Homework #1, 290Q

1- a) Show that basis functions $\varphi_k(t) = \sin c(t - kT)$ are orthogonal over $(-\infty, \infty)$ with

$$\int \varphi_k(t)\varphi_l(t)st = T\delta_{kl}$$

Therefore sampling can be viewed as an orthogonal expansion of a bandlimited signal where the weights are samples of the signal.

b)Energy of a bandlimited signal x(t) is

$$E = \int_{-\infty}^{\infty} |x(t)|dt$$

Using part a show that

$$E = \frac{1}{2B}\sum_n \left| x(\frac{n}{2B}) \right|^2$$

Where B is the signal bandwidth. So the signal energy can be determined by summing the square of mag of samples.

2- Consider a process with PSD of

$$S_{xx}(\omega) = \frac{2}{\omega_c} \frac{\sigma^2{}_x}{1 + (\frac{\omega}{\omega_c})^2}$$

Show that prefiltering with an ideal low pass filter ($\omega_s / 2$) causes a mean square error σ_f^2 given by

$$\frac{\sigma_f^2}{\sigma_x^2} = \frac{2}{\pi} a\tan\frac{\omega_s}{2\omega_c} \approx \frac{4\omega_c}{\pi\omega_s} \qquad \omega_s / 2 >> \omega_c$$

3- **Restoring Lost Samples**: We can recover missed samples if there is enough oversampling. Consider a bandlimited signal *x(t)* with bandwidth B, oversampled with the rate *2W* where $r = B/W \le 1$. Show that

$$x(0) = \frac{r}{r-1}\sum_n x(\frac{n}{2W}) \sin c(rn)$$

Hint: Use basic sampling reconstruction formula with period *W*.