UNIVERSITY OF CALIFORNIA
College of Engineering
Department of Electrical Engineering
and Computer Sciences

C. SPANOS
K. POOLLA

Special Issues in Semiconductor Manufacturing

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PROBLEM SET No. 4
OFFICIAL SOLUTIONS

1. A process engineer is about to begin a comprehensive study to determine the effects of five variables on the etch rate of polysilicon.

a) If a 2^5 factorial design were used, how many runs would be made?

$2^5 = 32$

b) If σ^2 is the experimental error variance of an individual observation, what is the variance of the main effect?

Assuming 32 runs, then the variance of the main effect will be $2\sigma^2/16 = \sigma^2/8$.

c) What is the usual formula for a 99% confidence interval for the main effect of variable 1?

Assuming that we know sigma, then the 99% interval is given as:
$Effx_1 \pm 2.567\sigma/2.83$

d) On the basis of some previous work it is believed that σ=200 A/min. If the experimenter wants 95% confidence intervals for the main effect and interactions whose lengths are equal to 50 A/min, (i.e. the upper limit minus the lower limit is equal to 50 A/min), how many replications of the 2^5 factorial design will be required?

The width of the 95% intervals will be $\pm$ 1.96 * 200 * Sqrt(4/N), where N=32, 64, 96,...
In order to make this interval as close to 50, I choose N=992, or 31 replications of the 32 run experiment.

2. Tests were carried out on a newly designed carburetor. Four variables were studied as follows:

variable		−	+
A	tension on spring	low	high
B	air gap	narrow	open
C	size of aperture	small	large
D	rate of flow of gas	slow	rapid

The immediate object was to find the effects of these changes on the amount of unburned hydrocarbons in the engine exhaust gas. The following results were obtained:

A	B	C	D	unburned hydrocarbons
−	+	+	+	8.2
−	−	+	−	1.7
−	−	−	+	6.2
+	−	−	−	3.0
+	−	+	+	6.8
+	+	+	−	5.0
−	+	−	−	3.8
+	+	−	+	9.3

a) Stating any assumptions, analyze the data, using one Multivariate Analysis of Variance (MANOVA) table.

Here the basic assumptions are that of IIND residuals ()which means that all treatments have the same, normally distributed variance.

Table 1: Effect Test

Source	Nparm	DF	Sum of Squares	F Ratio	Prob>F
A	1	1	52.2050	5.1250	0.0051
B	1	1	29.2450	31.1250	0.0006
C	1	1	0.04500	1.1250	0.3667
D	1	1	36.125000	903.1250	0.0001

Table 2: Whole–Model Test Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio
Model	4	47.620000	11.905029	237.6250
Error	3	0.120000	0.0400	Prob>F 0.0003
C Total	7	47.740000		

b) Can you test any or all the hypotheses of model additivity?

Yes. Although this is a rather small fraction of a full factorial (it is a 2^{4-1}), and it has a generator of ABCD = -I. This means that three level interactions are confounded with main effects, and pairs of two level interactions are confounded to each other. More specifically: AB = -CD, AC = -BD, and AD = -BC. Although I cannot estimate the values of each of these interactions, the additivity of the model can be tested by testing the significance of the following estimates:

Effect Test

Source	Nparm	DF	Sum of Squares	F Ratio	Prob>F
A*B	1	1	0.02000000	0.0017	0.9693
A*C	1	1	0.02000000	0.0017	0.9693
A*D	1	1	0.08000000	0.0067	0.9386

Based on this analysis, it is clear that this model is additive (i.e. there are no significant interaction terms).

c) A MANOVA table uses the F distribution. Can you think of equivalent tests that make use of the student-t distribution? (just formulate the problem - do not do calculations).

Yes. In order to test the significance of an effect, I would simply compare the mean values of the result for all the + values of the effect, to the mean values of the same result attained for the - values of the effect. A standard two population t-test would be used for this. This is the result of such an analysis:

Table 3: Parameter Estimates

Term	Estimate	Std Errort	Ratio	Prob>|t|
Intercept	5.5	0.07071	77.78	0.0000
A[-1-1]	-0.525	0.07071	-7.42	0.0051
B[-1-1]	-1.075	0.07071	-15.20	0.0006
C[-1-1]	0.075	0.07071	1.06	0.3667
D[-1-1]	-2.125	0.07071	-30.05	0.0001

d) Comment on the differences and similarities of this analysis compared to the simpler, factorial analysis method.

The basic difference is that the factorial analysis uses a more simplistic method of dealing with the estimate of the unknown experimental error.

3. Estimate the effects of the following 2^{5-1} fractional factorial design, stating your assumptions.

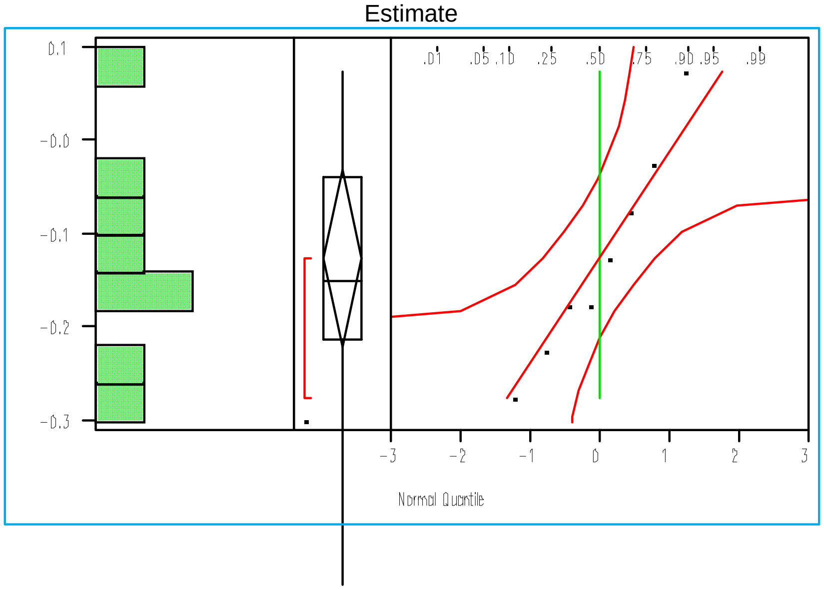
A	B	C	D	E	observation y
−	−	−	−	+	14.8
+	−	−	−	−	14.5
−	+	−	−	−	18.1
+	+	−	−	+	19.4
−	−	+	−	−	18.4
+	−	+	−	+	15.7
−	+	+	−	+	27.3
+	+	+	−	−	28.2
−	−	−	+	−	16.0
+	−	−	+	+	15.1
−	+	−	+	+	18.9
+	+	−	+	−	22.0
−	−	+	+	+	19.8
+	−	+	+	−	18.9
−	+	+	+	−	29.9
+	+	+	+	+	27.4

Plot the effects on probability paper, draw tentative conclusions, and verify by plotting residuals. (This is problem 22 from Box, pp. 445.)

hw4_solterms

Term	Estimate
A	-0.125
B	3.625
A*B	0.475
C	2.925
A*C	-0.525
B*C	1.375
D	0.725
A*D	-0.025
B*D	-0.075
C*D	0.075
E	-0.475
A*E	-0.275
B*E	-0.175
C*E	-0.175
D*E	-0.225

Estimate



Moments

Mean	-0.125000
Std Dev	0.113389
Std Error Mean	0.040089
Upper 95% Mean	-0.030203
Lower 95% Mean	-0.219797
N	8.000000
Sum Weights	8.000000

My conclusions are that effects B, A*B, C, A*C, B*C, D and E are significant. the rest appear to be normally distributed around zero...