UNIVERSITY OF CALIFORNIA
College of Engineering
Department of Electrical Engineering
and Computer Sciences

C. SPANOS & K. POOLLA

EECS 290H
SPRING 1999

PROBLEM SET No. 1
Official Solutions

1. An oncoming 200mm wafer has 5 randomly spaced point defects on it. The chip size is 2.0cm^2, and the final target yield is 90%. If there are 10 additional processing levels, calculate the maximum number of defects that we can afford to accumulate on each level. (Use the simple exponential yield model).

As the wafer comes in for processing, it already has a defect density of 5/πx10^2 = 0.0159 defects/cm^2. This means that the yield of the oncoming wafer will be:

$$Y_{in} = e^{-AD} = e^{-2 \times 0.0159} = 96.9\%$$

Now, each additional step will have a limited yield too, that will depend on the defect density of that step. Let us say that the defect density of each of the subsequent eight steps will be D_s. The yield of all the eight steps will then be:

$$Y_{step}^{8} = (e^{-AD_s})^{10}$$

Now, the final yield (including the original yield and that of the eight steps) will be:

$$0.90 = Y_{in} \times Y_{step}^{10} = 0.969 \times (e^{-AD_s})^{10} \Rightarrow \frac{ln(0.9288)}{10 \times A} = -D_s \Rightarrow D_s = \frac{0.0037}{cm^2}$$

If I multiply the defect density with the area of the wafer (3.14159x10^{2}cm^2), then I find that the number of defects I can accumulate during each of the steps is 1.16.

2. A manufacturing facility has a yield that is controlled purely by random defects. The density of these random defects depends on the design rule used. More specifically, for a 0.25μm design rule, the density is 0.1/cm^2, while for a 0.18μm design rule, the density is 0.5/cm^2. (Use the simple yield model).
   a) A given product takes 1.0cm^2. Further, 90% of this area is using 0.25μm design rules, while the rest 10% is using the 0.18μm design rules. Estimate the yield of this product.

To estimate the yield of this product I will estimate the yield of each of the two components and then calculate the product of the two yields. The yield of the 0.25 μm design rule part will be:

$$Y_1 = e^{-A_1 D_1} = e^{-0.9 \times 0.1} = 0.941$$

The yield of the 0.5μm design rule part will be:

$$Y_2 = e^{-A_2 D_2} = e^{-0.1 \times 0.5} = 0.952$$

The total yield will then be:

$$Y = Y_1 \times Y_2 = 0.87$$

b) This product can be redesigned (shrunk) to take only 0.5cm^2, but now 50% of the chip is using the 0.25μm design rules. Estimate the yield of the redesigned product.

I calculate again the yields of the two different areas. The area for Y_1 is now 0.25cm^2, so Y_1 will be 0.975. The area of Y_2 is now 0.25cm^2, so Y_2 will be 0.883. The product of the two is now 0.861 so yield does not change appreciably, even though a larger percentage of the die moves to the more aggressive design rule! (But you do get many more good devices per wafer! - see below)

c) What would be the ratio of good die per wafer of the redesigned product to that of the original product?

This is a very important question, because we can actually assume that the cost of processing one wafer is the same, regardless of the design rules. The number of potential die depends, of course, on the wafer area. Let us assume that the wafer area is W_A, same in both cases. The average good die per wafer (GDPW) in the each case will be:

$$\left.\begin{aligned} GDPW_1 &= \frac{W_A}{1cm^2} \times 0.870 \\ GDPW_2 &= \frac{W_A}{0.5cm^2} \times 0.861 \end{aligned}\right\} \Rightarrow \frac{GDPW_2}{GDPW_1} = 1.98$$

Since GDPW actually doubles, this means that the redesign actually cut my cost per good die by a factor of two, or increased my output by a factor of two at the same cost. This is VERY desirable.

3. Suppose that you use 200mm wafers, and also assume that you can only get functional dice only within the inner 190mm diameter (outer 5mm margin is full of defects). On the one product that you have run so far, a chip 5mm x 5mm, the yield is 80%.
   a) Using the simple Poisson distribution, find the defect density (in the good area of the wafer) and plot the yield as a function of S, where S is the square root of the area of the die in production. Plot the Total and the Good Die per wafer as a function of S on the same graph.

Looking only at the yield in the "good" area for 0.5cm x 0.5cm die, I can estimate the defect density (all areas are in cm^2 and all defect densities are in cm^{-2}. Also, in all the following formulas, S is in cm):

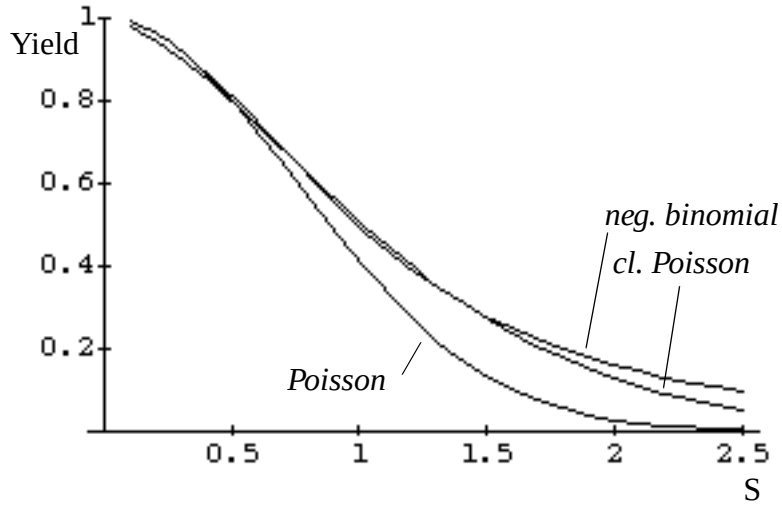
$$D = -\frac{ln(0.8)}{0.25} = 0.893cm^{-2}$$

I now simply plot these formulas for yield (Y), total die per wafer (TDPW) and good die per wafer (GDPW):

$$Y = e^{-S^2 \times 0.893}$$

$$TDPW = \frac{\pi \times 19^2/4}{S^2}$$

$$GDPW = \frac{\pi \times 19^2/4}{S^2} \times e^{-S^2 \times 0.893}$$



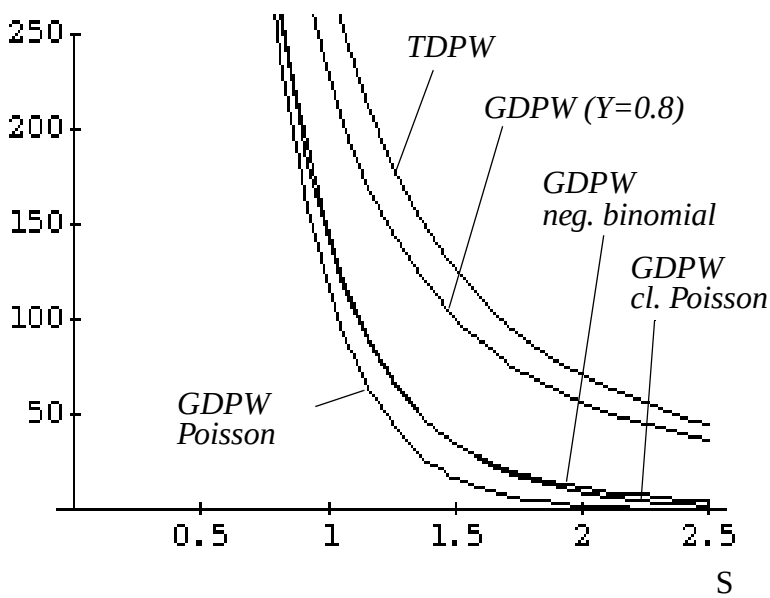
b) Repeat the calculations and plots in a) using the clustered Poisson model (b=0.2) and the Negative Binomial model (α=1.5).

Here I simply have to calculate yield (Y) using the appropriate Clustered Poisson (b=0.2):

$$Y = 0.8^{[(S/0.5)^2]^{1-b}}$$

$$TDPW = \frac{\pi \times 19^2/4}{S^2}$$

$$GDPW = \frac{\pi \times 19^2/4}{S^2} \times \left(0.8^{[(S/0.5)^2]^{1-b}}\right)$$



or the appropriate negative binomial distribution (a=1.5):

$$Y = \left[1 + \frac{0.893 \times S^2}{a}\right]^{-a}$$

$$TDPW = \frac{\pi \times 19^2/4}{S^2}$$

$$GDPW = \frac{\pi \times 19^2/4}{S^2} \times \left[1 + \frac{0.893 \times S^2}{a}\right]^{-a}$$

The point here is that with the appropriate choice of the "fudge factor" α or b, the clustered Poisson model and the negative binomial model can be made to look almost exactly the same!

c) Suppose that an alternative explanation for the data were that some fraction, f, of the wafer was perfect and the rest was totally dead. This is the "black-white" model that assumes a perfect deterministic clustering of defects. What is f? Plot the Good Die per wafer for this model on the same graph as in a)-b).

If we had perfect clustering of defects, then this would mean that a fixed fraction f=20% of the available portion of the wafer is bad. This translates to a fixed bad "island" inside the good region of the wafer that covers 20% of the original good area. This means that <u>the yield is 80%, regardless of die size</u>, so the formula for the GDPW for this situation would be (see previous page for plot):

$$GDPW = \frac{\pi \times 19^2/4}{S^2} \times 0.8$$

As you can see, this formula tends to be rather optimistic.

d) What defect density reduction would you have to achieve to yield 50% of the available die at S=15mm according to models a), b) and c)?

To achieve a yield of 50% under the Poisson model one needs to achieve a D of 0.308 defects / cm^2.

Note that under the clustered Poisson model, defect density depends on area. To achieve a yield of 50% at S=1.5cm under the clustered Poisson model, I calculate the equivalent yield for S=0.5cm and I find the respective defect density (for S=0.5cm):

$$Y(1.5^2) = Y(0.5^2)^{[(1.5/0.5)^2]^{0.8}} = 0.50 \Rightarrow Y(0.5^2) = 0.89 \Rightarrow D(0.5^2) = \frac{-ln(0.89)}{0.5^2} = 0.478cm^{-2}$$

So, the defect density must be reduced by about 50%...

To achieve a yield of 50% at S=1.5cm under the negative binomial model, I simply solve the equation:

$$0.5 = \left[1 + \frac{D \times 1.5^2}{15}\right]^{-1.5} \Rightarrow 1.59 = 1 + \frac{D \times 1.5^2}{15} \Rightarrow D = 0.392cm^{-2}$$

Finally, to achieve 50% yield under the "fixed fraction" model, 50% of the originally good area of the wafer must be bad.