EE 290H **Homework # 5**

Issued: March 09, 1999 Due: March 18, 1999

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1 *Curve Fitting*

The purpose of this exercise is to observe the *bias-variance* tradeoff that is endemic in all system identification problems.

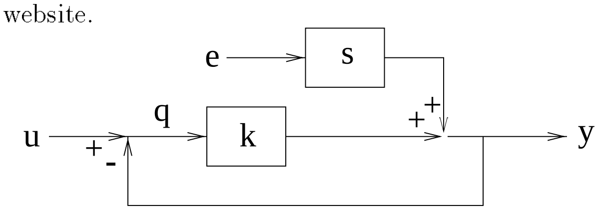
We are given input-output data from some process. We have reason to believe that this process is not *dynamic*. This data may be found at `www-inst.EECS.Berkeley.EDU/ ee290h/homeworks`.

Find a good polynomial fit to this data. Remember to use only part of the data for modeling and the rest for model verification.

2 *Maximum Likelihood Estimation*

In this exercise, you will work through the details of a maximum likelihood estimation. You will first try a clever *ad hoc* method, and discover the dangers of intuition in estimation. Then, you will revisit the problem using maximum likelihood estimation.

Shown below is the block diagram of a simple feedback process. The noise process e is white and $\mathcal{N}(0,1)$. The parameters s and k are to be estimated. Suppose we have available an L sample input-output data record $\{u_t^d, y_t^d\}_{t=1}^L$. This data was generated using the true parameter values $k^o = 7$ and $s^o = 2$, and using a white Gaussian input with zero mean and variance $\sigma_u^2 = 1.5$. The data may be found at the EE 290H website.



(a) Consider the following "clever" idea. Since we have u and y available, we know the signal q. We could then use least squares to get an estimate k_{LS}. Find a formula for this estimate and evaluate it using the given data.

(b) You will notice that it isn't very good at all. Show that for large data records. i.e. $L \to \infty$,

$$k_{LS} \to k^o - \frac{(s^o)^2(1+k^o)}{(s^o)^2 + \frac{\sigma_u^2}{\sigma_e^2}}$$

Thus this clever idea works well if the noise-to-signal ratio is small. But in our case, you should expect

$$k_{LS} \approx= 1.18$$

which means the idea isn't clever at all. So lets try maximum likelihood estimation now.

(c) Find the formula relating y to e and u.

(d) Find the density function $p_Y(y, s, k)$ and the log-likelihood function.

(e) Find the formulae for the maximum likelihood estimates of k and s, and also for the estimated parameter variances.

(f) Evaluate our answers above using the given data. Maximum likelihood estimation works very well !

(g) Show that as $L \to \infty$ (i.e. as we have more and more data),

$$\hat{s}_{ML} \to s^o, \quad \text{and} \quad \hat{k}_{ML} \to k^o$$

In proving some of the results in Problem 2, you will need the following facts.

Suppose u and e are zero mean white noise processes with variances σ_u^2 and σ_e^2 respectively. Also suppose that u and e are uncorrelated with each other. Then

$$\begin{aligned} \lim_{L\to\infty} \frac{1}{L} u' e &= \lim_{L\to\infty} \frac{1}{L} \sum_{t=1}^{L} u_t e_t = 0 \\ \lim_{L\to\infty} \frac{1}{L} u' u &= \lim_{L\to\infty} \frac{1}{L} \sum_{t=1}^{L} u_t u_t = \sigma_u^2 \end{aligned}$$