- **Background**: Projective geometry (2D, 3D), Parameter estimation, Algorithm evaluation.
- **Single View**: Camera model, Calibration, Single View Geometry.
- **Two Views**: Epipolar Geometry, 3D reconstruction, Computing F, Computing structure, Plane and homographies.
- **Three Views**: Trifocal Tensor, Computing T.
- **More Views**: N-Linearities, Multiple view reconstruction, Bundle adjustment, auto-calibration, Dynamic SfM, Cheirality, Duality

Multiple View Geometry course schedule

(subject to change)

Jan. 7, 9	Intro & motivation	Projective 2D Geometry
Jan. 14, 16	(no course)	Projective 2D Geometry
Jan. 21, 23	Projective 3D Geometry	Parameter Estimation
Jan. 28, 30	Parameter Estimation	Algorithm Evaluation
Feb. 4, 6	Camera Models	Camera Calibration
Feb. 11, 13	Single View Geometry	Epipolar Geometry
Feb. 18, 20	3D reconstruction	Fund. Matrix Comp.
Feb. 25, 27	Structure Comp.	Planes & Homographies
Mar. 4, 6	Trifocal Tensor	Three View Reconstruction
Mar. 18, 20	Multiple View Geometry	MultipleView Reconstruction
Mar. 25, 27	Bundle adjustment	Papers
Apr. 1, 3	Auto-Calibration	Papers
Apr. 8, 10	Dynamic SfM	Papers
Apr. 15, 17	Cheirality	Papers
Apr. 22, 24	Duality	Project Demos

Last week ...

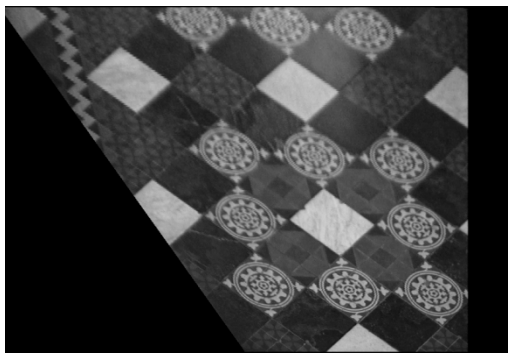
line at infinity (affinities)

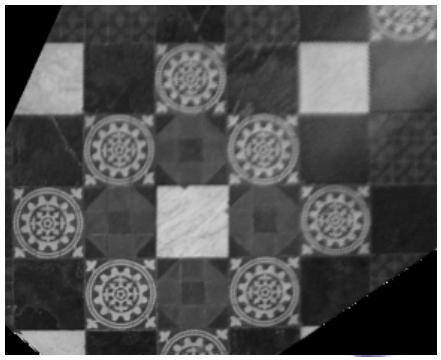
$$\mathbf{l}_\infty = (0,0,1)^\mathsf{T}$$

circular points (similarities)

$$\mathbf{C}_\infty^* = \mathbf{I}\mathbf{J}^\mathsf{T} + \mathbf{J}\mathbf{I}^\mathsf{T} \qquad \mathbf{C}_\infty^* = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

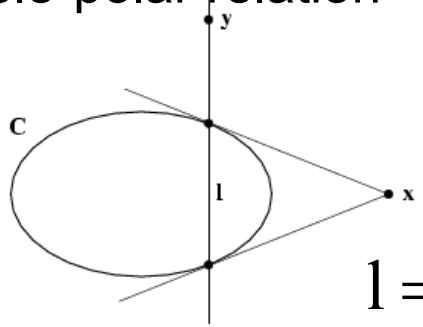
$$\mathbf{l}^\mathsf{T}\, \mathbf{C}_\infty^*\, \mathbf{m} = 0$$ (orthogonality)





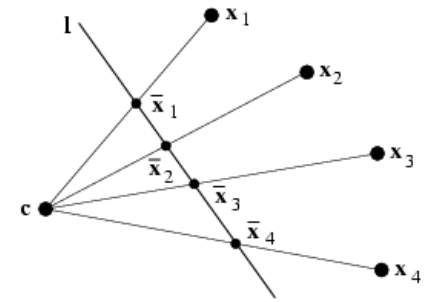
Last week …

pole-polar relation



$$\mathrm{l} = \mathbf{C}\,\mathrm{x} \qquad \mathrm{x} = \mathbf{C}^{*}\,\mathrm{l}$$

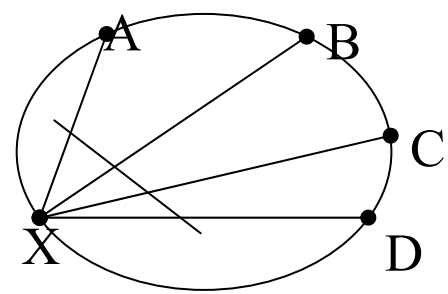
cross-ratio



conjugate points & lines

$$\mathrm{y}^{\mathsf{T}}\,\mathbf{C}\,\mathrm{x} = 0 \qquad \mathrm{m}^{\mathsf{T}}\,\mathbf{C}^{*}\,\mathrm{l} = 0$$

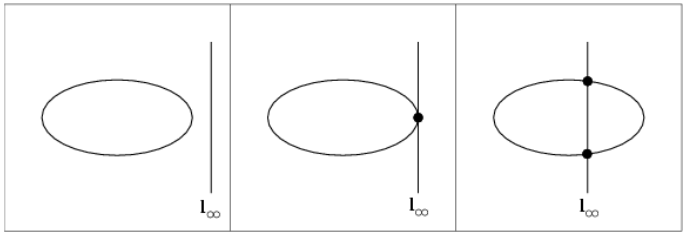
Chasles' theorem



projective conic classification

$$x^2 \pm \delta y^2 \pm \varepsilon w^2 = 0$$

affine conic classification

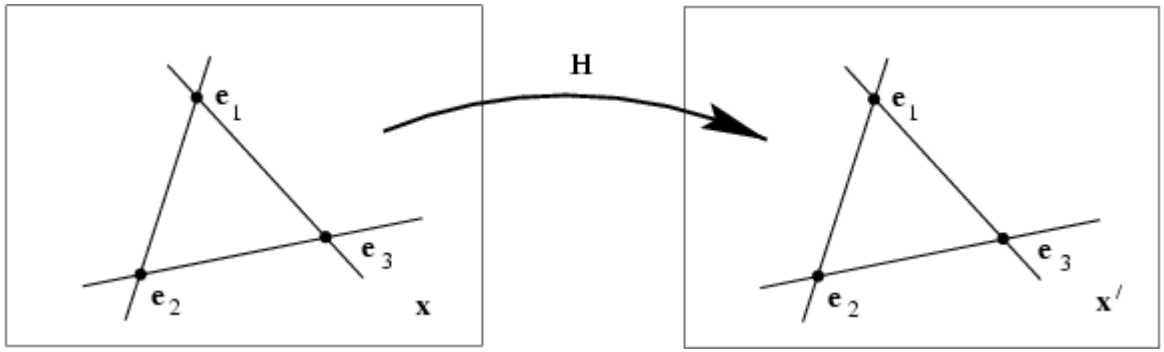


Fixed points and lines

$$\mathbf{H}\,\mathrm{e} = \lambda \mathrm{e}$$ (eigenvectors $\mathbf{H}$ =fixed points)

($\lambda_1 = \lambda_2 \Rightarrow$ pointwise fixed line)

$$\mathbf{H}^{-\mathsf{T}}\,\mathrm{l} = \lambda\,\mathrm{l}$$ (eigenvectors $\mathbf{H}^{-\mathsf{T}}$ =fixed lines)



Singular Value Decomposition

$$A_{m\times n} = U_{m\times m}\Sigma_{m\times n}V^{T}_{n\times n} \qquad m \geq n$$

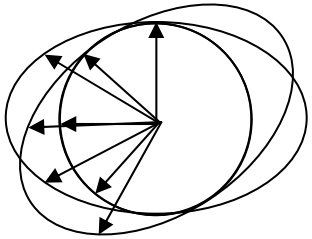
$$\Sigma = \begin{bmatrix} \sigma_1 & 0 & \cdots & 0 \\ 0 & \sigma_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_n \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \cdots & 0 \end{bmatrix}$$

$$\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_n \geq 0$$

$$U^{T}U = I$$

$$V^{T}V = I$$

$$A = U_1\,\sigma_1\,V_1^{T} + U_2\,\sigma_2\,V_2^{T} + \cdots + U_n\,\sigma_n\,V_n^{T}$$



$$U\Sigma V^{T}\mathbf{X}$$

Singular Value Decomposition

$$A = U\Sigma V^T$$

- Homogeneous least-squares

$$\min\|AX\| \text{ subject to } \|X\| = 1 \qquad \text{solution } X = V_n$$

- Span and null-space

$$\mathbf{S}_L = [U_1\ U_2];\ \mathbf{N}_L = [U_3 U_4]$$
$$\mathbf{S}_R = [V_1 V_2];\ \mathbf{N}_R = [V_3 V_4]$$

$$\Sigma = \begin{bmatrix} \sigma_1 & 0 & 0 & 0 \\ 0 & \sigma_2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

- Closest rank r approximation

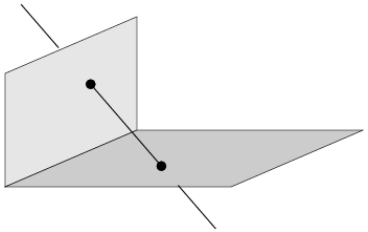
$$\tilde{A} = U\tilde{\Sigma} V^T \qquad \tilde{\Sigma} = diag(\sigma_1, \sigma_2, \cdots, \sigma_r, 0_{r+1}, \cdots, 0_n)$$

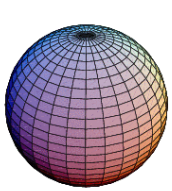
- Pseudo inverse

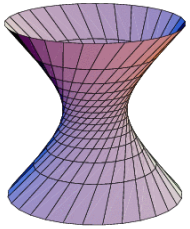
$$A^+ = V\Sigma^+ U^T \quad \Sigma^+ = diag(\sigma_1^{-1}, \sigma_2^{-1}, \cdots, \sigma_r^{-1}, 0, \cdots, 0)$$

Projective 3D Geometry

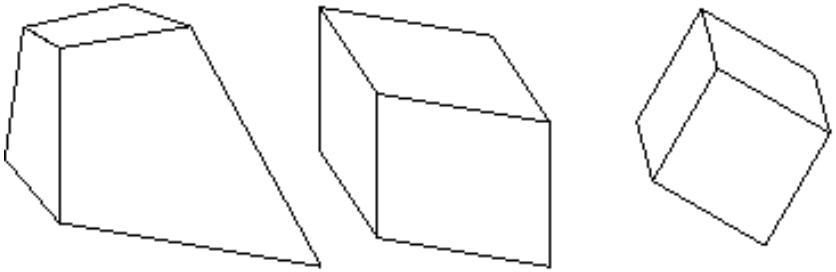
- Points, lines, planes and quadrics



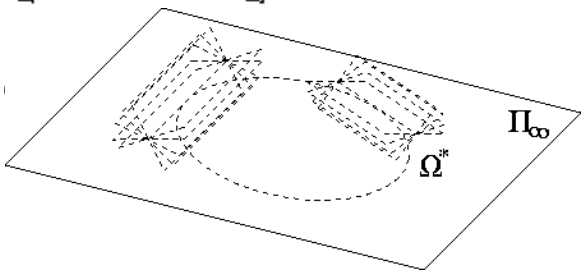




- Transformations



- Π_∞, ω_∞ and Ω



3D points

3D point

$(X, Y, Z)^{\mathrm{T}}$ in $\mathbf{R}^3$

$\mathrm{X} = (X_1, X_2, X_3, X_4)^{\mathrm{T}}$ in $\mathbf{P}^3$

$$\mathrm{X} = \left(\frac{X_1}{X_4}, \frac{X_2}{X_4}, \frac{X_3}{X_4}, 1 \right)^{\mathrm{T}} = (X, Y, Z, 1)^{\mathrm{T}} \qquad (X_4 \neq 0)$$

projective transformation

$\mathrm{X}' = \mathbf{H}\,\mathrm{X}$ (4x4-1=15 dof)

Planes

3D plane

$$\pi_1 X + \pi_2 Y + \pi_3 Z + \pi_4 = 0$$

$$\pi_1 X_1 + \pi_2 X_2 + \pi_3 X_3 + \pi_4 X_4 = 0$$

$$\pi^{\mathsf{T}} \mathrm{X} = 0$$

Transformation

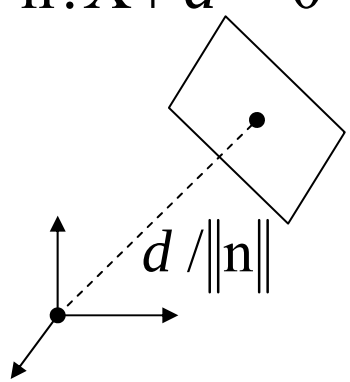
$$\mathrm{X}' = \mathbf{H}\,\mathrm{X}$$

$$\pi' = \mathbf{H}^{-\mathsf{T}}\,\pi$$

Euclidean representation

$$\mathrm{n}.\widetilde{\mathrm{X}} + d = 0 \qquad \mathrm{n} = (\pi_1, \pi_2, \pi_3)^{\mathsf{T}} \qquad \widetilde{\mathrm{X}} = (X, Y, Z)^{\mathsf{T}}$$

$$\pi_4 = d \qquad X_4 = 1$$



Dual: points ↔ planes, lines ↔ lines

Planes from points

Solve π from $X_1^T \pi = 0, X_2^T \pi = 0$ and $X_3^T \pi = 0$

$$\begin{bmatrix} X_1^T \\ X_2^T \\ X_3^T \end{bmatrix} \pi = 0 \quad \text{(solve } \pi \text{ as right nullspace of } \begin{bmatrix} X_1^T \\ X_2^T \\ X_3^T \end{bmatrix} \text{)}$$

Or implicitly from coplanarity condition

$$\det\begin{bmatrix} X_1 & (X_1)_1 & (X_2)_1 & (X_3)_1 \\ X_2 & (X_1)_2 & (X_2)_2 & (X_3)_2 \\ X_3 & (X_1)_3 & (X_2)_3 & (X_3)_3 \\ X_4 & (X_1)_4 & (X_2)_4 & (X_3)_4 \end{bmatrix} = 0$$

$$[X_1 \; X_2 \; X_3] = 0$$

$$X_1 D_{234} - X_2 D_{134} + X_3 D_{124} - X_4 D_{123} = 0$$

$$\pi = (D_{234}, -D_{134}, D_{124}, -D_{123})^T$$

Points from planes

Solve X from $\pi_1^\mathsf{T}\mathrm{X}=0$, $\pi_2^\mathsf{T}\mathrm{X}=0$ and $\pi_3^\mathsf{T}\mathrm{X}=0$

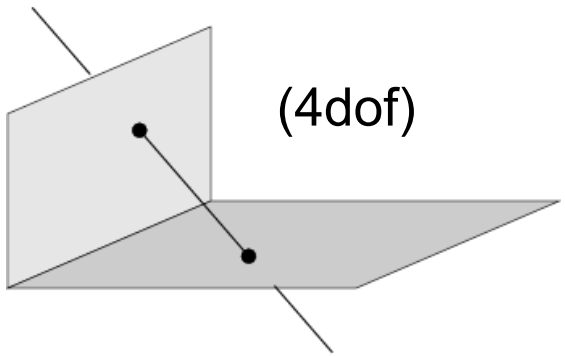
$$\begin{bmatrix}\pi_1^\mathsf{T}\\ \pi_2^\mathsf{T}\\ \pi_3^\mathsf{T}\end{bmatrix}\mathrm{X}=0 \quad \text{(solve } \mathrm{X} \text{ as right nullspace of } \begin{bmatrix}\pi_1^\mathsf{T}\\ \pi_2^\mathsf{T}\\ \pi_3^\mathsf{T}\end{bmatrix}\text{)}$$

Representing a plane by its span

$$\mathrm{X}=\mathbf{M}\,\mathrm{x} \qquad \mathbf{M}=\left[\mathrm{X}_1\mathrm{X}_2\mathrm{X}_3\right] \qquad \pi=\left(a,b,c,d\right)^\mathsf{T}$$

$$\pi^\mathsf{T}\,\mathbf{M}=0 \qquad \mathbf{M}=\begin{bmatrix}\mathrm{p}\\ \mathrm{I}\end{bmatrix} \qquad p=\left(-\frac{b}{a},-\frac{c}{a},-\frac{d}{a}\right)^\mathsf{T}$$

Lines



(4dof)

$$W = \begin{bmatrix} A^T \\ B^T \end{bmatrix} \qquad \lambda A + \mu B$$

$$W^* = \begin{bmatrix} P^T \\ Q^T \end{bmatrix} \qquad \lambda P + \mu Q$$

$$W^* W^T = W W^{*T} = 0_{2\times 2}$$

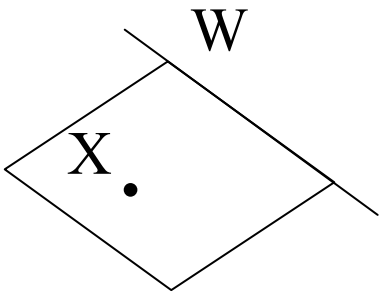
Fig. 3.1. *A line may be specified by its points of intersection with two orthogonal planes. Each intersection point has 2 degrees of freedom, which demonstrates that a line in* $\mathbb{P}^3$ *has a total of 4 degrees of freedom.*

Example: X-axis

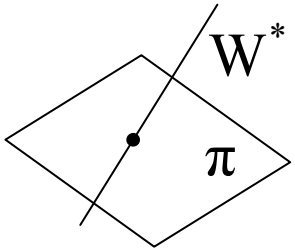
$$W = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \end{bmatrix} \qquad W^* = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Points, lines and planes

$$\mathbf{M} = \begin{bmatrix} W \\ X^{\mathsf{T}} \end{bmatrix} \qquad \mathbf{M}\,\pi = 0$$



$$\mathbf{M} = \begin{bmatrix} W^{*} \\ \pi^{\mathsf{T}} \end{bmatrix} \qquad \mathbf{M}\,X = 0$$



Plücker matrices

Plücker matrix (4x4 skew-symmetric homogeneous matrix)

$$l_{ij} = A_i B_j - B_i A_j$$

$$\mathrm{L} = \mathrm{A}\mathrm{B}^{\mathrm{T}} - \mathrm{B}\mathrm{A}^{\mathrm{T}}$$

1. L has rank 2 $\quad \mathrm{L}\mathrm{W}^{*\mathrm{T}} = 0_{4\times 2}$
2. 4dof
3. generalization of $\mathrm{l} = \mathrm{x} \times \mathrm{y}$
4. L independent of choice A and B
5. Transformation $\quad \mathrm{L}' = \mathrm{H}\mathrm{L}\mathrm{H}^{\mathrm{T}}$

<u>Example</u>: x-axis

$$\mathrm{L} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}^{\mathrm{T}} = \begin{bmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

Plücker matrices

Dual Plücker matrix L^*

$$\mathrm{L}^* = \mathrm{P}\mathrm{Q}^\mathsf{T} - \mathrm{Q}\mathrm{P}^\mathsf{T}$$

$$\mathrm{L}^{*'} = \mathrm{H}^{-\mathsf{T}}\mathrm{L}\mathrm{H}^{-1}$$

Correspondence

$$l_{12} : l_{13} : l_{14} : l_{23} : l_{42} : l_{34} = l_{34}^* : l_{42}^* : l_{23}^* : l_{14}^* : l_{13}^* : l_{12}^*$$

Join and incidence

$\pi = \mathrm{L}^*\mathrm{X}$ (plane through point and line)

$\mathrm{L}^*\mathrm{X} = 0$ (point on line)

$\mathrm{X} = \mathrm{L}\pi$ (intersection point of plane and line)

$\mathrm{L}\pi = 0$ (line in plane)

$[\mathrm{L}_1, \mathrm{L}_2, \ldots]\pi = 0$ (coplanar lines)

Plücker line coordinates

$$☹ = \left[l_{12}, l_{13}, l_{14}, l_{23}, l_{42}, l_{34}\right]^{\mathrm{T}} \in \mathbf{P}^5$$

$$l_{12}l_{34} + l_{13}l_{42} + l_{14}l_{23} = 0$$ on *Klein quadric* $$☹^{\mathrm{T}}\mathrm{K}☹ = 0$$

$$☹, \hat{☹} \leftrightarrow (\mathrm{A}, \mathrm{B}), (\hat{\mathrm{A}}, \hat{\mathrm{B}})$$

$$\det\left[\mathrm{A}, \mathrm{B}, \hat{\mathrm{A}}, \hat{\mathrm{B}}\right] = l_{12}\hat{l}_{34} + l_{13}\hat{l}_{42} + l_{14}\hat{l}_{23} + l_{23}\hat{l}_{14} + l_{42}\hat{l}_{13} + l_{34}\hat{l}_{12} = 0$$

$$\left[l_{12} l_{13} l_{14} l_{23} l_{42} l_{34}\right] \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{l}_{12} \\ \hat{l}_{13} \\ \hat{l}_{14} \\ \hat{l}_{23} \\ \hat{l}_{42} \\ \hat{l}_{34} \end{bmatrix} = 0$$

$$☹^{\mathrm{T}}\mathrm{K}☹ = \left(☹ \mid \hat{☹}\right)$$

Plücker line coordinates

$$\left(☹ \,|\, ☹\right) = 0$$ (Plücker internal constraint)

$$\left(☹ \,|\, \hat{☹}\right) = \det\left[A, B, \hat{A}, \hat{B}\right] = 0$$ (two lines intersect)

$$\left(☹ \,|\, \hat{☹}\right) = \det\left[P, Q, \hat{P}, \hat{Q}\right] = 0$$ (two lines intersect)

$$\left(☹ \,|\, \hat{☹}\right) = \left(P^T A\right)\left(Q^T B\right) - \left(Q^T A\right)\left(P^T B\right) = 0$$ (two lines intersect)

Quadrics and dual quadrics

$X^T Q X = 0$ (Q : 4x4 symmetric matrix)

$$Q = \begin{bmatrix} \bullet & \bullet & \bullet & \bullet \\ \circ & \bullet & \bullet & \bullet \\ \circ & \circ & \bullet & \bullet \\ \circ & \circ & \circ & \bullet \end{bmatrix}$$

1. 9 d.o.f.
2. in general 9 points define quadric
3. det Q=0 ↔ degenerate quadric
4. pole – polar $\pi = QX$
5. (plane ∩ quadric)=conic $C = M^T Q M$ $\pi : X = Mx$
6. transformation $Q' = H^{-T} Q H^{-1}$

$\pi^T Q^* \pi = 0$

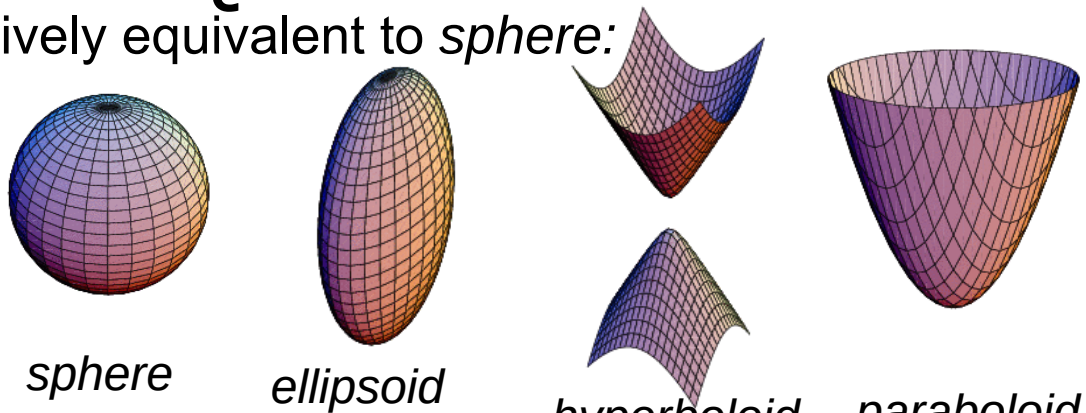
1. relation to quadric $Q^* = Q^{-1}$ (non-degenerate)
2. transformation $Q'^* = H Q^* H^T$

Quadric classification

Rank	Sign.	Diagonal	Equation	Realization
4	4	(1,1,1,1)	$X^2+ Y^2+ Z^2+1=0$	No real points
	2	(1,1,1,-1)	$X^2+ Y^2+ Z^2=1$	Sphere
	0	(1,1,-1,-1)	$X^2+ Y^2= Z^2+1$	Hyperboloid (1S)
3	3	(1,1,1,0)	$X^2+ Y^2+ Z^2=0$	Single point
	1	(1,1,-1,0)	$X^2+ Y^2= Z^2$	Cone
2	2	(1,1,0,0)	$X^2+ Y^2= 0$	Single line
	0	(1,-1,0,0)	$X^2= Y^2$	Two planes
1	1	(1,0,0,0)	$X^2=0$	Single plane

Quadric classification

Projectively equivalent to *sphere:*

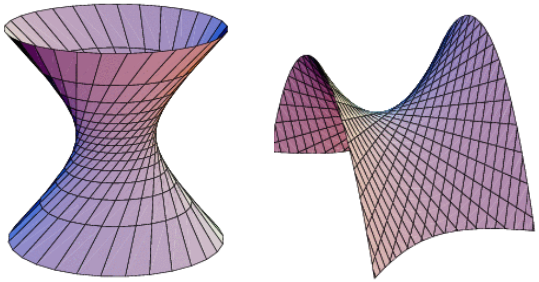


sphere *ellipsoid* *hyperboloid of two sheets* *paraboloid*

Fig. 3.2. **Non-ruled quadrics.** *This shows plots of a sphere, ellipsoid, hyperboloid of two sheets and paraboloid. They are all projectively equivalent.*

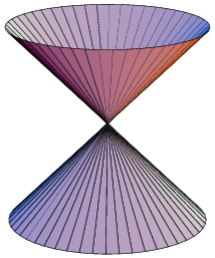
Ruled quadrics:

Fig. 3.3. **Ruled quadrics.** *Two examples of a hyperboloid of one sheet are given. These surfaces are given by equations* $X^2 + Y^2 = Z^2 + 1$ *and* $XY = Z$ *respectively, and are projectively equivalent. Note that these two surfaces are made up of two sets of disjoint straight lines, and that each line from one set meets each line from the other set. The two quadrics shown here are projectively (though not affinely) equivalent.*

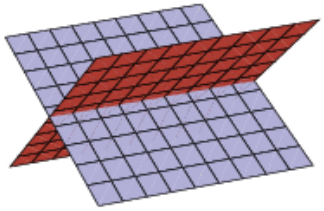


hyperboloids of one sheet

Degenerate ruled quadrics:



cone



two planes

Fig. 3.4. **Degenerate quadrics.** *The two most important degenerate quadrics are shown, the cone and two planes. Both these quadrics are ruled. The matrix representing the cone has rank 3, and the null-vector represents the nodal point of the cone. The matrix representing the two (non-coincident) planes has rank 2, and the two generators of the rank 2 null-space are two points on the intersection line of the planes.*

Twisted cubic

twisted cubic

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \mathrm{A}\begin{pmatrix} 1 \\ \theta \\ \theta^2 \\ \theta^3 \end{pmatrix} = \begin{pmatrix} a_{11} + a_{12}\theta + a_{13}\theta^2 + a_{14}\theta^3 \\ a_{21} + a_{22}\theta + a_{23}\theta^2 + a_{24}\theta^3 \\ a_{31} + a_{32}\theta + a_{33}\theta^2 + a_{34}\theta^3 \\ a_{41} + a_{42}\theta + a_{43}\theta^2 + a_{44}\theta^3 \end{pmatrix}$$

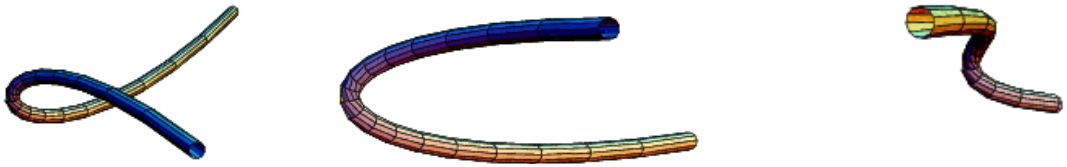


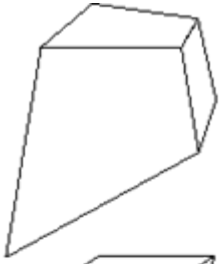
Fig. 3.5. *Various views of the twisted cubic* $(t^3, t^2, t,)^\top$. *The curve is thickened to a tube to aid in visualization.*

1. 3 intersection with plane (in general)
2. 12 dof (15 for A – 3 for reparametrisation (1 θ $\theta^2\theta^3$)
3. 2 constraints per point on cubic, defined by 6 points
4. projectively equivalent to (1 θ $\theta^2\theta^3$)
5. Horopter & degenerate case for reconstruction

Hierarchy of transformations

Projective
15dof

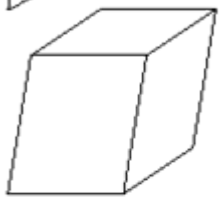
$$\begin{bmatrix} A & t \\ v^T & v \end{bmatrix}$$



Intersection and tangency

Affine
12dof

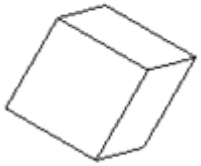
$$\begin{bmatrix} A & t \\ 0^T & 1 \end{bmatrix}$$



Parallellism of planes,
Volume ratios, centroids,
The plane at infinity π_∞

Similarity
7dof

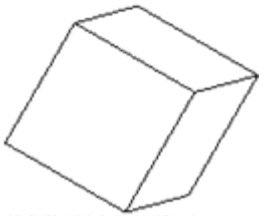
$$\begin{bmatrix} sR & t \\ 0^T & 1 \end{bmatrix}$$



The absolute conic Ω_∞

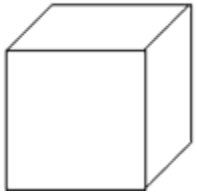
Euclidean
6dof

$$\begin{bmatrix} R & t \\ 0^T & 1 \end{bmatrix}$$



Volume

Table 3.2. **Geometric properties invariant to commonly occurring transformations of 3-space.** *The matrix* $\mathbf{A}$ *is an invertible* 3×3 *matrix,* $\mathbf{R}$ *is a 3D rotation matrix,* $\mathbf{t} = (t_X, t_Y, t_Z)^T$ *a 3D translation,* $\mathbf{v}$ *a general 3-vector,* v *a scalar, and* $\mathbf{0} = (0, 0, 0)^T$ *a null 3-vector. The distortion column shows typical effects of the transformations on a cube. Transformations higher in the table can produce all the actions of the ones below. These range from Euclidean, where only translations and rotations occur, to projective where five points can be transformed to any other five points (provided no three points are collinear, or four coplanar).*



Screw decomposition

Any particular translation and rotation is equivalent to a rotation about a screw axis and a translation along the screw axis.

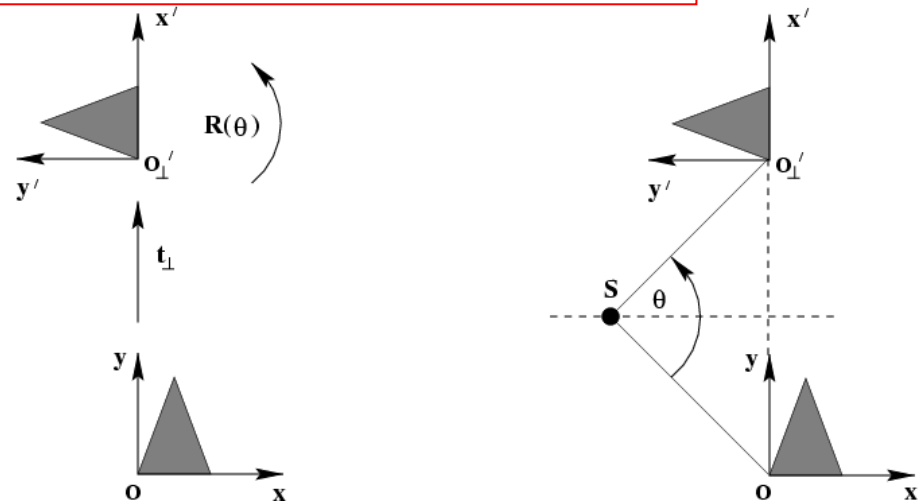


Fig. 3.6. **2D Euclidean motion and a "screw" axis.** *(a) The frame $\{x, y\}$ undergoes a translation $\mathbf{t}_\perp$ and a rotation by θ to reach the frame $\{x', y'\}$. The motion is in the plane orthogonal to the rotation axis. (b) This motion is equivalent to a single rotation about the screw axis* **S**. *The screw axis lies on the perpendicular bisector of the line joining corresponding points, such that the angle between the lines joining* **S** *to the corresponding points is θ. In the figure the corresponding points are the two frame origins and θ has the value 90°.*

2D Euclidean Motion and the screw decomposition

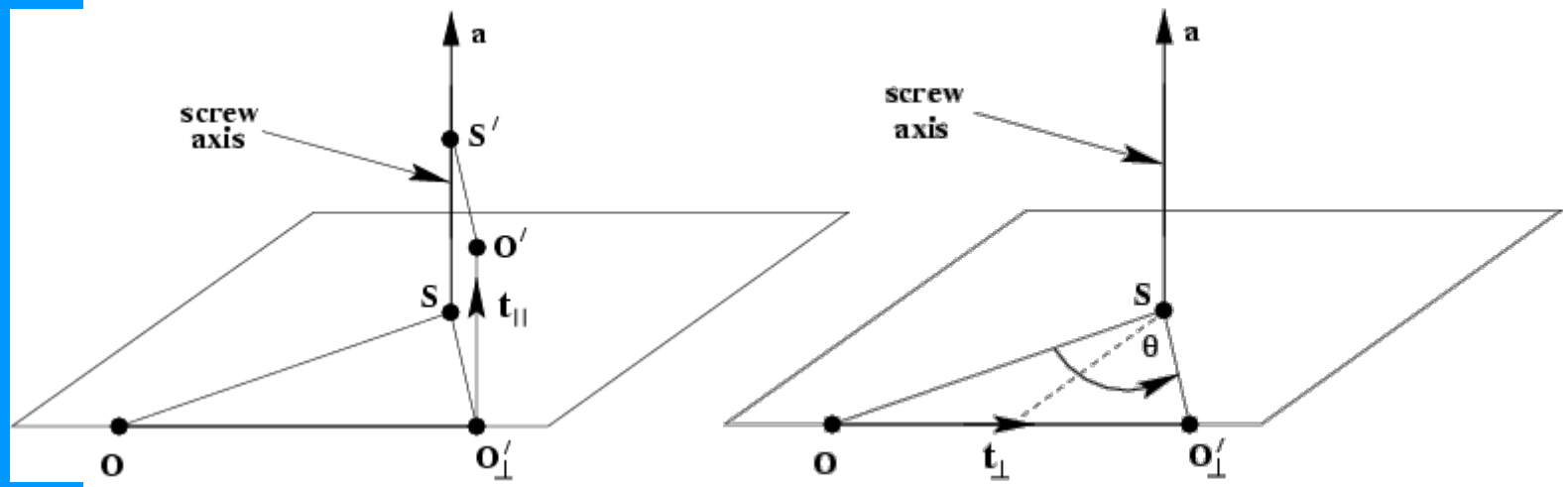


Fig. 3.7. **3D Euclidean motion and the screw decomposition.** *Any Euclidean rotation* $\mathbb{R}$ *and translation* $\mathbf{t}$ *may be achieved by (a) a rotation about the screw axis, followed by (b) a translation along the screw axis by* $\mathbf{t}_{||}$. *Here* $\mathbf{a}$ *is the (unit) direction of the rotation axis (so that* $\mathbb{R}\mathbf{a} = \mathbf{a}$*), and* $\mathbf{t}$ *is decomposed as* $\mathbf{t} = \mathbf{t}_{||} + \mathbf{t}_{\perp}$, *which are vector components parallel and orthogonal respectively to the rotation axis direction The point* $\mathbf{S}$ *is closest to* $\mathbf{O}$ *on the screw axis (so that the line from* $\mathbf{S}$ *to* $\mathbf{O}$ *is perpendicular to the direction of* $\mathbf{a}$*). Similarly* $\mathbf{S}'$ *is the point on the screw axis closest to* $\mathbf{O}'$.

screw axis // rotation axis

$$t = t_{//} + t_{\perp}$$

The plane at infinity

$$\pi'_\infty = \mathbf{H}_A^{-\mathsf{T}} \pi_\infty = \begin{bmatrix} \mathbf{A}^{-\mathsf{T}} & 0 \\ -\mathbf{A}\,\mathrm{t} & 1 \end{bmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix} = \pi_\infty$$

The plane at infinity π_∞ is a fixed plane under a projective transformation H iff H is an affinity

1. canical position $\pi_\infty = (0,0,0,1)^{\mathsf{T}}$
2. contains directions $\mathrm{D} = (X_1, X_2, X_3, 0)^{\mathsf{T}}$
3. two planes are parallel $\Leftrightarrow$ line of intersection in π_∞
4. line // line (or plane) $\Leftrightarrow$ point of intersection in π_∞

The absolute conic

The absolute conic Ω_∞ is a (point) conic on π_∞.

In a metric frame:

$$\left.\begin{array}{c} X_1^2 + X_2^2 + X_3^2 \\ X_4 \end{array}\right\} = 0$$

or conic for directions: $(X_1, X_2, X_3)\mathrm{I}(X_1, X_2, X_3)^\mathsf{T}$

(with no real points)

The absolute conic Ω_∞ is a fixed conic under the projective transformation $\mathbf{H}$ iff $\mathbf{H}$ is a similarity

1. Ω_∞ is only fixed as a set
2. Circle intersect Ω_∞ in two points
3. Spheres intersect π_∞ in Ω_∞

The absolute conic

Euclidean:

$$\cos\theta = \frac{\left(d_1^T d_2\right)}{\sqrt{\left(d_1^T d_1\right)\left(d_2^T d_2\right)}}$$

Projective:

$$\cos\theta = \frac{\left(d_1^T \Omega_\infty d_2\right)}{\sqrt{\left(d_1^T \Omega_\infty d_1\right)\left(d_2^T \Omega_\infty d_2\right)}}$$

$$d_1^T \Omega_\infty d_2 = 0$$ (orthogonality=conjugacy)

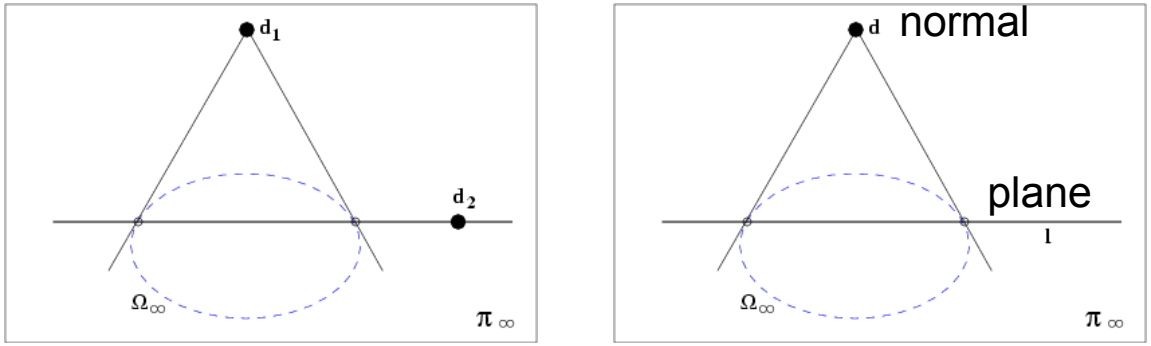
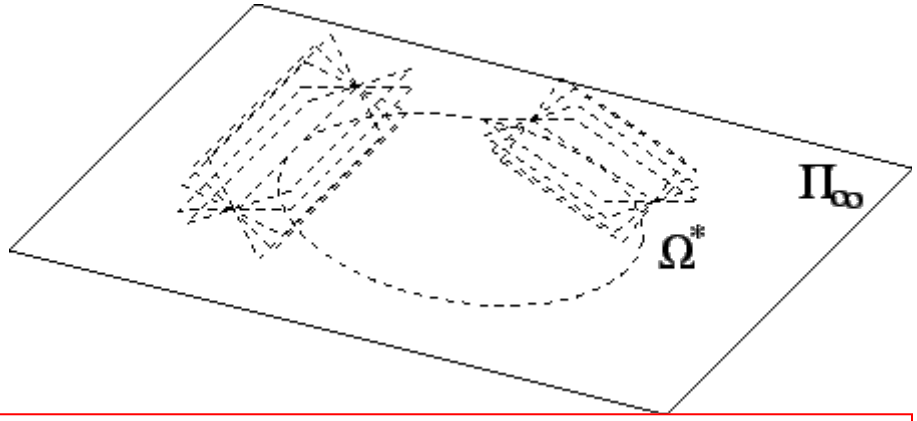


Fig. 3.8. **Orthogonality and Ω_∞.** *(a) On π_∞ orthogonal directions $\mathbf{d}_1$, $\mathbf{d}_2$ are conjugate with respect to Ω_∞. (b) A plane normal direction $\mathbf{d}$ and the intersection line $\mathbf{l}$ of the plane with π_∞ are in pole–polar relation with respect to Ω_∞.*

The absolute dual quadric

$$\Omega^*_\infty = \begin{bmatrix} I & 0 \\ 0^T & 0 \end{bmatrix}$$

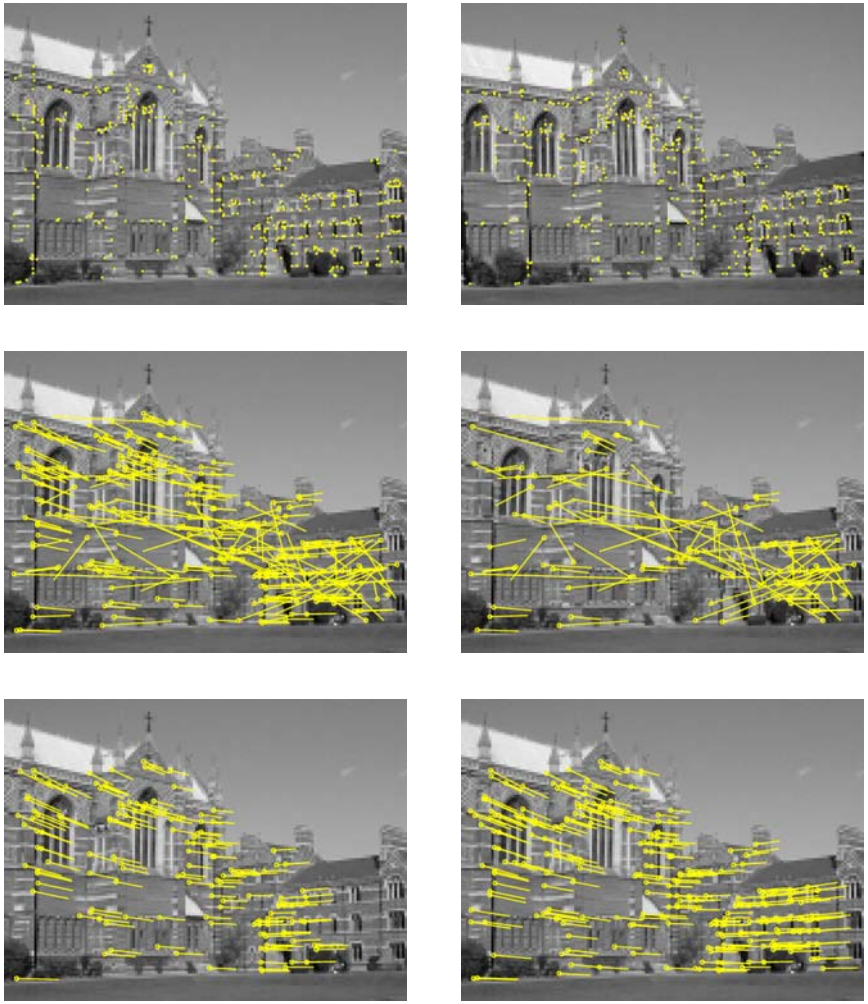


The absolute conic Ω^*_∞ is a fixed conic under the projective transformation $\mathbf{H}$ iff $\mathbf{H}$ is a similarity

1. 8 dof
2. plane at infinity π_∞ is the nullvector of Ω^*_∞
3. Angles:

$$\cos\theta = \frac{\pi_1^T \Omega^*_\infty \pi_2}{\sqrt{\left(\pi_1^T \Omega^*_\infty \pi_1\right)\left(\pi_2^T \Omega^*_\infty \pi_2\right)}}$$

Next classes: Parameter estimation



Direct Linear Transform
Iterative Estimation
Maximum Likelihood Est.
Robust Estimation