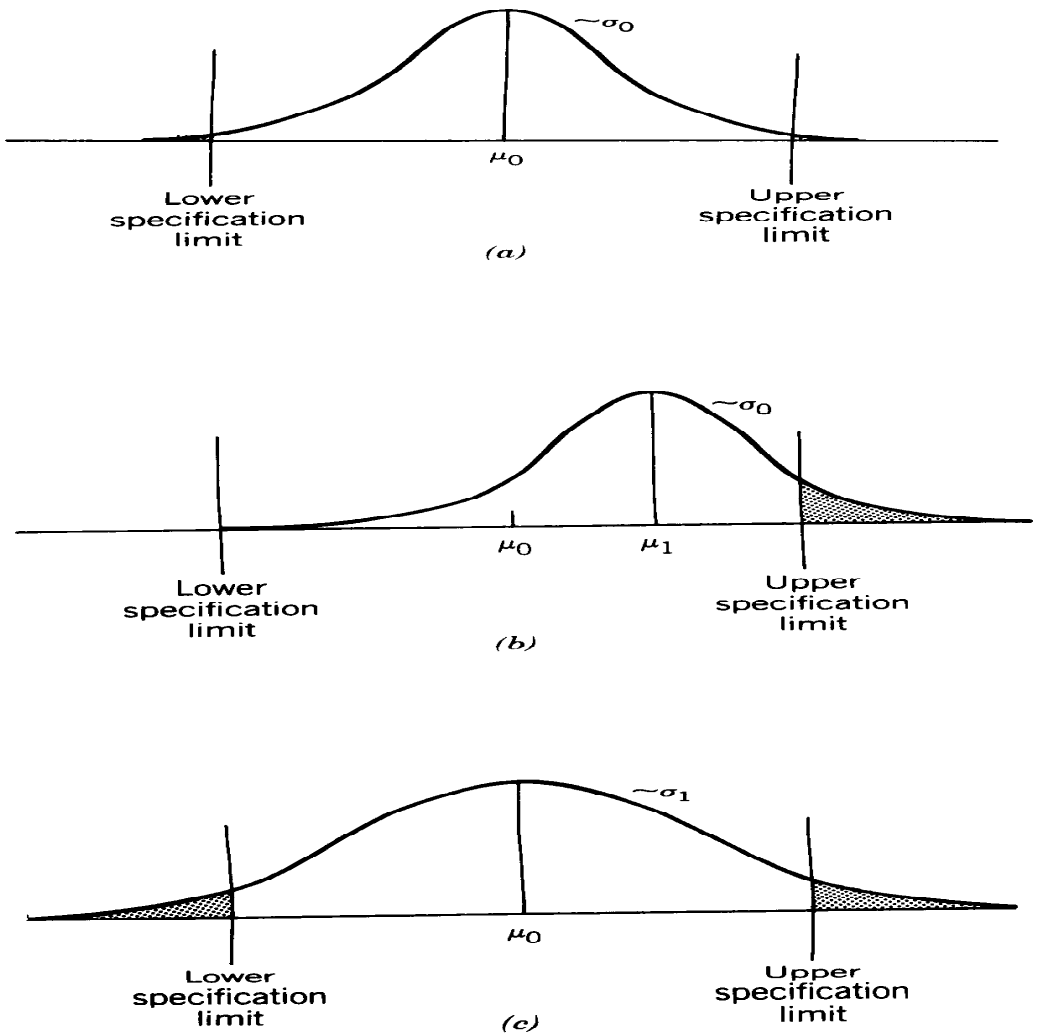
Spanos

Control Charts for Variables

$\bar{x}$-R, $\bar{x}$-s charts, non-random patterns, process capability estimation

Control Chart for $\bar{x}$ and R

Often, there are two things that might go wrong in a process; its mean or its variance (or both) might change.



Statistical Basis for the Charts

A normally distributed variable x with known μ and σ, can be controlled:

<u>average</u>:

$$\bar{x} = \frac{x_1 + x_2 + \ldots + x_n}{n} \quad \text{with} \quad \mu \pm Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$$

where $\pm Z_{\frac{\alpha}{2}}$ is the distance from μ (in # of σ),

that would capture (1- α)% of the normal.

<u>standard deviation</u>:

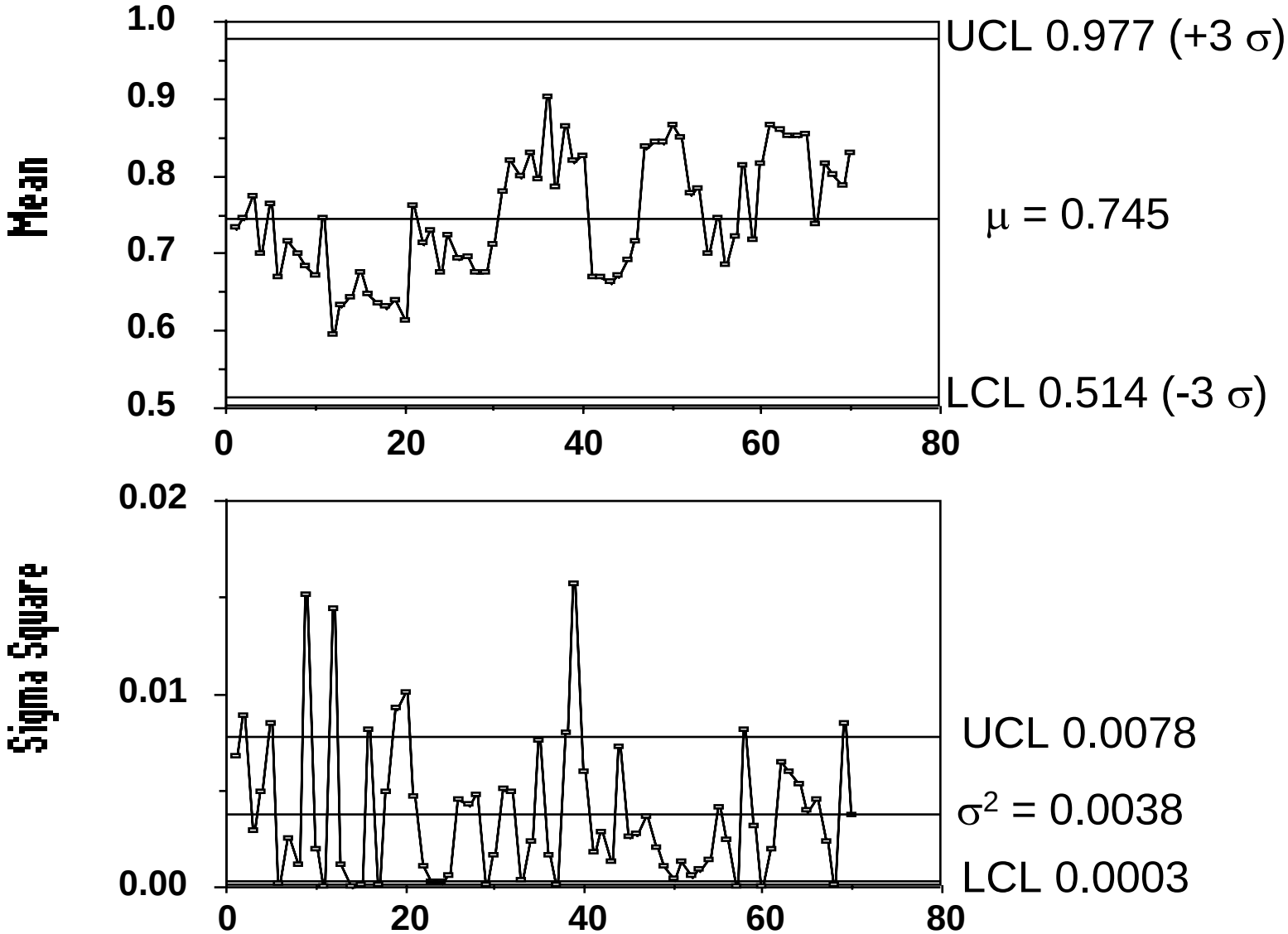
$$s^2 = \frac{\sum_{i=1}^{n} (x_i - \mu)^2}{n-1} \sim \frac{\sigma^2 \chi^2_{(n-1)}}{n-1} \quad \text{with:}$$

$$\frac{\sigma^2 \chi^2_{1-\alpha/2,(n-1)}}{n-1} < s^2 < \frac{\sigma^2 \chi^2_{\alpha/2,(n-1)}}{n-1}$$

where $\chi^2_{1-\alpha/2,(n-1)}$ and $\chi^2_{\alpha/2,(n-1)}$ are the numbers that capture between them 1-α of the $\chi^2_{(n-1)}$ distribution

Control Chart for $\bar{x}$ and s

(with mean and variance known).



Statistical Basis for the Charts (cont.)

In practice we do not know the mean or the sigma. The mean can be estimated by the grand average. If the sample size is small, we can use the *range* to describe spread.

$$\bar{\bar{x}} = \frac{\bar{x}_1 + \bar{x}_2 + \ldots + \bar{x}_m}{m}$$

$$R = x_{max} - x_{min}$$

Range R is related to the sigma in terms of a constant (depending on sample size) that is listed in statistical tables:

$$\bar{R} = \frac{R_1 + R_2 + \ldots + R_m}{m}$$

$$\hat{\sigma} = \frac{\bar{R}}{d_2}$$

Statistical Basis for the Charts (cont.)

The control limits for the $\bar{x}$-R chart are as follows:

$$UCL = \bar{\bar{x}} + A_2 \bar{R}$$

center at $\bar{\bar{x}}$

$$LCL = \bar{\bar{x}} - A_2 \bar{R}$$

$$UCL = \bar{R} D_4$$

center at $\bar{R}$

$$LCL = \bar{R} D_3$$

$$A_2 = \frac{3}{d_2 \sqrt{n}} \qquad D_{3,4} = 1 \pm 3 \frac{d_3}{d_2}$$

(d_2 and d_3 are tabulated constants that depend on n)

Range and Mean charts for Photoresist Control

Range

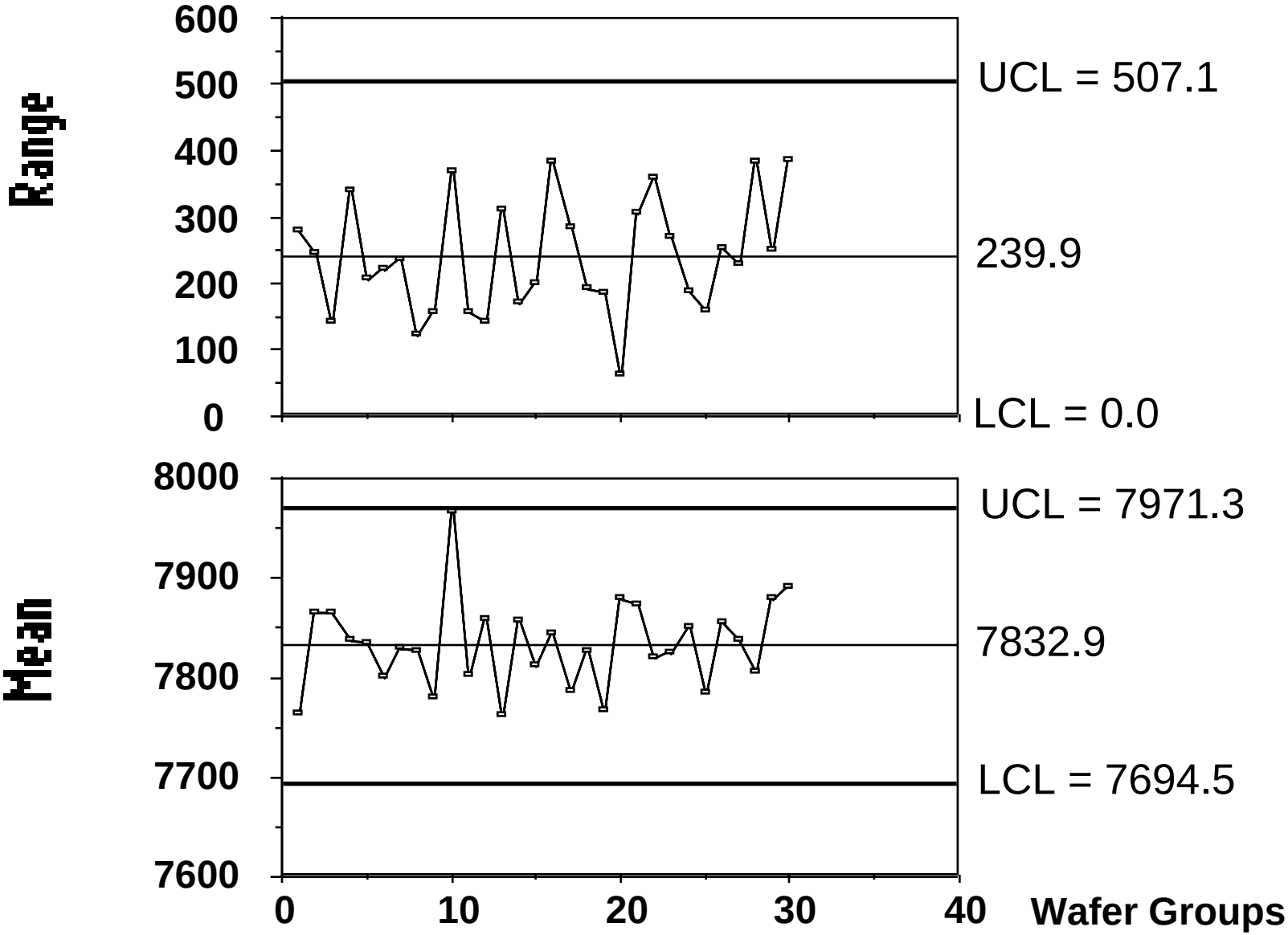
n=5 and from table, D_3=0.0 and D_4=2.11. Average Range is 239.4, so the range center line is 239.4, the LCL is 0.0 and the UCL is 507.1. These control limits will give us the equivalent of 3 sigma control. (α = 0.0027).

$\overline{x}$

The global average is 7832.9 and from table, A_2 is 0.577, so LCL is 7694.5 and UCL is 7971.3. These control limits will give us 3 sigma control. (α = 0.0027).

Example: Photoresist Coating (cont)

Range and $\bar{x}$ chart for all wafer groups.



The Grouping of The Parameters is Crucial

Known as *rational subgrouping*, the choice of grouping is very important.

In general, only *random* variation should be allowed within the subgroup.

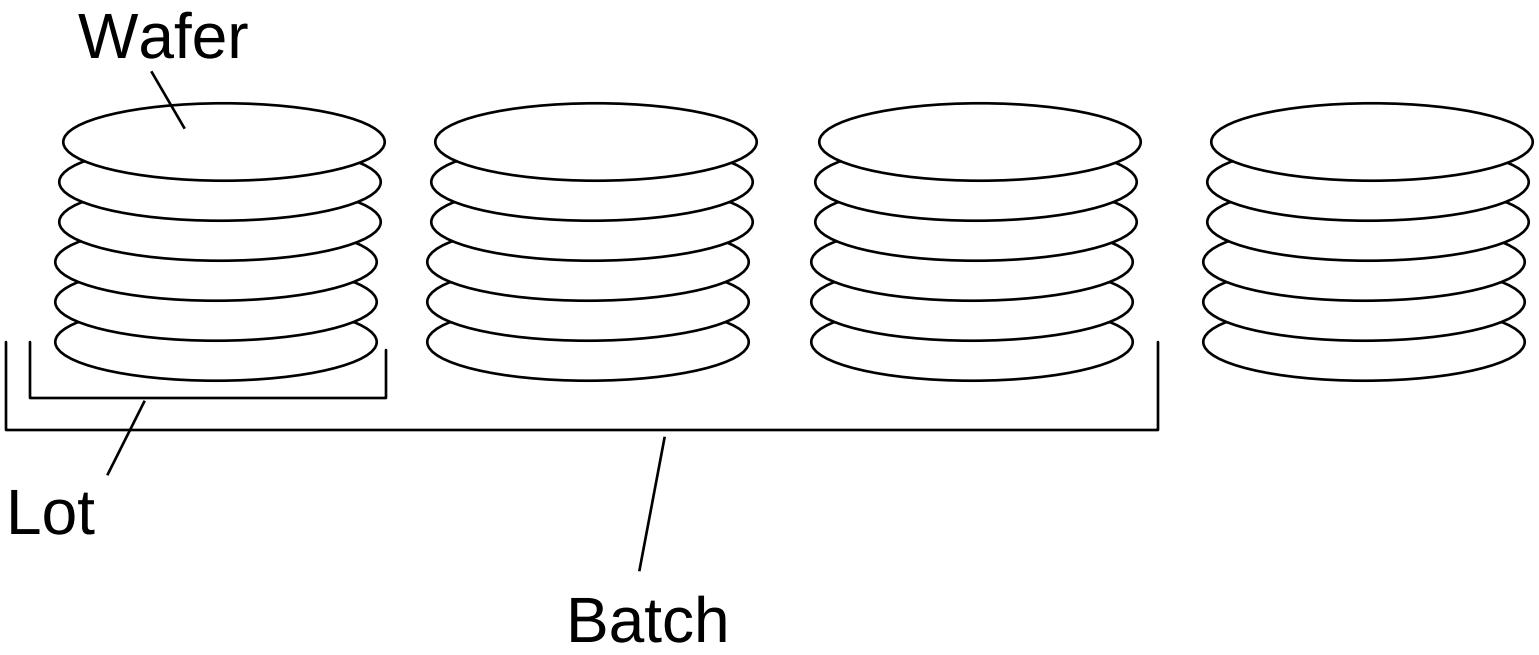
(i.e. grouping wafers within the boat of a diffusion tube is inappropriate - gas depletion effect is systematic.)

The range of the appropriate group should be used to estimate the variance of the process.

(i.e. the range across a lot should not be used to estimate the variance of a parameter measured between lots - *within* lot statistics are different from *between* lot statistics).

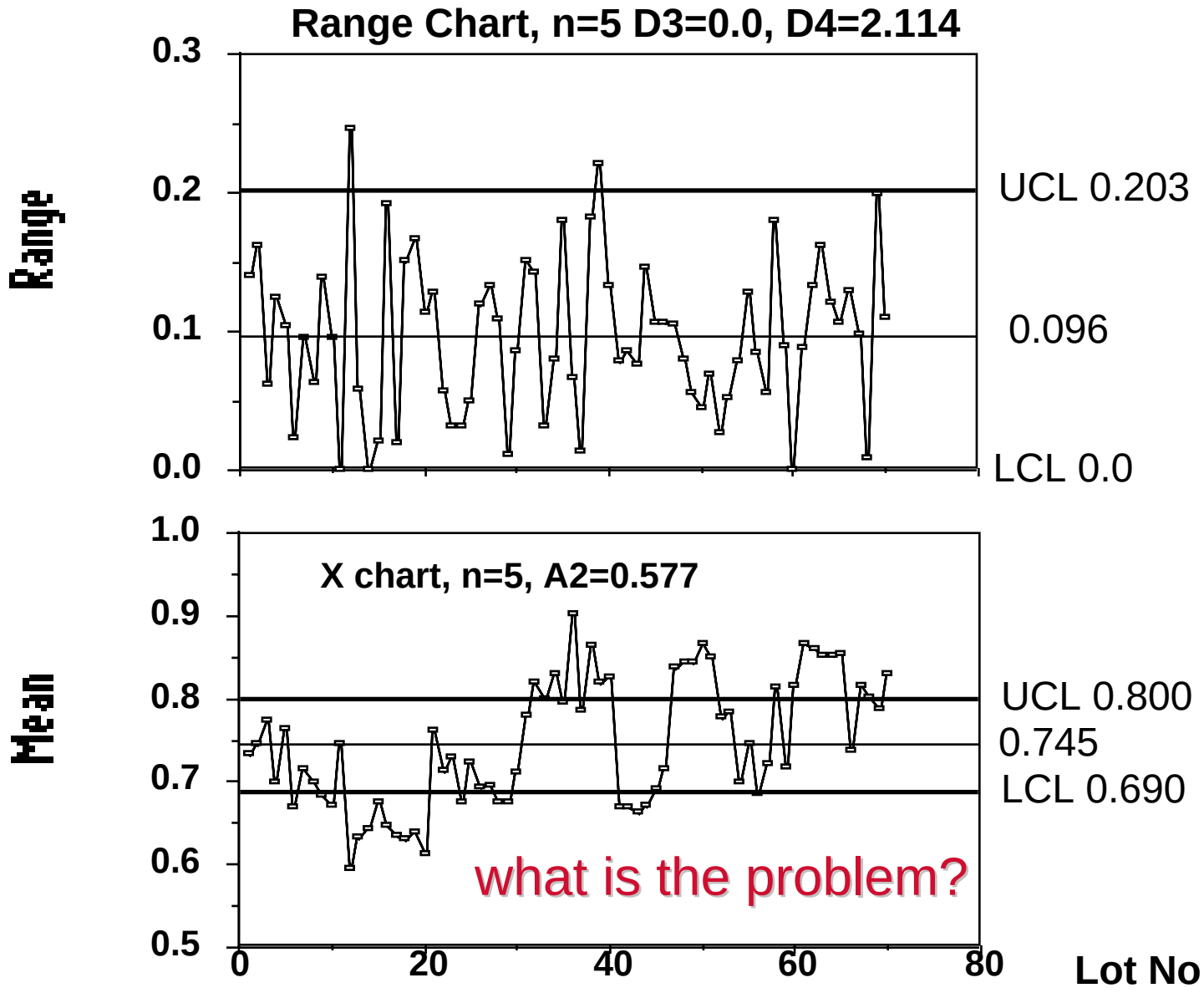
Rational Subgrouping

Rule of thumb: use only groups with **IIND** data, if possible. (**I**ndependently, **I**dentically, **N**ormally **D**istributed).



The natural grouping of semiconductor data might not lead to IIND subgroups!

Example: charts for line-width control

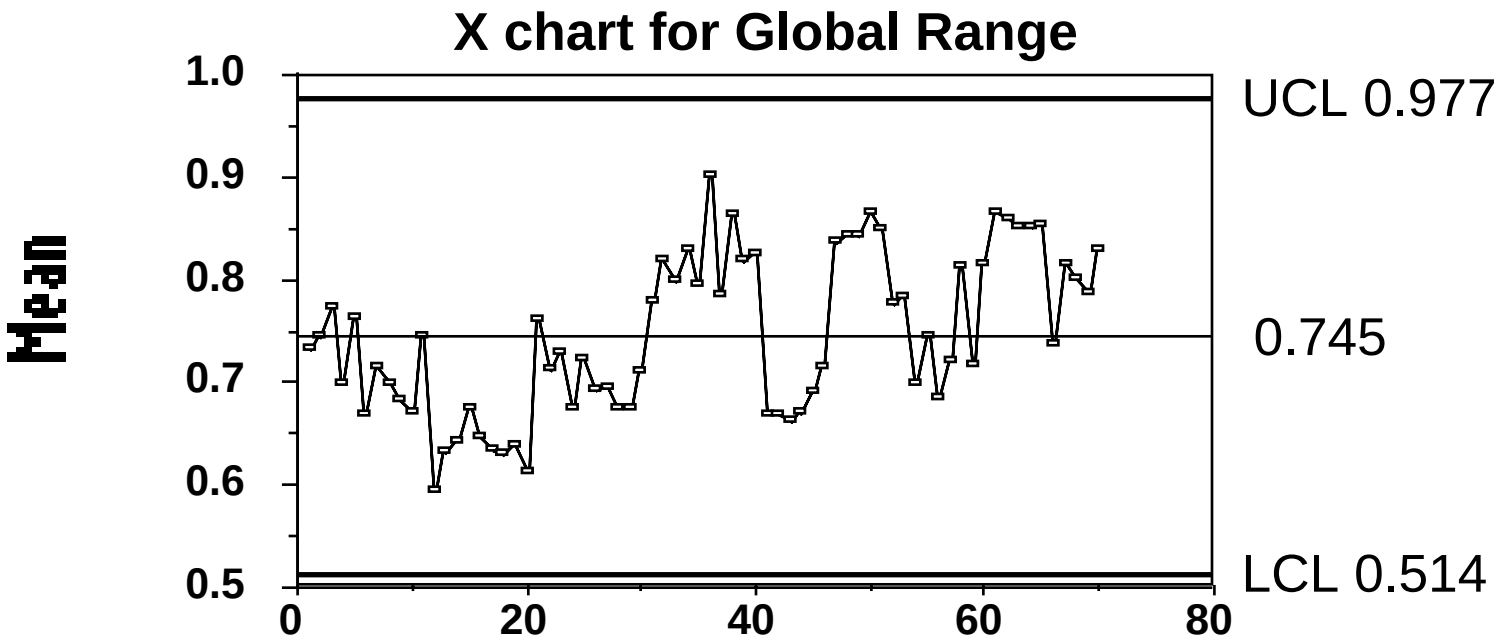


Rational Subgrouping (cont.)

Remember, we are using R to estimate the *global* sigma. The rational subgroups we chose might bias this estimation.

In this case, the within lot variation is much less than the global variation.

If we estimate the sigma from the global range, we get:



If this looks too good, it might be because of the "mixture" patterns in it!

Specification Limits vs. Control Limits

The specification limits of a process reflect our need. These limits are set by the management as objectives.

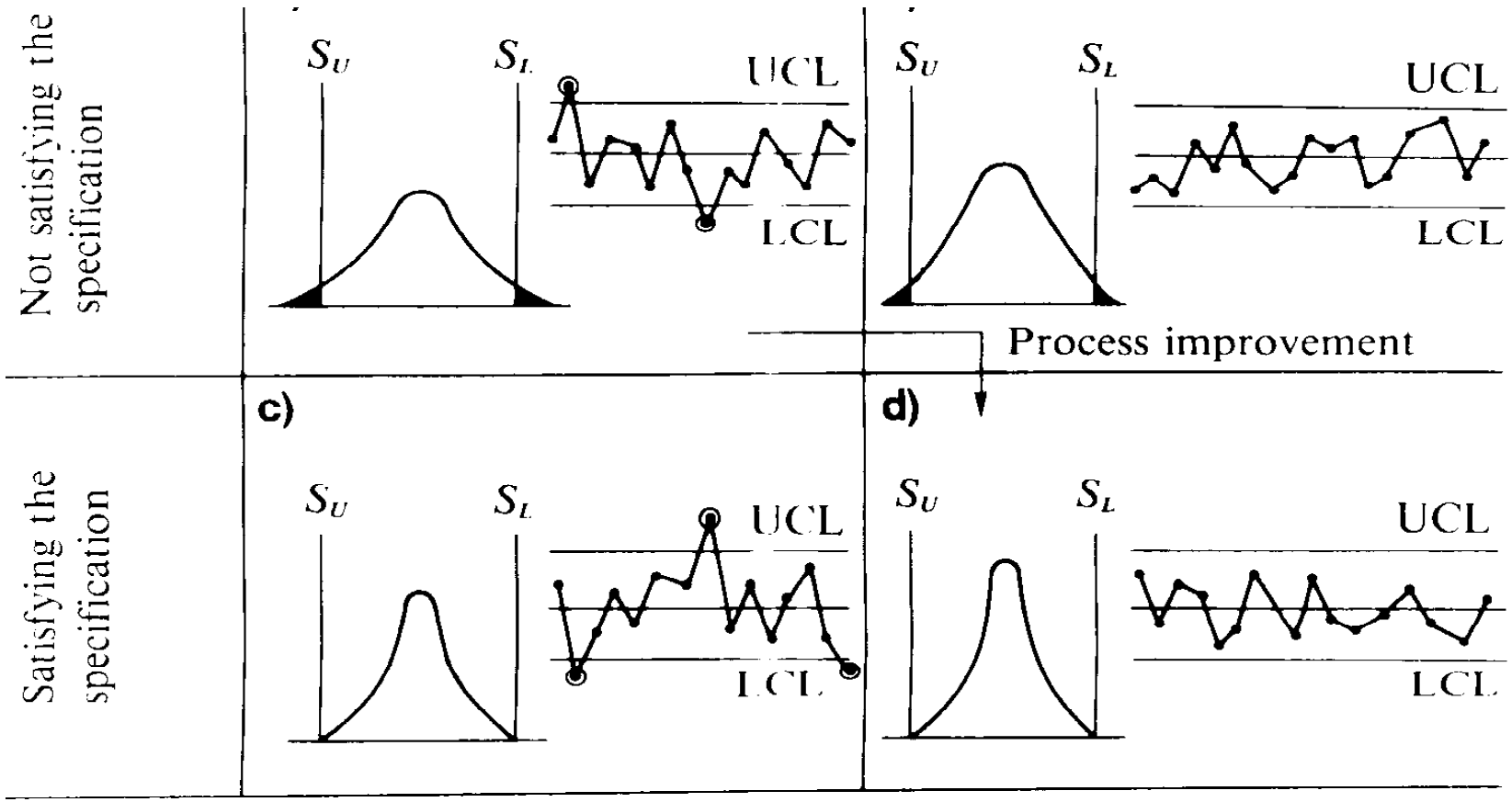
The control limits of a process tell us what the process can do when it is operating properly. These limits are set by the quality of the machinery and the skills of the operators.

Process Capability is a figure of merit that tells us whether a process is suitable for our manufacturing objectives.

Process Capability

Process specifications and control limits are, in general, different concepts.

A process might be in control without meeting the specs, or might be meeting the specs without being is control...



Process Capability Estimation

Calculate what the process (when in control) can do and compare it with specifications.

A control chart provides good estimates of σ, so it can be used for process capability evaluation.

Process Capability Ratio (PCR, C_P)

$$C_p = (USL-LSL) / 6\sigma$$

<u>Example</u>

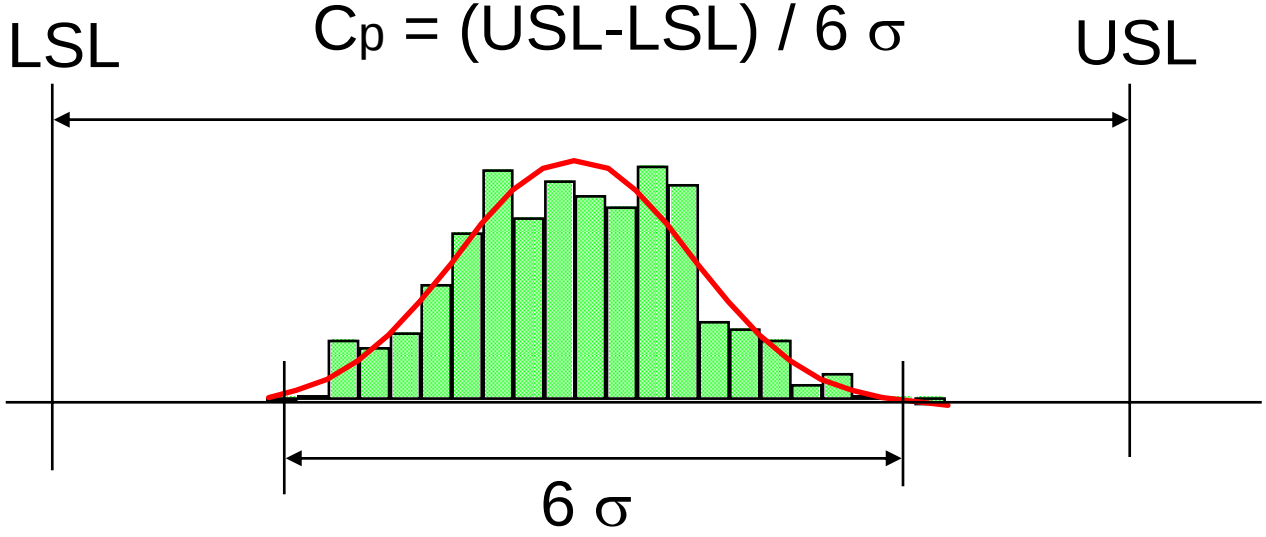
The line-width control data show a process capability of 1.08, since the specs for DLN are .5 to 1.0 µm and σ is .077.

The symbol C_{PK} is used for the process capability when the spec limits are not symmetrical around the process spread:

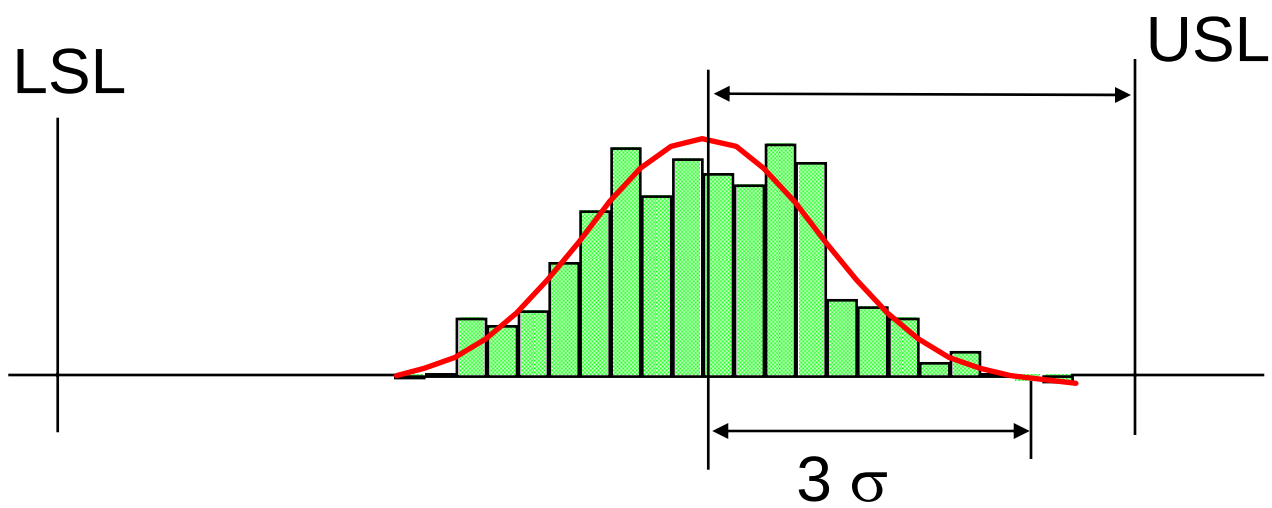
$$C_{PK} = \min \{ (USL-x)/3\sigma, (x-LSL)/3\sigma \}$$

Process Capability (cont)

$C_p = (USL-LSL) / 6\sigma$

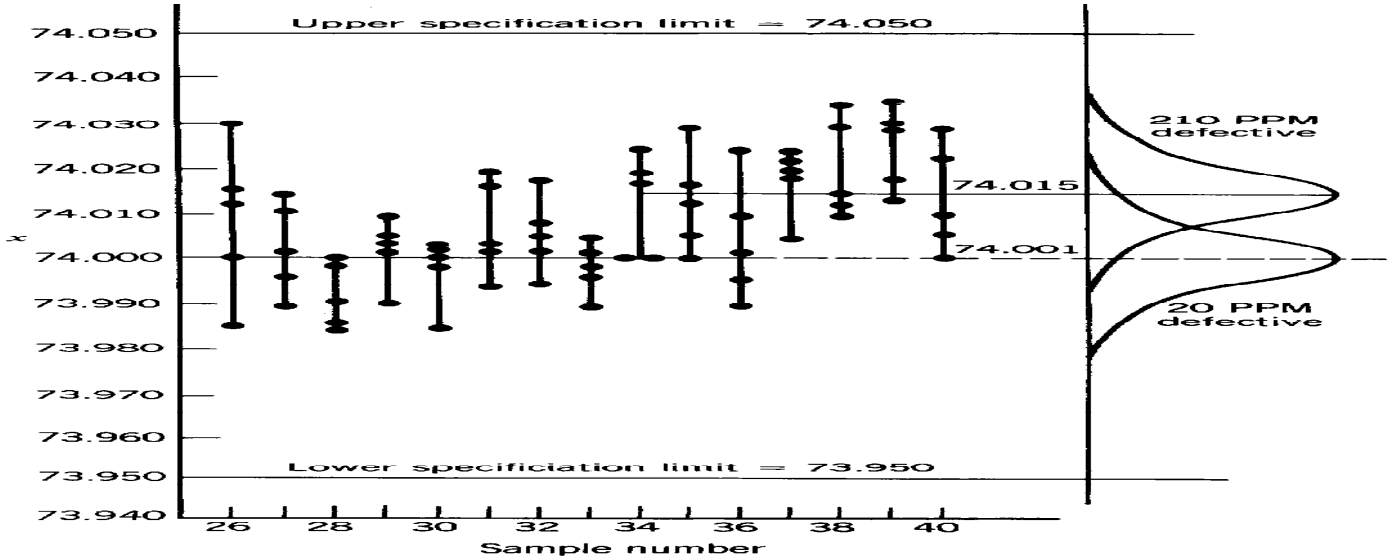


$C_{pk} = \min \{ (USL-x)/3\sigma, (x-LSL)/3\sigma \}$

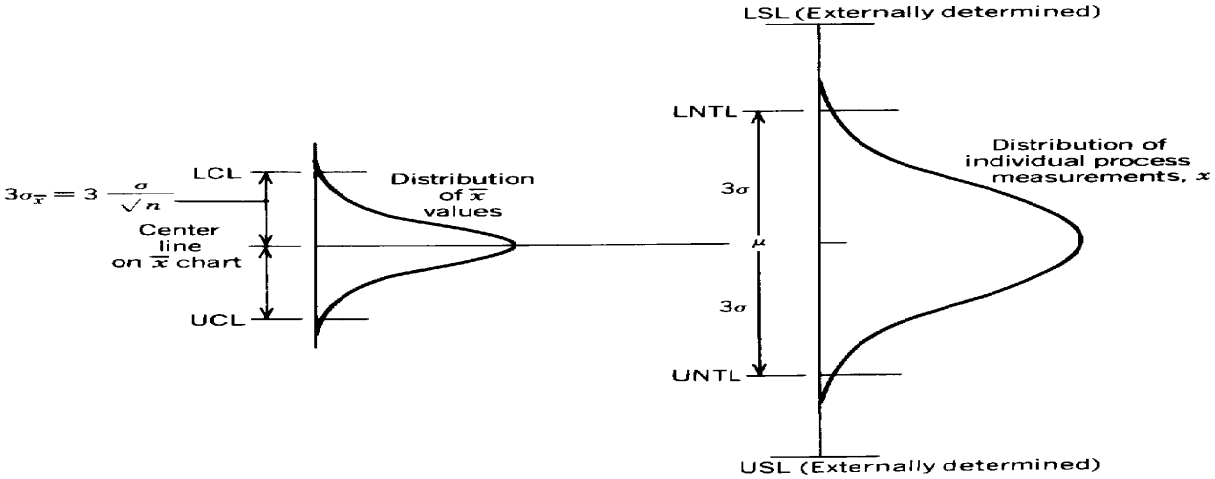


Never Mix Specification and Control Limits!

Only individuals should be plotted against the specs...

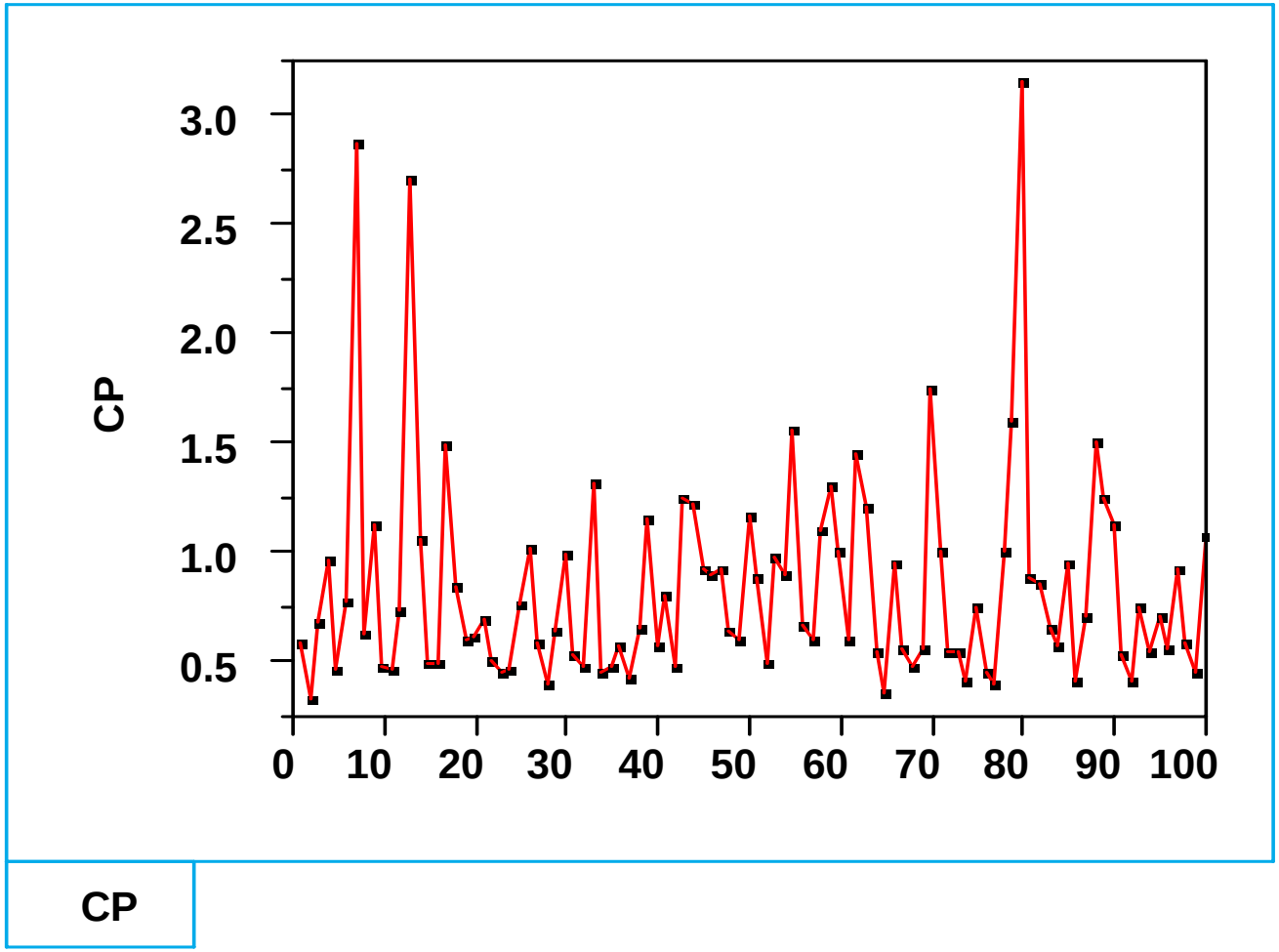


Relationship of natural tolerance, control and spec limits:



Never Attempt to Estimate Cp or Cpk from small groups!

Plot of CP estimate, n=5 (true CP=0.67)



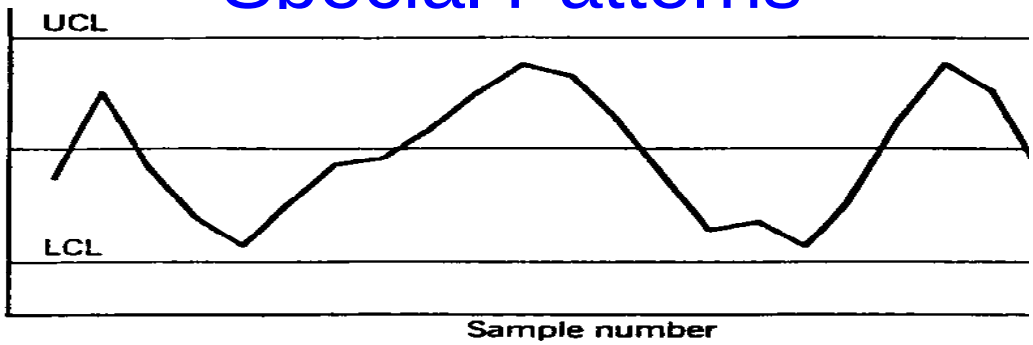
Special patterns in charts

While a point outside the control limits is a good indicator of non-randomness, other patterns are possible:

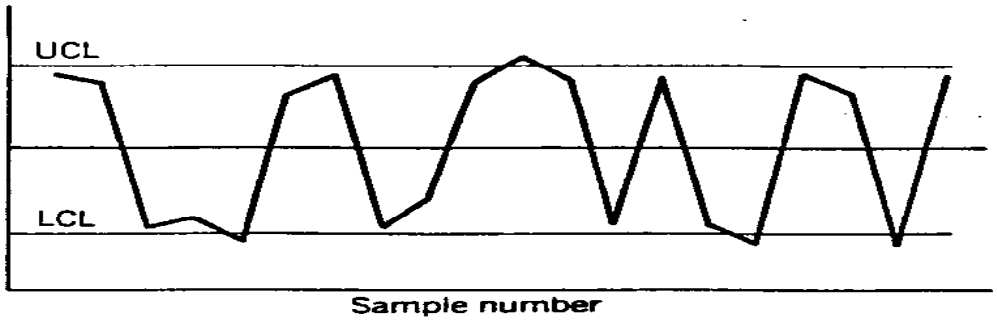
- Cyclic (periodic signal)
- Mixtures (two or more sources)
- Shifts (abrupt change)
- Trends (gradual change)
- Stratification (variability too small)

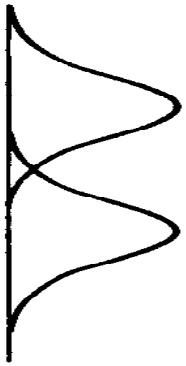
Additional rules can be used to detect them. (Watch for an increase in false alarm rates).

Special Patterns

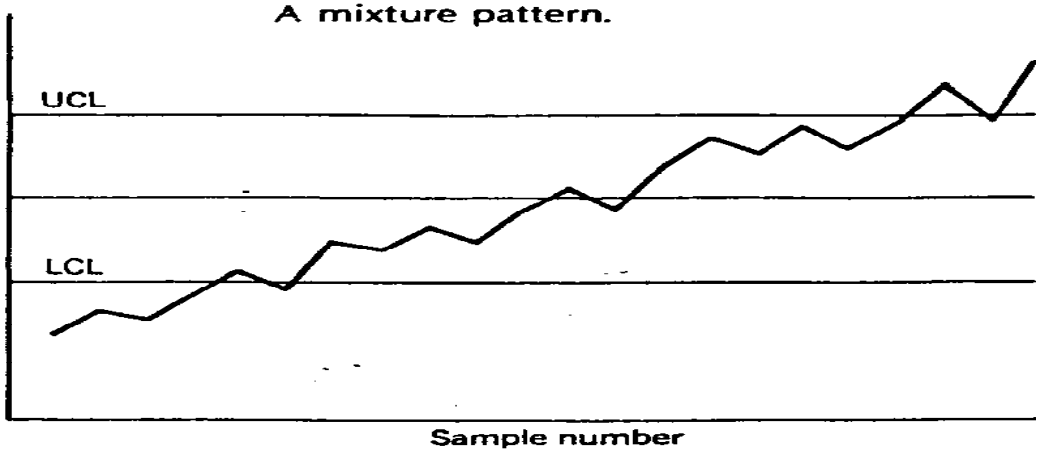


Cycles on a control chart.



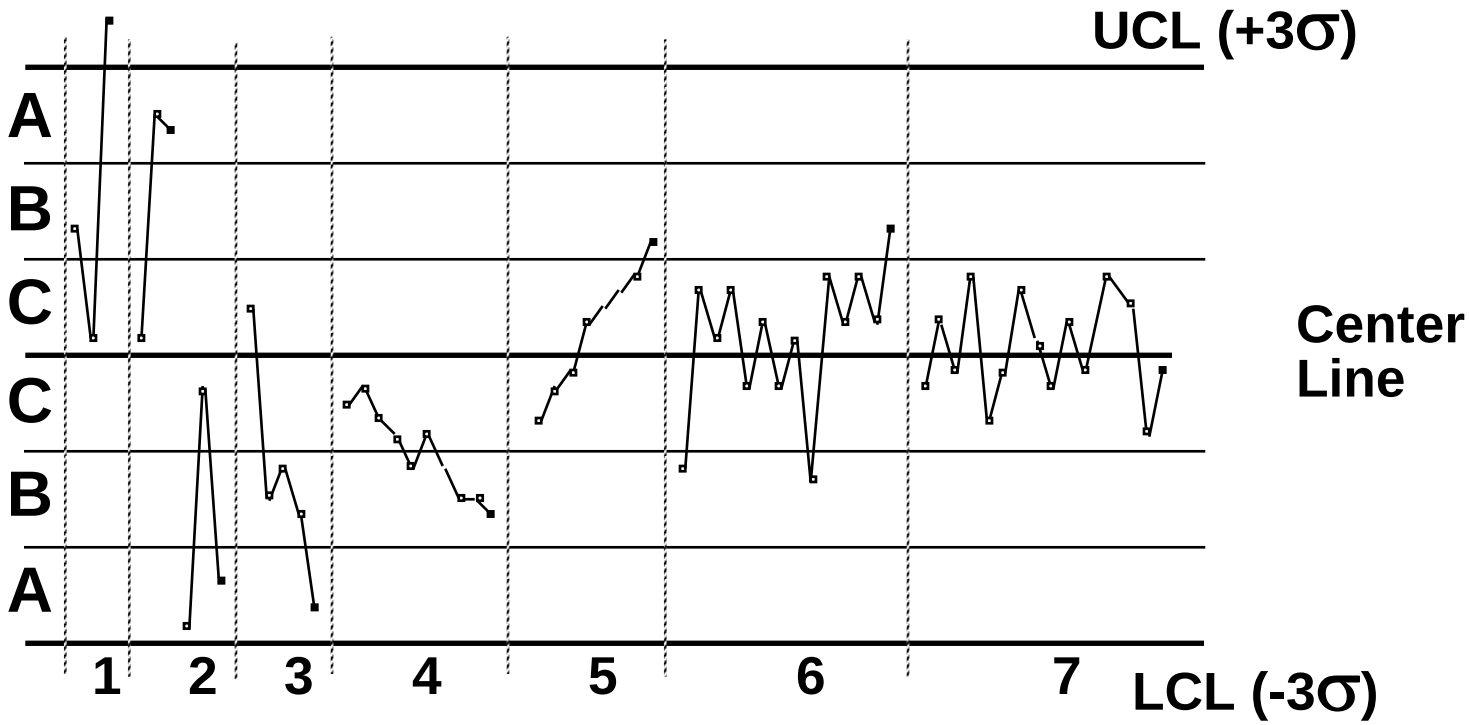


A mixture pattern.



Trend.

The Western Electric Rules



1. Any point beyond 3s UCL or LCL.
2. 2/3 cons. points on same side, in A or beyond
3. 4/5 cons. points on same side, in B or beyond.
4. 9/9 cons. points on same side of center line.
5. 6/6 cons. points increasing or decreasing.
6. 14/14 cons. points alternating up and down.
7. 15/15 cons. points on either side in zone C.

Robustness of the $\bar{x}$-R control chart

So far we have assumed that our process is fluctuating according to a normal distribution:

$$X \sim N(\mu, \sigma^2)$$

This assumption is not important for the $\bar{x}$ chart (thanks to the central limit theorem).

The R chart is much more sensitive to this assumption.

If the underlying distribution is not normal, watch for signs of correlation between x and R.

The $\overline{x}$-R Operating Characteristic Function

What is the probability of *not* detecting a shift of μ by k σ?

$$\beta = \Phi\left[\frac{UCL-(\mu_o+k\,\sigma)}{\sigma/\sqrt{n}}\right] - \Phi\left[\frac{LCL-(\mu_o+k\,\sigma)}{\sigma/\sqrt{n}}\right]$$

or, for 3σ control linits:

$$\beta = \Phi\,(\,3 - k\sqrt{n}\,) - \Phi\,(\,-3 - k\sqrt{n}\,)$$

The probability that the shift will be detected on the mth sample is:

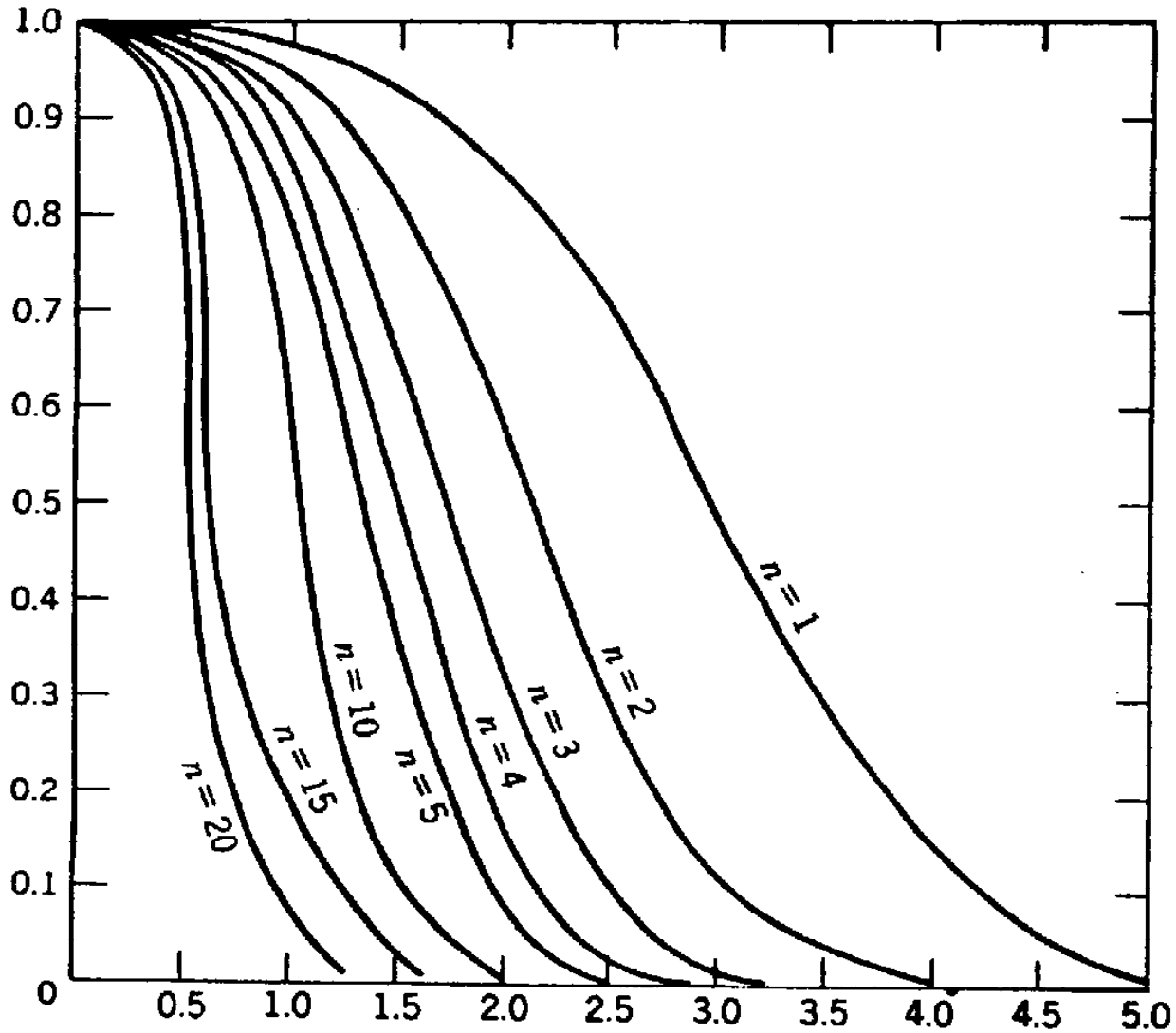
$$\beta^{m-1}(1-\beta)$$

And the average number of samples that we need for detecting this shift is:

$$ARL = \frac{1}{1-\beta}$$

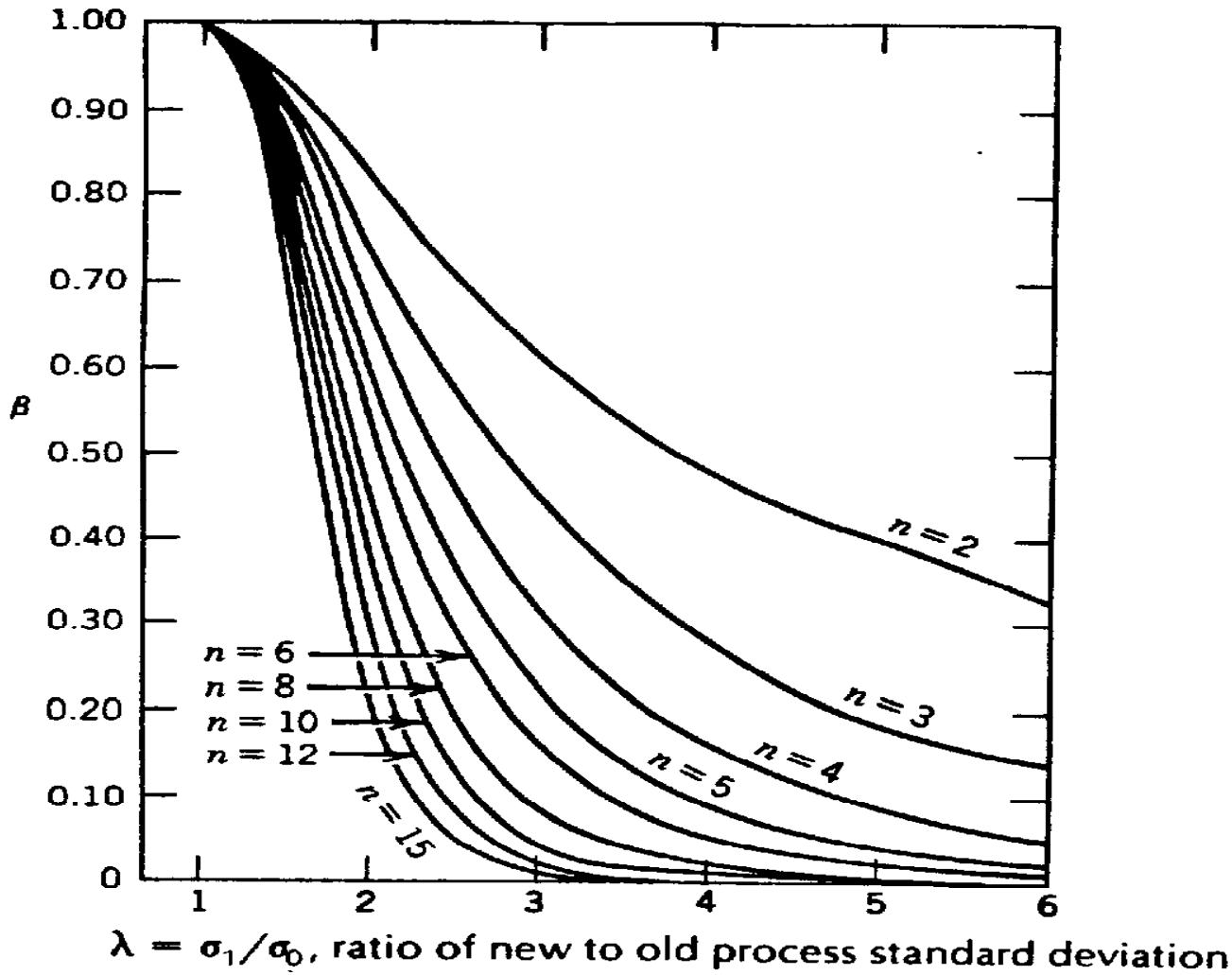
also known as the **Average Run Length**

The $\bar{x}$-R Operating Characteristic Function (cont.)



The $\bar{x}$-R Operating Characteristic Function (cont.)

The OC for the R part of the chart shows that it cannot catch small shifts in sigma:



$\bar{x}$ - s control charts

When n is larger (>10), then using the standard deviation s gives better results. Although s^2 is a good, *unbiased* estimator of the variance,

$$s^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}$$

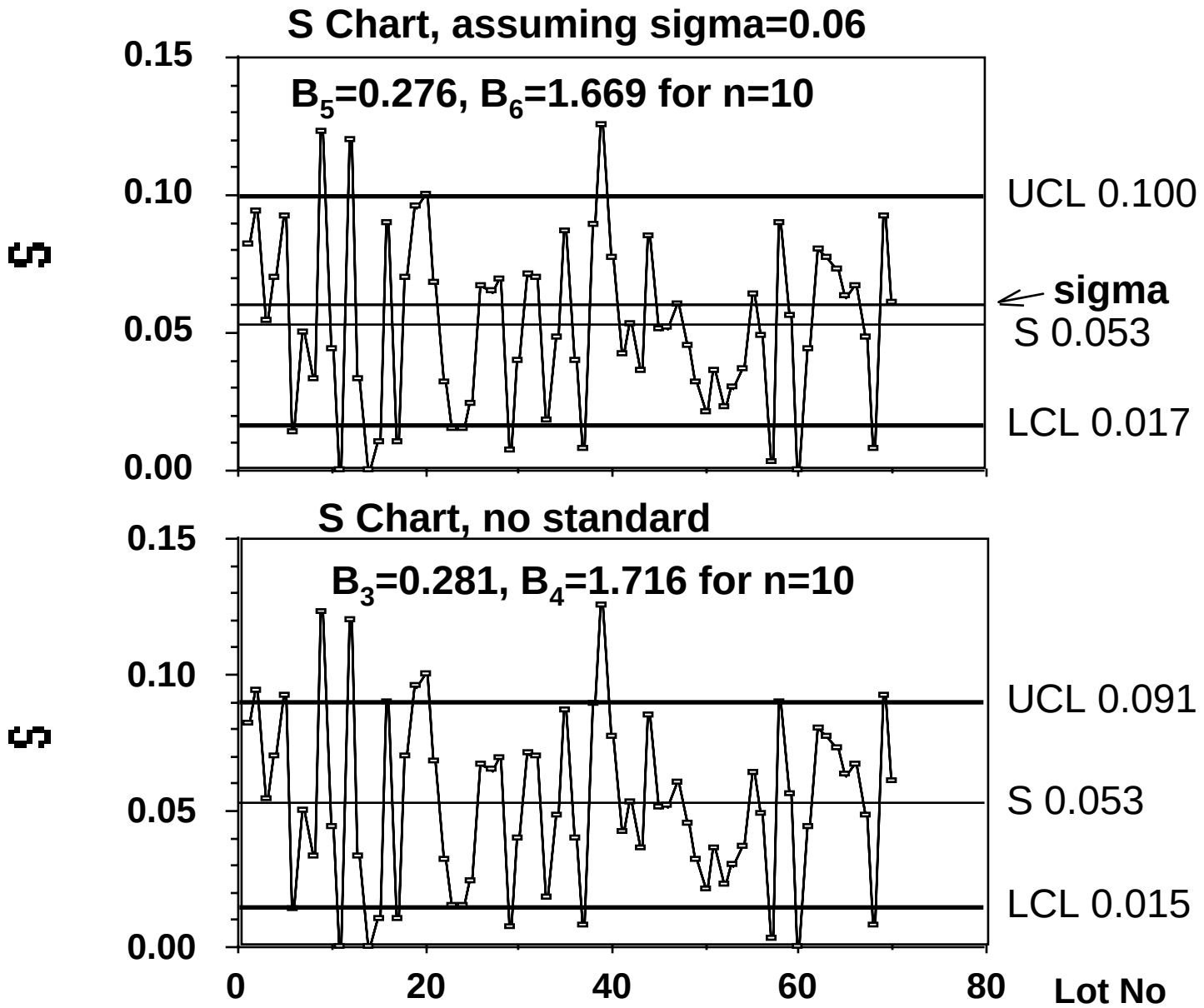
s is a *biased* estimator of sigma. A correction factor is a function of n and can be found is tables. In summary:

$$\bar{s} = \frac{1}{m}\sum_{i=1}^{m} s_i \text{ and } \frac{\bar{s}}{C_4} \text{ unb. estim of sig}$$

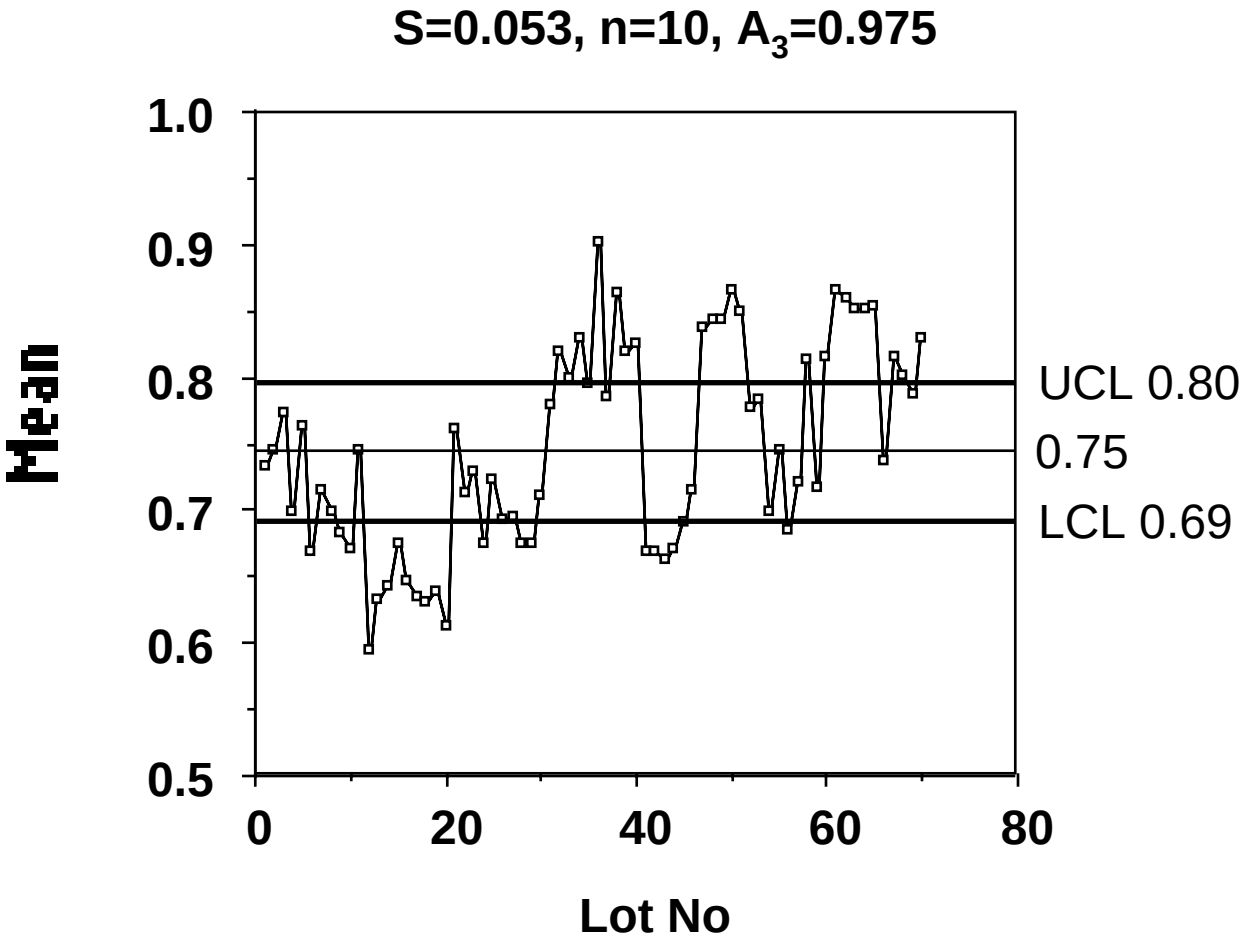
$$CL_s = B_{3,4}\,\bar{s} \quad B_{3,4} = 1 \pm \frac{3}{C_4}\sqrt{1-C_4^2}$$

$$CL_{\bar{x}} = \bar{\bar{x}} \pm A_3\bar{s} \quad A_3 = \frac{3}{C_4\sqrt{n}}$$

$\bar{x}$ and s Control Charts (when n is large)

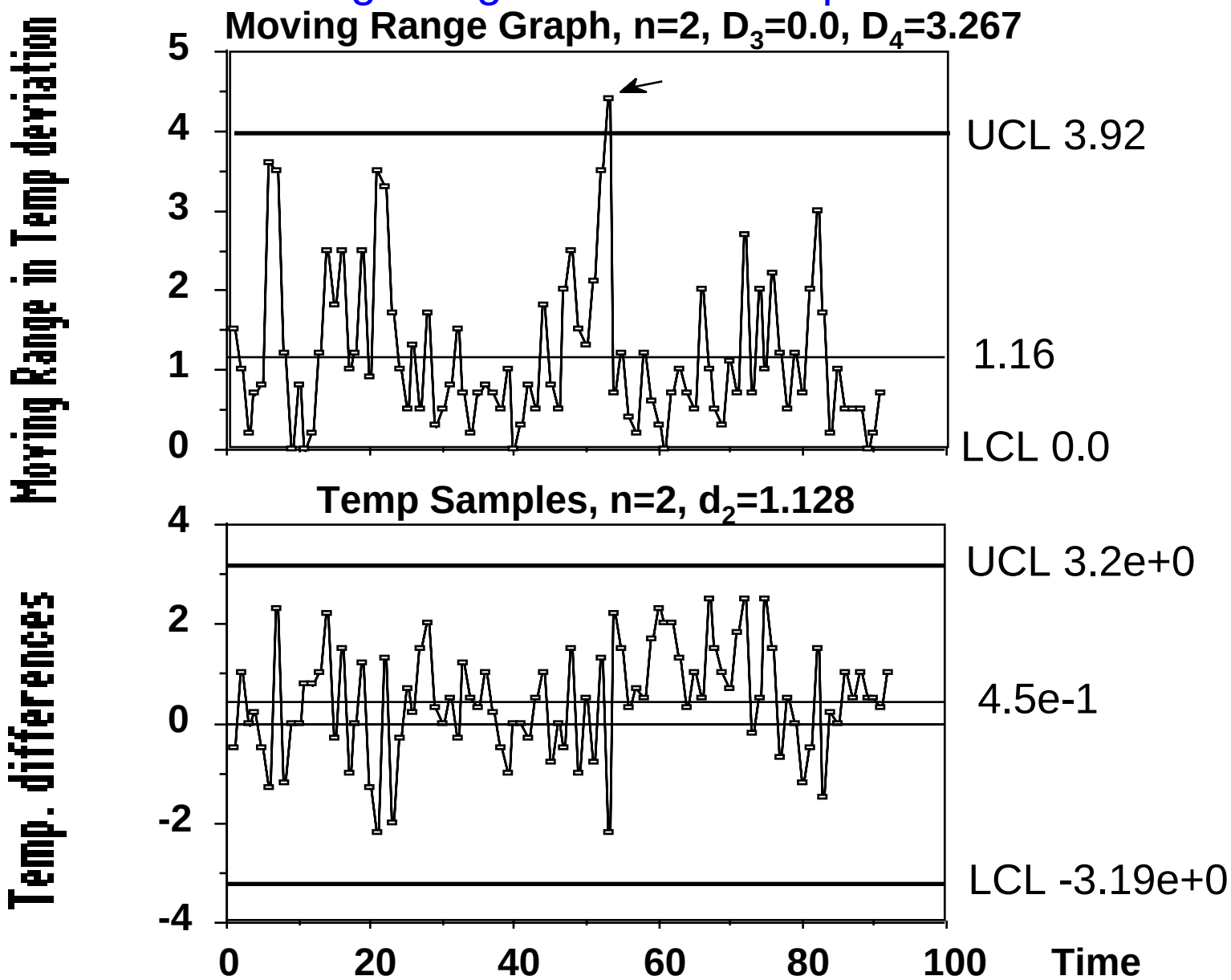


The control limits for $\bar{x}$ can now be calculated from s.



Control Charts for Individual Units

Moving Range chart for Temp. Control:



Questions Most Frequently Asked

Q How many points are needed to establish control limits?

A Typically 20-30 *in-control* points will do.

Q How do I know whether points are *in-control* if limits have not been set?

A *Out-of-control* points should be: a) explained and excluded. b) left in the graph if cannot be explained.

When done, we should have about 1/ARL unexplained out-of-control points (for 3σ control ~1/370 samples.) These points are accepted as false alarms.

Q How often limits must be recalculated?

A Every time it is obvious from the chart that the process has reached a new, acceptable state of statistical control.

Choosing the proper control chart

Variables

- new process or product
- old process with chronic trouble
- diagnostic purposes
- destructive testing
- very tight specs
- decide adjustments

Attributes

- reduce process fallout
- multiple step process evaluation
- cannot measure variables
- historical summation of the process

Individuals

- physically difficult to group
- full testing or auto measurements
- data rate too slow

Summary, so far

- Continuous variables need two (2) charts:
  - chart monitoring the mean
  - chart monitoring the spread
- Rational subgrouping must be done correctly.
- Charts are using Control Limits - Spec limits are very different (conceptually and numerically).
- The concept of process capability brings the two sets of limits together.
- A chart is a hypothesis test. It suffers from type I and type II errors.
- Violating the control limits is just one type of alarm, out of many.

SPC
Calculations for Control Limits

Notation:

UCL	—Upper Control Limit	$\bar{x}$	—Average of Measurements
LCL	—Lower Control Limit	$\bar{\bar{x}}$	—Average of Averages
CL	—Center Line	R	—Range
n	—Sample Size	$\bar{R}$	—Average of Ranges
PCR	—Process Capability Ratio	USL	—Upper Specification Limit
$\hat{\sigma}$	—Process Standard Deviation	LSL	—Lower Specification Limit

Variables Data ($\bar{x}$ and R Control Charts)

$\bar{x}$ Control Chart

$\text{UCL} = \bar{\bar{x}} + A_2\bar{R}$

$\text{LCL} = \bar{\bar{x}} - A_2\bar{R}$

$\text{CL} = \bar{\bar{x}}$

R Control Chart

$\text{UCL} = \bar{R}\,D_4$

$\text{LCL} = \bar{R}\,D_3$

$\text{CL} = \bar{R}$

Capability Study

$C_p = (\text{USL} - \text{LSL})/(6\hat{\sigma})$; where $\hat{\sigma} = \bar{R}/d_2$

n	A_2	D_3	D_4	d_2
2	1.880	0.000	3.267	1.128
3	1.023	0.000	2.574	1.693
4	0.729	0.000	2.282	2.059
5	0.577	0.000	2.114	2.326
6	0.483	0.000	2.004	2.534
7	0.419	0.076	1.924	2.704
8	0.373	0.136	1.864	2.847
9	0.337	0.184	1.816	2.970
10	0.308	0.223	1.777	3.078

Attribute Data (p, np, c, and u Control Charts)

Control Chart Formulas

	p (fraction)	np (number of nonconforming)	c (count of nonconformances)	u (count of nonconformances/unit)
CL	$\bar{p}$	$n\bar{p}$	$\bar{c}$	$\bar{u}$
UCL	$\bar{p} + 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$	$n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$	$\bar{c} + 3\sqrt{\bar{c}}$	$\bar{u} + 3\sqrt{\frac{\bar{u}}{n}}$
LCL	$\bar{p} - 3\sqrt{\frac{\bar{p}(1-\bar{p})}{n}}$	$n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$	$\bar{c} - 3\sqrt{\bar{c}}$	$\bar{u} - 3\sqrt{\frac{\bar{u}}{n}}$
Notes	If n varies, use $\bar{n}$ or individual n_i	n must be a constant	n must be a constant	If n varies, use $\bar{n}$ or individual n_i

Observations in Sample, n	Chart for Averages: Factors for Control Limits A	A_2	A_3	Chart for Standard Deviations: Factors for Center Line c_4	$1/c_4$	Factors for Control Limits B_3	B_4	B_5	B_6	Chart for Ranges: Factors for Center Line d_2	$1/d_2$	Factors for Control Limits d_3	D_1	D_2	D_3	D_4
2	2.121	1.880	2.659	0.7979	1.2533	0	3.267	0	2.606	1.128	0.8865	0.853	0	3.686	0	3.267
3	1.732	1.023	1.954	0.8862	1.1284	0	2.568	0	2.276	1.693	0.5907	0.888	0	4.358	0	2.574
4	1.500	0.729	1.628	0.9213	1.0854	0	2.266	0	2.088	2.059	0.4857	0.880	0	4.698	0	2.282
5	1.342	0.577	1.427	0.9400	1.0638	0	2.089	0	1.964	2.326	0.4299	0.864	0	4.918	0	2.114
6	1.225	0.483	1.287	0.9515	1.0510	0.030	1.970	0.029	1.874	2.534	0.3946	0.848	0	5.078	0	2.004
7	1.134	0.419	1.182	0.9594	1.0423	0.118	1.882	0.113	1.806	2.704	0.3698	0.833	0.204	5.204	0.076	1.924
8	1.061	0.373	1.099	0.9650	1.0363	0.185	1.815	0.179	1.751	2.847	0.3512	0.820	0.388	5.306	0.136	1.864
9	1.000	0.337	1.032	0.9693	1.0317	0.239	1.761	0.232	1.707	2.970	0.3367	0.808	0.547	5.393	0.184	1.816
10	0.949	0.308	0.975	0.9727	1.0281	0.284	1.716	0.276	1.669	3.078	0.3249	0.797	0.687	5.469	0.223	1.777
11	0.905	0.285	0.927	0.9754	1.0252	0.321	1.679	0.313	1.637	3.173	0.3152	0.787	0.811	5.535	0.256	1.744
12	0.866	0.266	0.886	0.9776	1.0229	0.354	1.646	0.346	1.610	3.258	0.3069	0.778	0.922	5.594	0.283	1.717
13	0.832	0.249	0.850	0.9794	1.0210	0.382	1.618	0.374	1.585	3.336	0.2998	0.770	1.025	5.647	0.307	1.693
14	0.802	0.235	0.817	0.9810	1.0194	0.406	1.594	0.399	1.563	3.407	0.2935	0.763	1.118	5.696	0.328	1.672
15	0.775	0.223	0.789	0.9823	1.0180	0.428	1.572	0.421	1.544	3.472	0.2880	0.756	1.203	5.741	0.347	1.653
16	0.750	0.212	0.763	0.9835	1.0168	0.448	1.552	0.440	1.526	3.532	0.2831	0.750	1.282	5.782	0.363	1.637
17	0.728	0.203	0.739	0.9845	1.0157	0.466	1.534	0.458	1.511	3.588	0.2787	0.744	1.356	5.820	0.378	1.622
18	0.707	0.194	0.718	0.9854	1.0148	0.482	1.518	0.475	1.496	3.640	0.2747	0.739	1.424	5.856	0.391	1.608
19	0.688	0.187	0.698	0.9862	1.0140	0.497	1.503	0.490	1.483	3.689	0.2711	0.734	1.487	5.891	0.403	1.597
20	0.671	0.180	0.680	0.9869	1.0133	0.510	1.490	0.504	1.470	3.735	0.2677	0.729	1.549	5.921	0.415	1.585
21	0.655	0.173	0.663	0.9876	1.0126	0.523	1.477	0.516	1.459	3.778	0.2647	0.724	1.605	5.951	0.425	1.575
22	0.640	0.167	0.647	0.9882	1.0119	0.534	1.466	0.528	1.448	3.819	0.2618	0.720	1.659	5.979	0.434	1.566
23	0.626	0.162	0.633	0.9887	1.0114	0.545	1.455	0.539	1.438	3.858	0.2592	0.716	1.710	6.006	0.443	1.557
24	0.612	0.157	0.619	0.9892	1.0109	0.555	1.445	0.549	1.429	3.895	0.2567	0.712	1.759	6.031	0.451	1.548
25	0.600	0.153	0.606	0.9896	1.0105	0.565	1.435	0.559	1.420	3.931	0.2544	0.708	1.806	6.056	0.459	1.541

For $n > 25$.

$$A = \frac{3}{\sqrt{n}} \qquad A_3 = \frac{3}{c_4\sqrt{n}} \qquad c_4 \cong \frac{4(n-1)}{4n-3}$$

$$B_3 = 1 - \frac{3}{c_4\sqrt{2(n-1)}} \qquad B_4 = 1 + \frac{3}{c_4\sqrt{2(n-1)}}$$

$$B_5 = c_4 - \frac{3}{\sqrt{2(n-1)}} \qquad B_6 = c_4 + \frac{3}{\sqrt{2(n-1)}}$$

Intro to SQC 5th Edition, D. Montgomery