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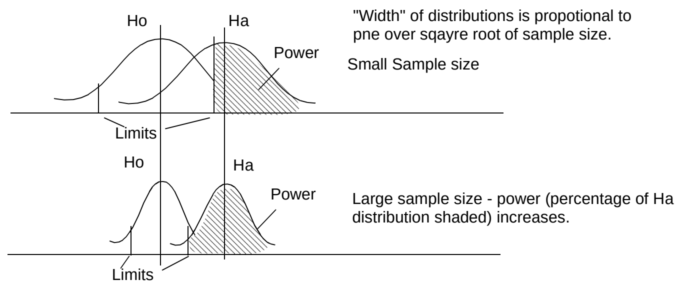
Official Solutions No. 3

1. Anser the following questions (briefly explain your answers)

a) What is a "test statistic" in the context of Hypothesis testing?

A test statsitic is the "score" one calculates from the data. Usefull test statistics follow known, well defined distributions under the null, and possibly known distribution(s) under the alternative hypothesis. The test consists of seeing how likely it is, that the test statistic actually came from the null hypothesis distribution.

b) How is the power of a test related to the sample size of a test, assuming that a test statistic is an average of independent samples, and it follows the normal distribution?



The power of the test should be proportional to the square root of the sample size. This is because the two distributions (the null and the alternate) will reduce their variance at one over the square root of the sample size.

c) Is the power (P) of a test independent of the type I error?

No. One can trade power for type one error and vice versa. The power of the test depends on the criterion for Ho rejection, and this criterion (limit) is directly dependent on the type I error a.

d) If the sample size is correctly calculated to be 10, would a better experiment result if a sample size of 20 is used?

Hmm. That depends on what we mean by a "better" experiment. If a better experiment gives higher power with the same type I error, then this is true. This applies, however, only if the samples are independent. If the samples are not independent, then a 20 sample experiment is simply more expensive... Even if the samples <u>*are*</u> *independent, however, better power comes at a cost of more measurements. One has to carefully consider if the additional cost is justified.*

2. Two types of photoresist (A and B) were used on a total of 8 randomly selected wafers, 4 for A and 4 for B. After patterned with a DUV stepper (using the same mask for all), the average linewidth was measured in mm:

A	0.18	0.17	0.18	0.19
B	0.21	0.18	0.19	0.22

Obtain a 95% confidence interval for the ratio of sigmas. State your assumptions carefully.

The Assumptions are that my data are Indepednently, Identically and Normally Distributed (IIND). The calculations are straightforward:

$$\frac{s_A^2/\sigma_A^2}{s_B^2/\sigma_B^2} \sim F_{3,\,3} \Rightarrow \frac{1}{F_{3,\,3(\alpha/2)}} \le \frac{s_A^2/\sigma_A^2}{s_B^2/\sigma_B^2} \le F_{3,\,3(\alpha/2)} \Rightarrow \frac{1}{F_{3,\,3\langle 0.025\rangle}}\frac{s_B^2}{s_A^2} \le \frac{\sigma_B^2}{\sigma_A^2} \le F_{3,\,3(0.025)}\frac{s_B^2}{s_A^2} \Rightarrow$$

$$\frac{1}{15.44}\cdot\frac{0.000333}{0.000066} \le \frac{\sigma_B^2}{\sigma_A^2} \le 15.44\cdot\frac{0.000333}{0.000066} \Rightarrow 0.3238 \le \frac{\sigma_B^2}{\sigma_A^2} \le 77.2 \Rightarrow 0.5690 \le \frac{\sigma_A}{\sigma_B} \le 8.786$$

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3. You are considering introducing a new etch recipe, hoping that it will reliably shirnk your after-etch CD. (remember that each nm you shave off the CD gives you about $7/microprocessor you sell!). You have determined that the sigma of the process is 25nm, and it will not be affected by the new process. Using a mask with 0.22mm lines, and the old etch recipe, the average measured polysilicon linewidth is 0.19mm. You will be willing to switch to the new etch recipe only if you see a 20nm improvement (i.e. your measured pattern goes from 0.19mm to 0.17mm) with a=0.05 and b=0.10. How many samples done with the new process do you need for this experiment?

According to the null and to the alternate hypothesis, the limit "x" that will be used to implement the (one sided) test has as follows:

$$z_{1-0.05} = \frac{x-0.19}{\sigma/\sqrt{n}}$$

$$z_{0.90} = \frac{x-0.17}{\sigma/\sqrt{n}}$$

Substituting the proper z values from the normal probability tables, we form one equation with only *n (the sample size) as the unknown.*

$$-1.64 \cdot \frac{0.025}{\sqrt{n}} + 0.19 = 1.29 \cdot \frac{0.025}{\sqrt{n}} + 0.17 \Rightarrow n = 13.41$$

From the above I conclune that n must be at least 14.

4. We now have three types of resist to compare: A, B and C. We would like to use an ANOVA table in order to test the hypothesis of equivalence of the three treatments. A few wafers from the first group have already been measured. Look at the 4 measurements below and estimate the total number of wafers we need to measure if we would like to detect deviations in the order of 0.05mm between groups with a power of 80%, while the type I error is kept at 5%.

A:	0.32	0.27	0.30	0.27

Explain all implicit assumptions.

OK, so this is a tough problem. The idea here is that if we assume (to simply the solution) an equal number of samples n in each group, then, assuming that the groups are not the same, we have the following. First, we estimate the sigma of the process to be 0.0245 and the $s^2 = 6\ 10^{-4}$. *According to the known ANOVA formulas, if we had completed the experiment , then* s_t^2 *would be estimating the following number:*

$$s_T^2 = \frac{\sum_{t=1}^{3} n(\bar{y}_t - \bar{y})^2}{3-1} \sim \left(\sigma^2 + \frac{\sum_{t=1}^{3} n\tau_t^2}{3-1} \right)$$

The question is what would be the sampling distribution of s_t^2*? A good approximation is to use thje non-central* x^2 *distribution with 2 degrees of freedom for this. So, under the null and the alternate hypothesis, the formula that will give me the solution is:*

$$F_{2,\,3(n-1),\,a} = \frac{\sigma^2 + n0.01^2}{\sigma^2} F_{2,\,3(n-1),\,1-\beta}$$