

EE247

Lecture 8

- Continuous-time filter design considerations
 - Active bandpass filter design (continued)
 - Gm-C bandpass filter using simple diff. pair (continued)
 - Various Gm-C filter implementations
- Performance comparison of various continuous-time filter topologies
- Switched-capacitor filters
 - Emulating a resistor by using a switched capacitor
 - Tradeoffs in choosing sampling rate
 - Effect of sample and hold
 - Switched-capacitor network electronic noise

Summary

Lecture 7

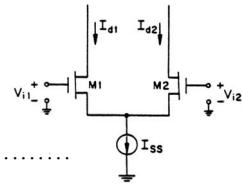
- Automatic on-chip filter tuning (continued from previous lecture)
 - Continuous tuning (continued)
 - DC tuning of resistive timing element
 - Periodic digitally assisted filter tuning
 - Systems where filter is followed by ADC & DSP, existing hardware can be used to periodically update filter freq. response
- Continuous-time filter design considerations
 - Monolithic highpass filters
 - Active bandpass filter design
 - Lowpass to bandpass transformation
 - Example: 6th order bandpass filter
 - Gm-C bandpass filter using simple diff. pair

Linearity of the Source-Coupled Pair CMOS Gm-Cell

$$IM3 \approx \frac{3a_3}{4a_1} \hat{v}_i^2 + \frac{25a_5}{8a_1} \hat{v}_i^4 \dots \dots \dots$$

Substituting for a_1, a_3, \dots

$$IM3 \approx \frac{3}{32} \left(\frac{\hat{v}_i}{(V_{GS} - V_{th})} \right)^2 + \frac{25}{1024} \left(\frac{\hat{v}_i}{(V_{GS} - V_{th})} \right)^4 \dots \dots \dots$$



$$\hat{v}_{i\max} \approx 4(V_{GS} - V_{th}) \times \sqrt{\frac{2}{3} \times IM3}$$

$$IM3 = 1\% \text{ & } (V_{GS} - V_{th}) = 1V \Rightarrow \hat{v}_{in}^{rms} \approx 230mV$$

- Note that max. signal handling capability function of gate-overdrive voltage

Dynamic Range for Source-Coupled Pair Based Filter

$$IM3 = 1\% \text{ & } (V_{GS} - V_{th}) = 1V \Rightarrow V_{in}^{rms} \approx 230mV$$

- Minimum detectable signal determined by total noise voltage
- It can be shown for the **6th** order Butterworth bandpass filter fundamental noise contribution is given by:

$$\sqrt{v_o^2} \approx \sqrt{3Q \frac{kT}{C_{intg}}}$$

$$\text{Assuming } Q = 10 \quad C_{intg} = 5pF$$

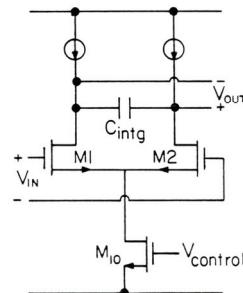
$$v_{noise}^{rms} \approx 160\mu V$$

$$\text{since } v_{max}^{rms} = 230mV$$

$$\text{Dynamic Range} = 20 \log \frac{230 \times 10^{-3}}{160 \times 10^{-6}} \approx 63dB$$

Simplest Form of CMOS Gm-Cell

- Pros
 - Capable of very high frequency performance (highest?)
 - Simple design
- Cons
 - Tuning affects max. signal handling capability (can overcome)
 - Limited linearity (possible to improve)
 - Tuning affects power dissipation



Ref: H. Khorramabadi and P.R. Gray, "High Frequency CMOS continuous-time filters," *IEEE Journal of Solid-State Circuits*, Vol.-SC-19, No. 6, pp.939-948, Dec. 1984.

Gm-Cell Source-Coupled Pair with Degeneration

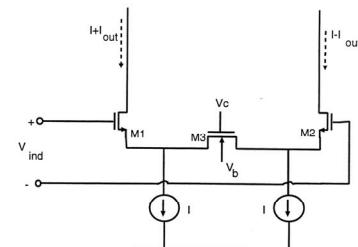
$$I_d = \frac{\mu C_{ox} W}{2 L} \left[2(V_{gs} - V_{th})V_{ds} - V_{ds}^2 \right]$$

$$g_{ds} = \frac{\partial I_d}{\partial V_{ds}} \approx \mu C_{ox} \frac{W}{L} (V_{gs} - V_{th}) \Big|_{V_{ds} \text{ small}}$$

$$g_{eff} = \frac{I}{\frac{1}{g_{ds}^{M3}} + \frac{2}{g_m^{M1,2}}}$$

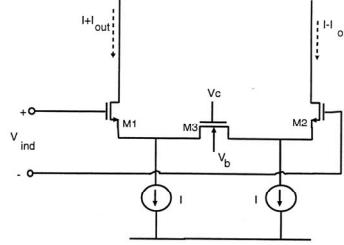
for $g_m^{M1,2} \gg g_{ds}^{M3}$

$$g_{eff} \approx g_{ds}^{M3}$$



M3 operating in triode mode → source degeneration → determines overall gm
Provides tuning through varing Vc (DC voltage source)

Gm-Cell Source-Coupled Pair with Degeneration



- Pros

- Moderate linearity
- Continuous tuning provided by varying V_c
- Tuning does not affect power dissipation

- Cons

- Extra poles associated with the source of M1,2,3
→ Low frequency applications only

Ref: Y. Tsividis, Z. Czarnul and S.C. Fang, "MOS transconductors and integrators with high linearity," *Electronics Letters*, vol. 22, pp. 245-246, Feb. 27, 1986

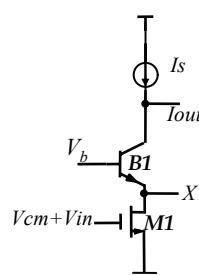
BiCMOS Gm-Cell Example

- MOSFET operating in triode mode (M1):

$$I_d = \frac{\mu C_{ox} W}{2 L} [2(V_{gs} - V_{th})V_{ds} - V_{ds}^2]$$

$$g_m^{M1} = \frac{\partial I_d}{\partial V_{gs}} = \mu C_{ox} \frac{W}{L} V_{ds}$$

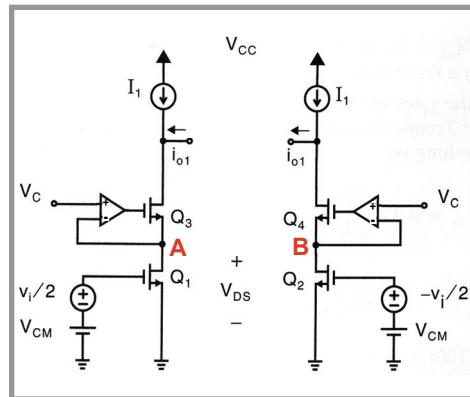
- Note that if V_{ds} is kept constant → g_m stays constant
- Linearity performance → keep g_m constant as V_{in} varies → function of how constant V_{ds}^{M1} can be held
 - Need to minimize gain @ node X
- Since for a given current, g_m of BJT is larger compared to MOS- preferable to use BJT
- Extra pole at node X could limit max. freq.



Varying V_b changes V_{ds}^{M1}
→ Changes g_m^{M1}
→ adjustable overall stage g_m

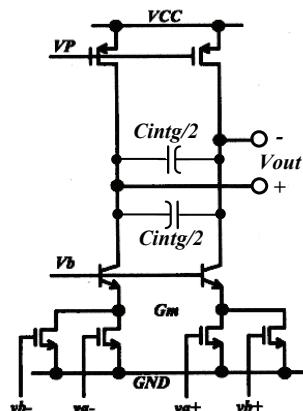
Alternative Fully CMOS Gm-Cell Example

- BJT replaced by a MOS transistor with boosted g_m
- Lower frequency of operation compared to the BiCMOS version due to more parasitic capacitance at nodes A & B

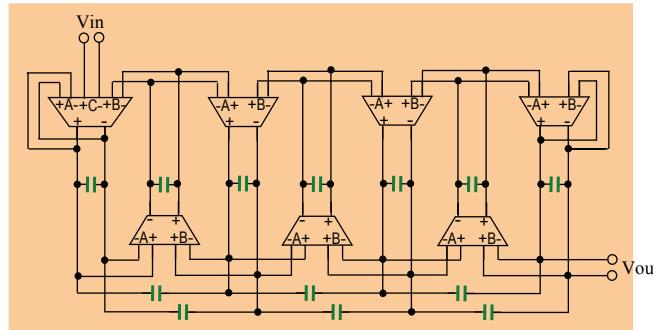


BiCMOS Gm-C Integrator

- Differential- needs common-mode feedback ckt
- Freq.tuned by varying V_b
- Design tradeoffs:
 - Extra poles at the input device drain junctions
 - Input devices have to be small to minimize parasitic poles
 - Results in high input-referred offset voltage → could drive ckt into non-linear region
 - Small devices → high 1/f noise



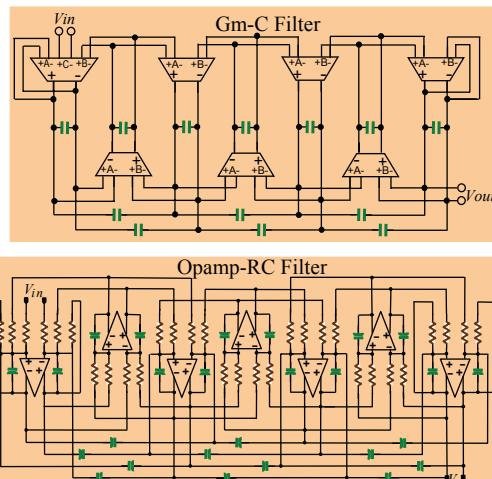
7th Order Elliptic Gm-C LPF For CDMA RX Baseband Application



- Gm-Cell in previous page used to build a 7th order elliptic filter for CDMA baseband applications (650kHz corner frequency)
- In-band dynamic range of <50dB achieved

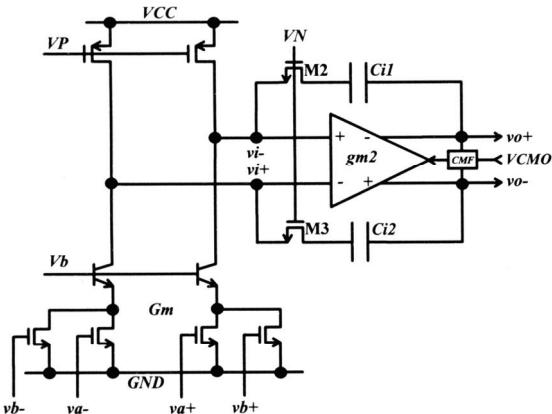
Comparison of 7th Order Gm-C versus Opamp-RC LPF

- Gm-C filter requires 4 times less intg. cap. area compared to Opamp-RC
→ For low-noise applications where filter area is dominated by C_s , could make a significant difference in the total area
- Opamp-RC linearity superior compared to Gm-C
- Power dissipation tends to be lower for Gm-C since OTA load is C and thus no need for buffering



BiCMOS Gm-OTA-C Integrator

- Used to build filter for disk-drive applications
- Since high frequency of operation, time-constant sensitivity to parasitic caps significant.
- Opamp used
- M2 & M3 added → provides phase lead to compensate for phase lag due to amp extra poles



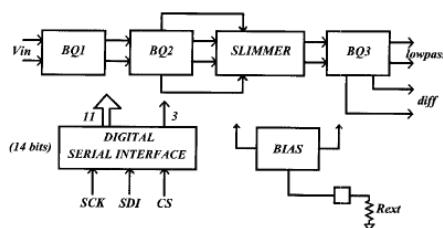
Ref: C. Laber and P. Gray, "A 20MHz 6th Order BiCMOS Parasitic Insensitive Continuous-time Filter & Second Order Equalizer Optimized for Disk Drive Read Channels," *IEEE Journal of Solid State Circuits*, Vol. 28, pp. 462-470, April 1993.

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Filters: Continuous-Time & Switched-Capacitor

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6th Order BiCMOS Continuous-time Filter & Second Order Equalizer for Disk Drive Read Channels



- Gm-C-opamp of the previous page used to build a 6th order filter for Disk Drive
- Filter consists of cascade of 3 biquads with max. Q of 2 each
- Tuning → DC tuning of gm-cells (Lect. 7 page 3) + trimming of Cs
- Performance in the order of 40dB SNDR achieved for up to 20MHz corner frequency

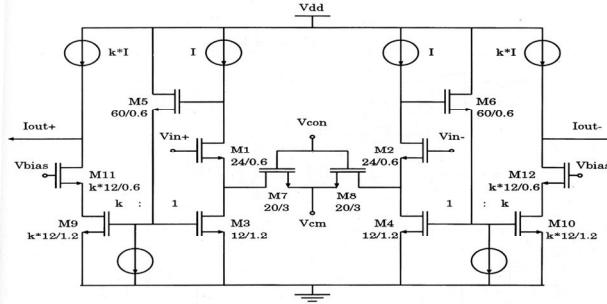
Ref: C. Laber and P. Gray, "A 20MHz 6th Order BiCMOS Parasitic Insensitive Continuous-time Filter & Second Order Equalizer Optimized for Disk Drive Read Channels," *IEEE Journal of Solid State Circuits*, Vol. 28, pp. 462-470, April 1993.

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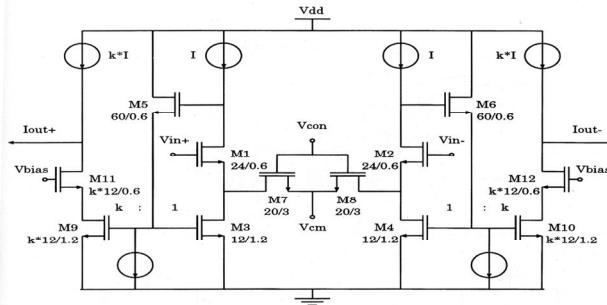
Gm-Cell Source-Coupled Pair with Degeneration



- Gm-cell intended for low Q disk drive filter
- $M7,8$ operating in triode mode provide source degeneration for $M1,2$
→ determine the overall g_m of the cell

Ref: I.Mehr and D.R.Welland, "A CMOS Continuous-Time Gm-C Filter for PRML Read Channel Applications at 150 Mb/s and Beyond", IEEE Journal of Solid-State Circuits, April 1997, Vol.32, No.4, pp. 499-513.

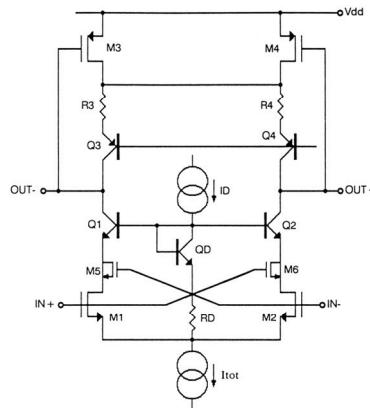
Gm-Cell Source-Coupled Pair with Degeneration



- Feedback provided by M5,6 maintains the gate-source voltage of M1,2 constant by forcing their current to be constant → helps deliver V_{in} across M7,8 with good linearity
- Current mirrored to the output via M9,10 with a factor of k → overall gm scaled by k
- Performance level of about 50dB SNDR at f_{corner} of 25MHz achieved

BiCMOS Gm-C Integrator

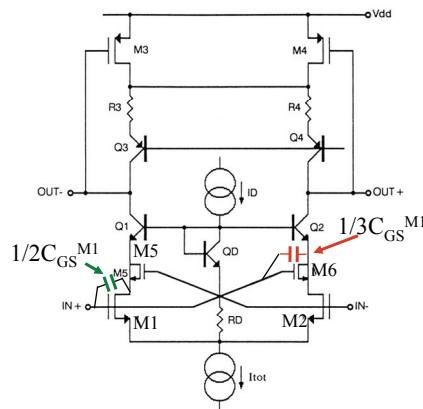
- Needs higher supply voltage compared to the previous design since quite a few devices are stacked vertically
- M1,2 → triode mode
- Q1,2 → hold V_{ds} of M1,2 constant
- Current ID used to tune filter critical frequency by varying V_{ds} of M1,2 and thus controlling gm of M1,2
- M3, M4 operate in triode mode and added to provide common-mode feedback



Ref: R. Alini, A. Baschirotto, and R. Castello, "Tunable BiCMOS Continuous-Time Filter for High-Frequency Applications," *IEEE Journal of Solid State Circuits*, Vol. 27, No. 12, pp. 1905-1915, Dec. 1992.

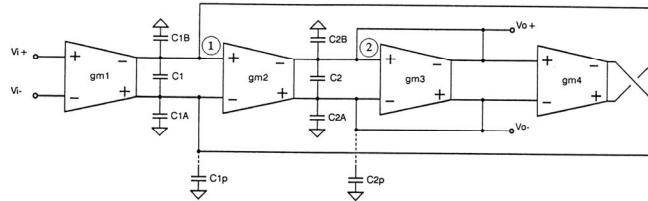
BiCMOS Gm-C Integrator

- M5 & M6 configured as capacitors- added to compensate for RHP zero due to C_{gd} of M1,2 (moves it to LHP) size of M5,6 → $1/3$ of M1,2



Ref: R. Alini, A. Baschirotto, and R. Castello, "Tunable BiCMOS Continuous-Time Filter for High-Frequency Applications," *IEEE Journal of Solid State Circuits*, Vol. 27, No. 12, pp. 1905-1915, Dec. 1992.

BiCMOS Gm-C Filter For Disk-Drive Application



- Using the integrators shown in the previous page
- Biquad filter for disk drives
- $gm1=gm2=gm4=2gm3$
- $Q=2$
- Tunable from 8MHz to 32MHz

Ref: R. Alini, A. Baschirotto, and R. Castello, "Tunable BiCMOS Continuous-Time Filter for High-Frequency Applications," *IEEE Journal of Solid State Circuits*, Vol. 27, No. 12, pp. 1905-1915, Dec. 1992.

Summary Continuous-Time Filters

- Opamp RC filters
 - Good linearity → High dynamic range (*60-90dB*)
 - Only discrete tuning possible
 - Medium usable signal bandwidth (*<10MHz*)
- Opamp MOSFET-C
 - Linearity compromised (typical dynamic range *40-60dB*)
 - Continuous tuning possible
 - Low usable signal bandwidth (*<5MHz*)
- Opamp MOSFET-RC
 - Improved linearity compared to Opamp MOSFET-C (D.R. *50-90dB*)
 - Continuous tuning possible
 - Low usable signal bandwidth (*<5MHz*)
- Gm-C
 - Highest frequency performance -at least an order of magnitude higher compared to other integrator-based active filters (*<100MHz*)
 - Typically, dynamic range not as high as Opamp RC but better than Opamp MOSFET-C (*40-70dB*)

Switched-Capacitor Filters

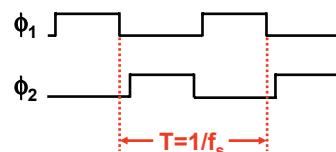
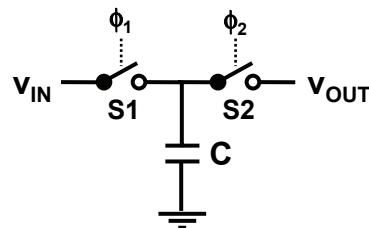
- SC filters are sampled-data type circuits operating with continuous signal amplitude & quantized time
- First produce including switched-capacitor filters → Intel 2912 Voice-band CODEC
- Other than filters, S.C. circuits are mostly used in oversampled data converters
- Almost all research resulting in S.C. filter technology was performed at UC Berkeley

Switched-Capacitor Filters

- Emulating resistor via switched-capacitor network
- Switched-capacitor 1st order filter
- Switch-capacitor filter considerations:
 - Issue of aliasing and how to prevent aliasing
 - Tradeoffs in choice of sampling rate
 - Effect of sample and hold
 - Switched-capacitor filter electronic noise

Switched-Capacitor Resistor

- Capacitor C is the “switched capacitor”
- Non-overlapping clocks ϕ_1 and ϕ_2 control switches S1 and S2, respectively
- v_{IN} is sampled at the falling edge of ϕ_1
 - Sampling frequency f_s
- Next, ϕ_2 rises and the voltage across C is transferred to v_{OUT}
- Why does this behave as a resistor?

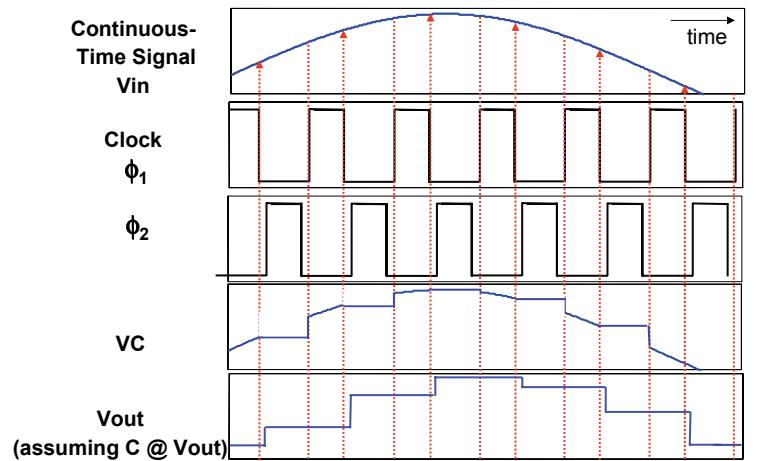


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Switched-Capacitor Resistor Waveforms



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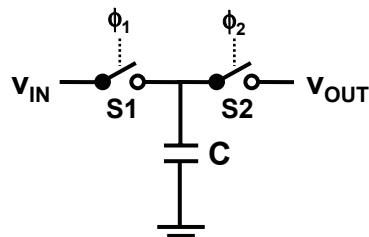
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Switched-Capacitor Resistors

- Charge transferred from v_{IN} to v_{OUT} during each clock cycle is:

$$Q = C(v_{IN} - v_{OUT})$$

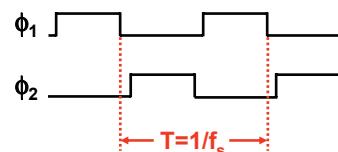


- Average current flowing from v_{IN} to v_{OUT} is:

$$i = Q/t = Q \cdot f_s$$

Substituting for Q :

$$i = f_s C(v_{IN} - v_{OUT})$$

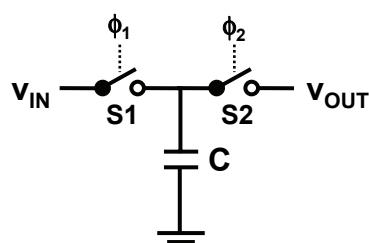


Switched-Capacitor Resistors

$$i = f_s C(v_{IN} - v_{OUT})$$

With the current through the switched-capacitor resistor proportional to the voltage across it, the equivalent "switched capacitor resistance" is:

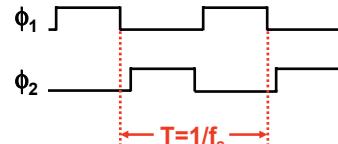
$$R_{eq} = \frac{V_{IN} - V_{OUT}}{i} = \frac{1}{f_s C}$$



Example:

$$\begin{aligned} f_s &= 100\text{KHz}, C = 0.1\text{pF} \\ \rightarrow R_{eq} &= 100\text{Mega}\Omega \end{aligned}$$

Note: Can build large time-constant in small area

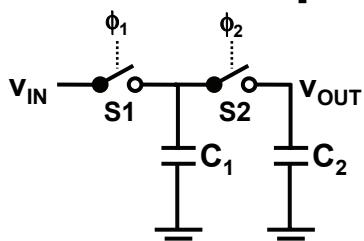
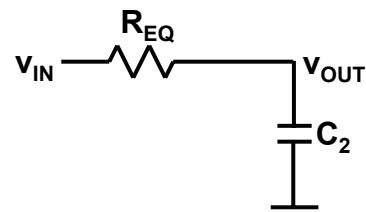


Switched-Capacitor Filter

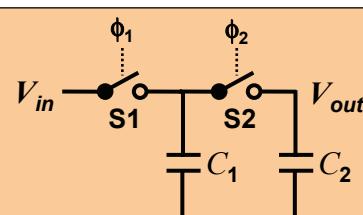
- Let's build a "switched- capacitor " filter ...
- Start with a simple RC LPF
- Replace the physical resistor by an equivalent switched-capacitor resistor
- 3-dB bandwidth:

$$\omega_{-3dB} = \frac{1}{R_{eq}C_2} = f_s \times \frac{C_1}{C_2}$$

$$f_{-3dB} = \frac{1}{2\pi} f_s \times \frac{C_1}{C_2}$$

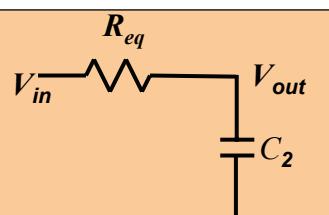


Switched-Capacitor Filter Advantage versus Continuous-Time Filter



$$f_{-3dB} = \frac{1}{2\pi} f_s \times \frac{C_1}{C_2}$$

- Corner freq. proportional to:
System clock (accurate to few ppm)
C ratio accurate $\rightarrow < 0.1\%$



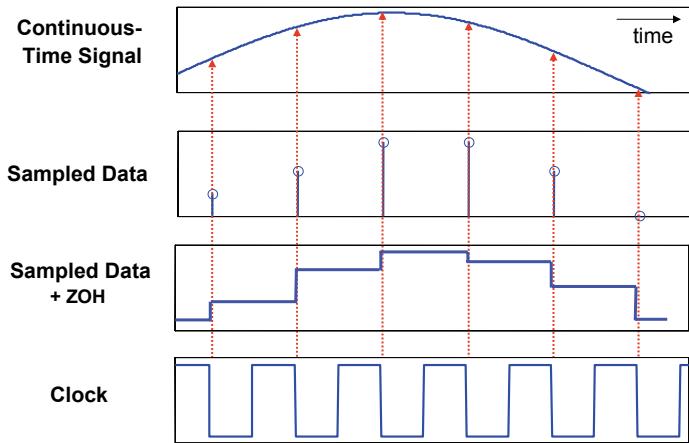
$$f_{-3dB} = \frac{1}{2\pi} \times \frac{1}{R_{eq}C_2}$$

- Corner freq. proportional to:
Absolute value of Rs & Cs
Poor accuracy $\rightarrow 20$ to 50%

>Main advantage of SC filters \rightarrow inherent critical frequency accuracy

Typical Sampling Process

Continuous-Time(CT) \Rightarrow Sampled Data (SD)



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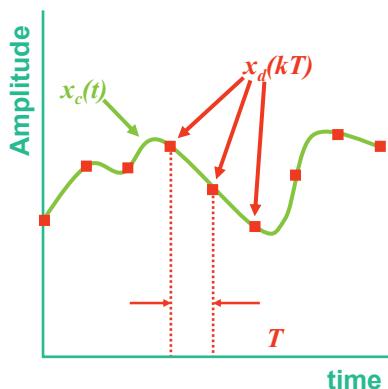
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Uniform Sampling

Nomenclature:

Continuous time signal	$x_c(t)$
Sampling interval	T
Sampling frequency	$f_s = 1/T$
Sampled signal	$x_d(kT) = x(k)$

- Samples are the waveform values at kT instances and undefined in between
- Problem: Multiple continuous time signals can yield exactly the same discrete time signal
- Let's examine samples taken at $1\mu s$ intervals of several sinusoidal waveforms ...

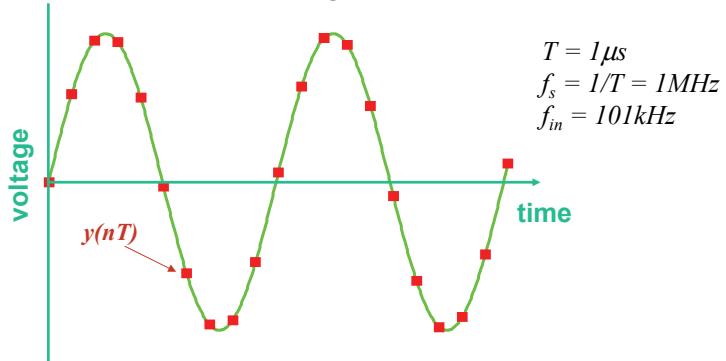


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Sampling Sine Waves

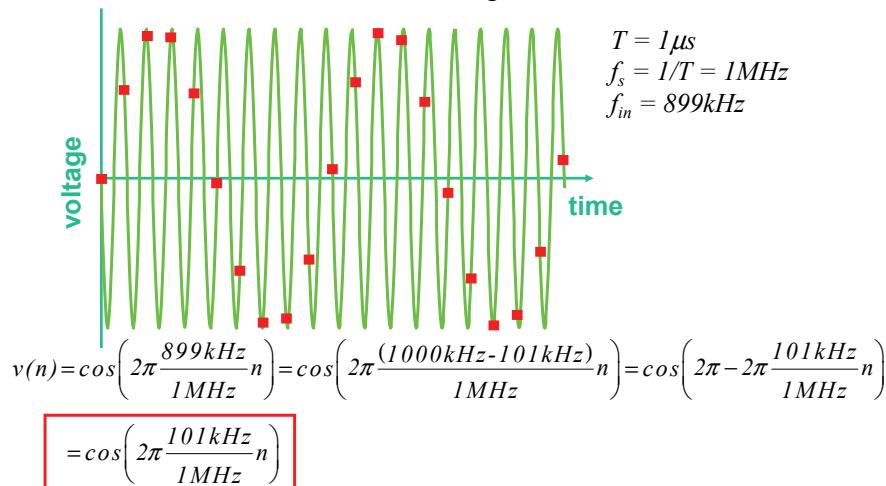


$$v(t) = \cos(2\pi f_{in} t)$$

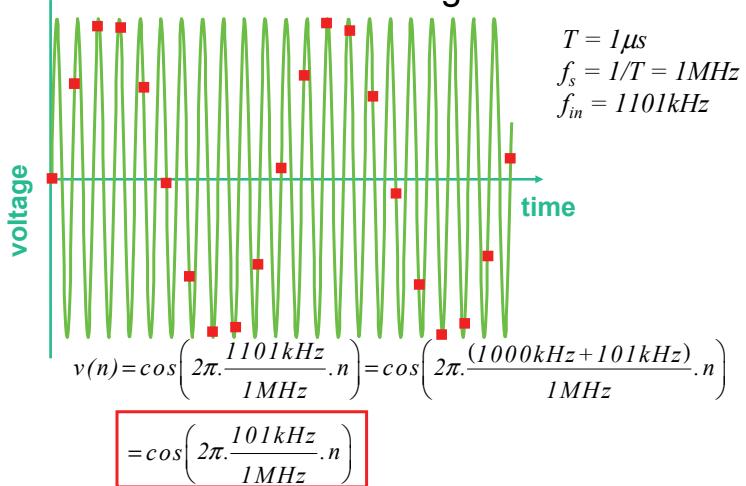
Sampled-data domain $\rightarrow t \rightarrow n.T$ or $t \rightarrow n/f_s$ (n integer)

$$v(n) = \cos\left(2\pi \frac{f_{in}}{f_s} n\right) = \cos\left(2\pi \frac{101kHz}{1MHz} n\right)$$

Sampling Sine Waves Aliasing



Sampling Sine Waves Aliasing



Sampling Sine Waves

Problem:

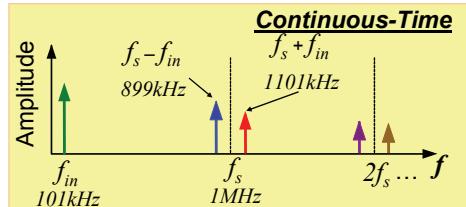
Sampled data domain → identical samples for:

$$\begin{aligned}
 v(t) &= \cos[2\pi f_{in}t] \\
 v(t) &= \cos[2\pi(f_{in}+n.f_s)t] \\
 v(t) &= \cos[2\pi(f_{in}-n.f_s)t] \\
 &\ast \text{ (n-integer)}
 \end{aligned}$$

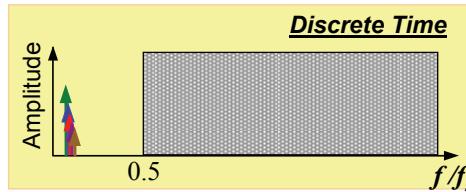
→ Multiple continuous time signals can yield exactly the same discrete time signal

Sampling Sine Waves Frequency Spectrum

Signal scenario
before sampling



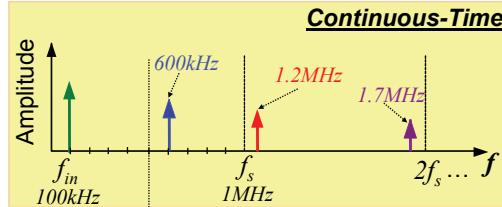
Signal scenario
after sampling



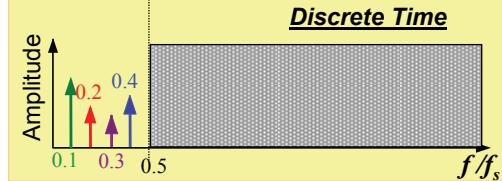
Key point: Signals @ $n f_s \pm f_{max_signal}$ fold back into band of interest → Aliasing

Sampling Sine Waves Frequency Spectrum

Signal scenario
before sampling



Signal scenario
after sampling



Key point: Signals @ $n f_s \pm f_{max_signal}$ fold back into band of interest → Aliasing

Aliasing

- Multiple continuous time signals can produce identical series of samples
- The folding back of signals from $nf_s \pm f_{sig}$ (n integer) down to f_{fin} is called aliasing
 - Sampling theorem: $f_s > 2f_{max_Signal}$
- If aliasing occurs, no signal processing operation downstream of the sampling process can recover the original continuous time signal

How to Avoid Aliasing?

- Must obey sampling theorem:

$$f_{max\text{-}signal} < f_s / 2$$

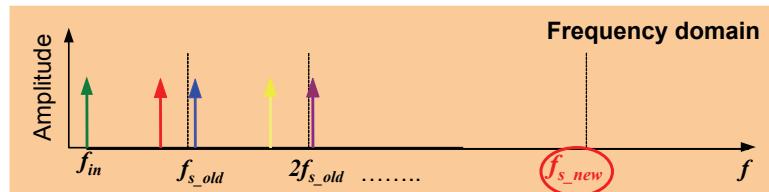
*Note:

Minimum sampling rate of $f_s = 2f_{max\text{-}Signal}$ is called Nyquist rate

- Two possibilities:

1. Sample fast enough to cover all spectral components, including "parasitic" ones outside band of interest
2. Limit $f_{max\text{-}Signal}$ through filtering → attenuate out-of-band components prior to sampling

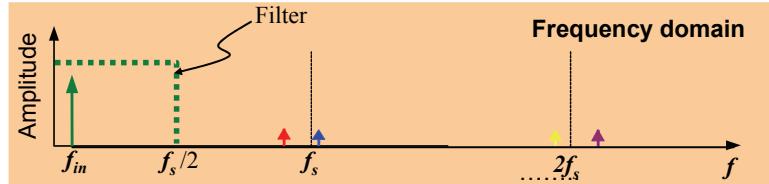
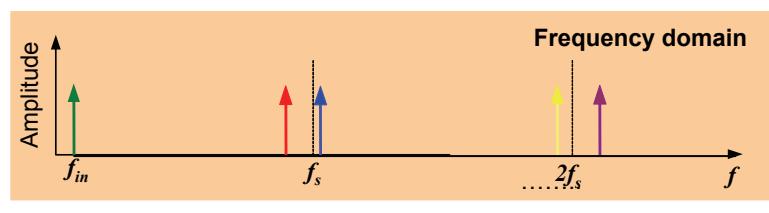
How to Avoid Aliasing? 1-Sample Fast



Push sampling frequency to $\times 2$ of the highest frequency signal to cover all unwanted signals as well as wanted signals

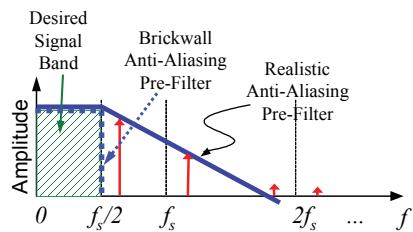
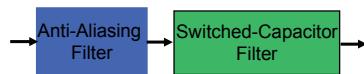
→ In vast majority of cases not practical

How to Avoid Aliasing? 2-Filter Out-of-Band Signal Prior to Sampling



Pre-filter signal to eliminate/attenuate signals above $f_s/2$ - then sample

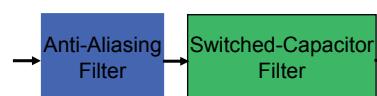
Anti-Aliasing Filter Considerations



Case1- $B = f_{sig}^{max} = f_s/2$

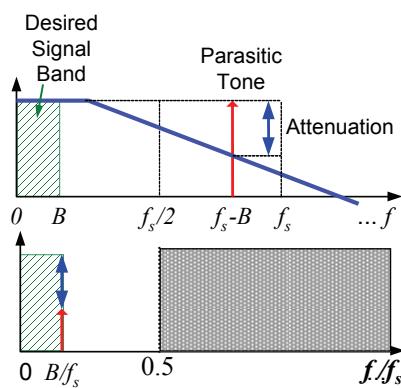
- Non-practical since an extremely high order anti-aliasing filter (close to an ideal brickwall filter) is required
- Practical anti-aliasing filter → Non-zero filter "transition band"
- In order to make this work, we need to sample much faster than 2x the signal bandwidth
→ "Oversampling"

Practical Anti-Aliasing Filter

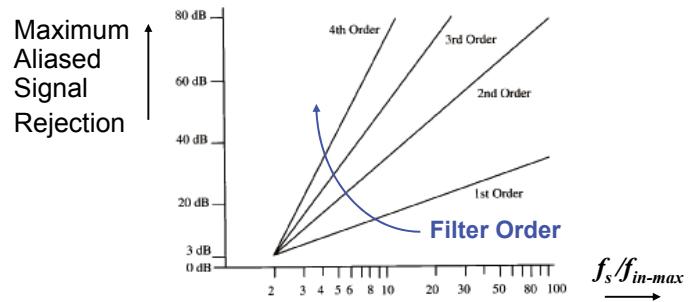


Case2 - $B = f_{max_Signal} \ll f_s/2$

- More practical anti-aliasing filter
- Preferable to have an anti-aliasing filter with:
 - The lowest order possible
 - No frequency tuning required (if frequency tuning is required then why use switched-capacitor filter, just use the prefilter!?)



Tradeoff Oversampling Ratio versus Anti-Aliasing Filter Order

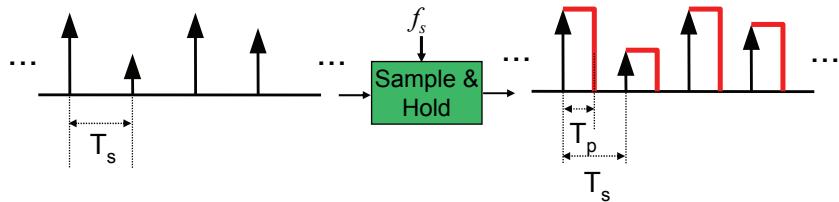


* Assumption → anti-aliasing filter is Butterworth type (not a necessary requirement)

→ Tradeoff: Sampling frequency versus anti-aliasing filter order

Ref: R. v. d. Plassche, *CMOS Integrated Analog-to-Digital and Digital-to-Analog Converters*, 2nd ed., Kluwer publishing, 2003, p.41

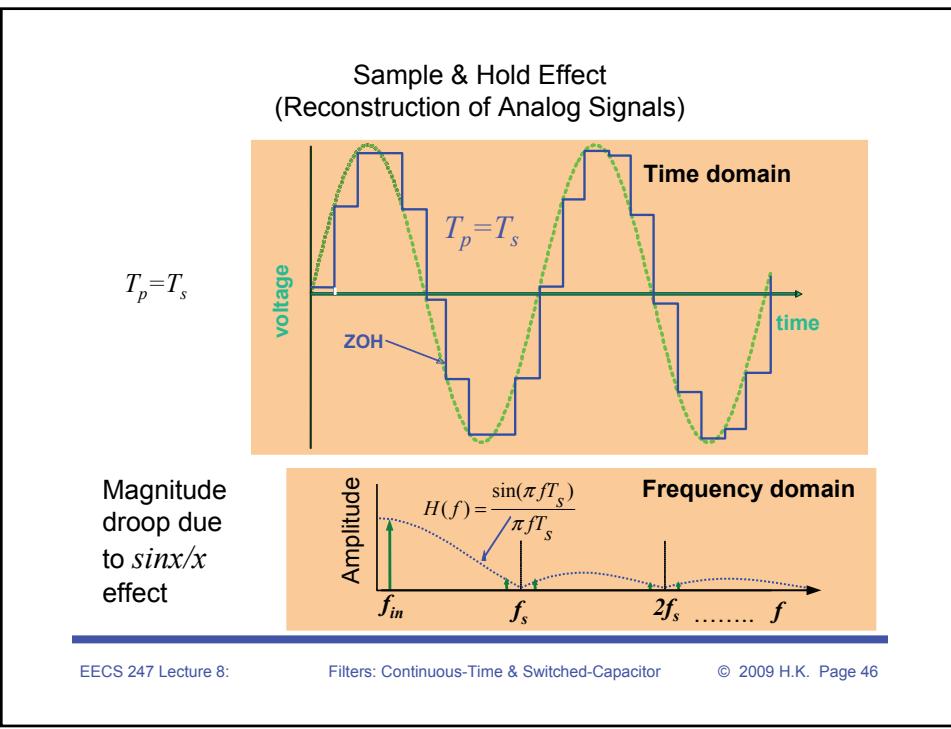
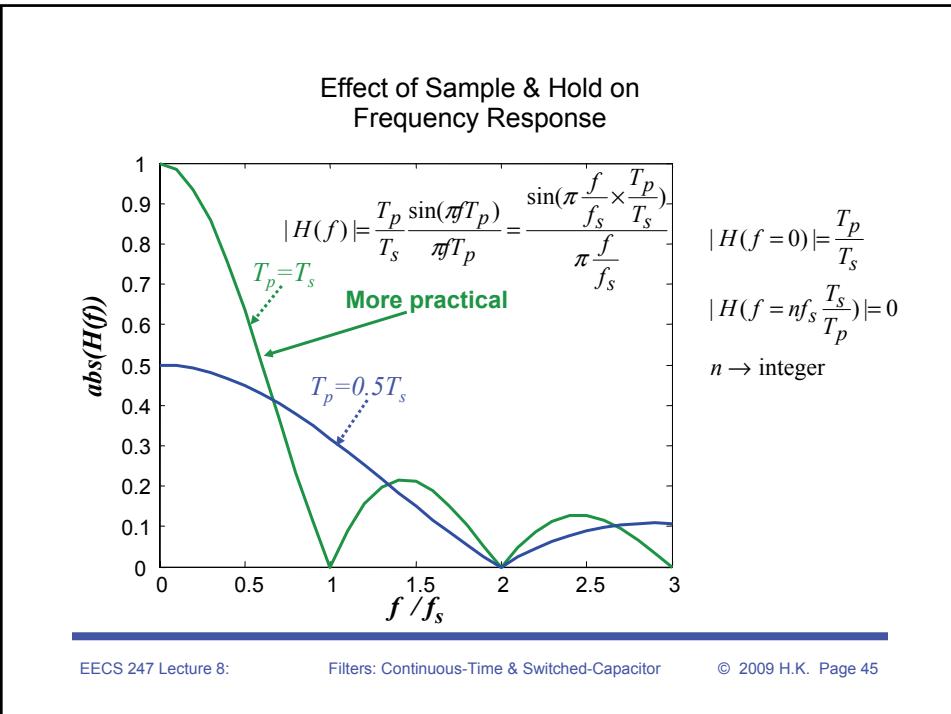
Effect of Sample & Hold



- Using the Fourier transform of a rectangular impulse:

$$|H(f)| = \frac{T_p}{T_s} \frac{\sin(\pi f T_p)}{\pi f T_p} \rightarrow \frac{\sin x}{x} \text{ shape}$$

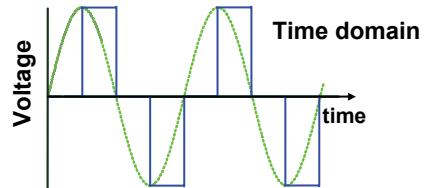
In literature also called Sinc function



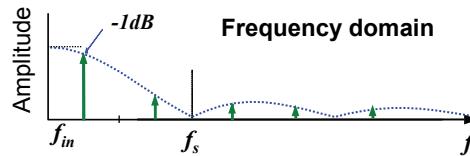
Sample & Hold Effect (Reconstruction of Analog Signals)

Magnitude droop
due to $\sin x/x$
effect:

$$\text{Case 1)} f_{sig} = f_s / 4$$



Droop = -1dB



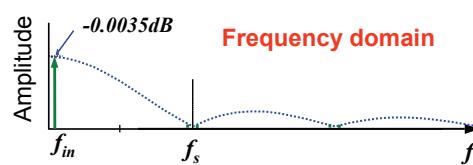
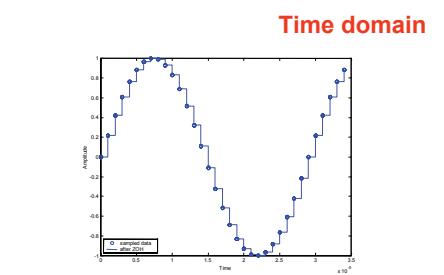
Sample & Hold Effect (Reconstruction of Analog Signals)

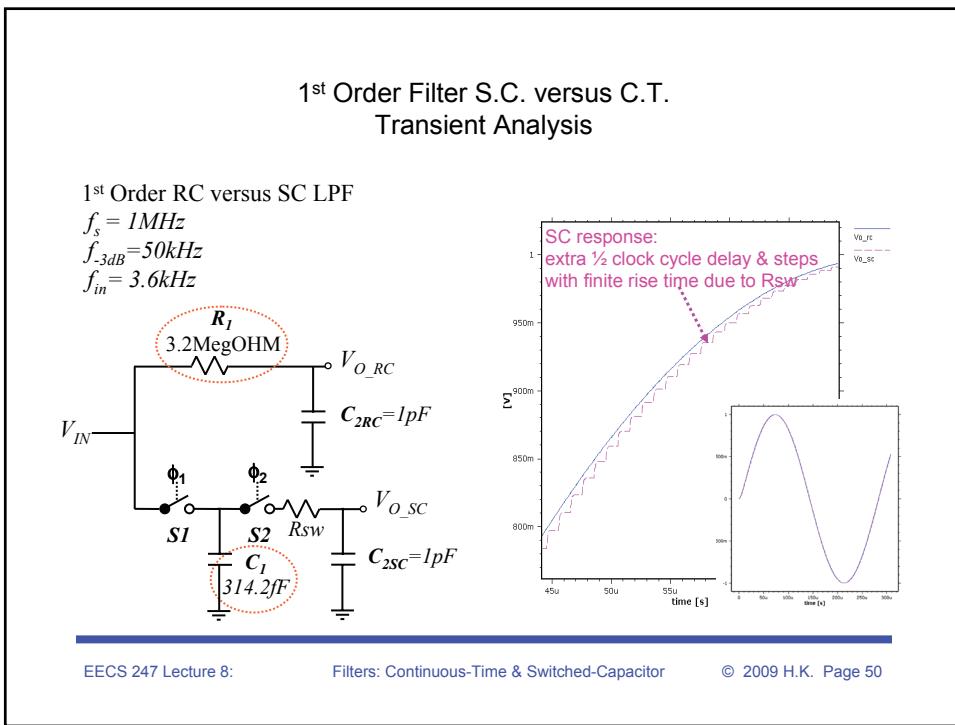
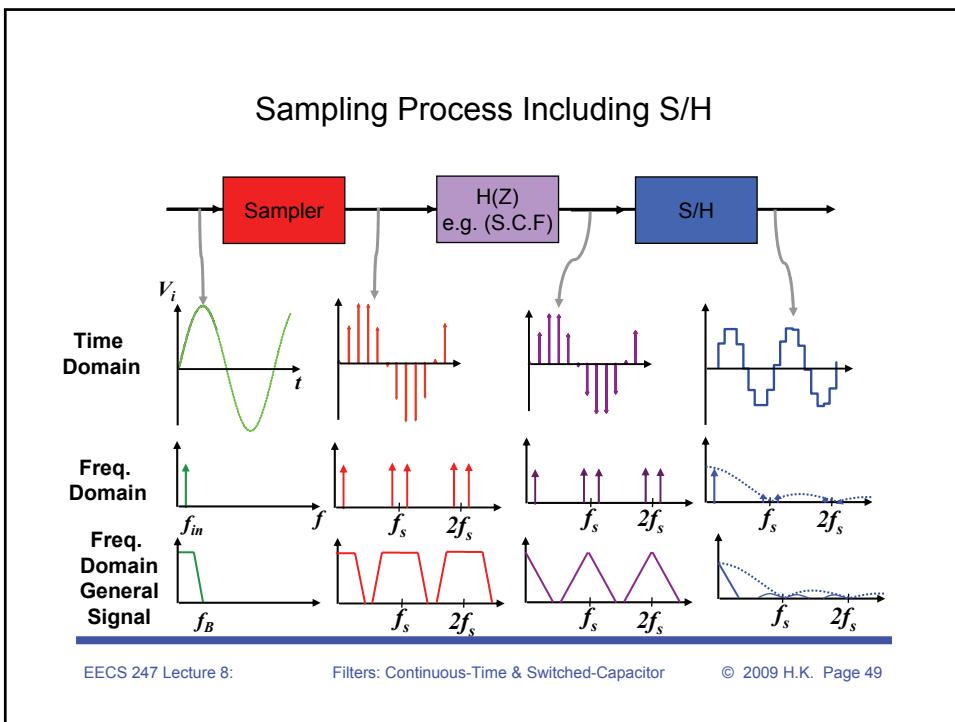
Magnitude droop due to
 $\sin x/x$ effect:

$$\text{Case 2)} f_{sig} = f_s / 32$$

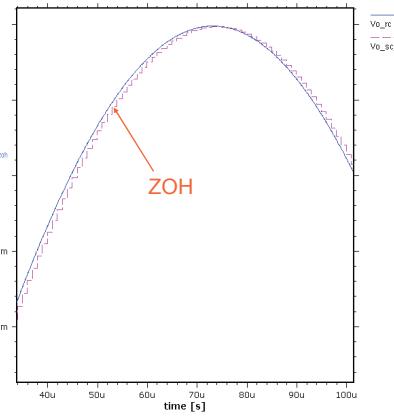
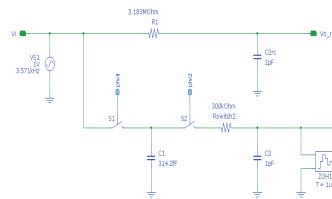
Droop = -0.0035dB

- **Insignificant droop**
→ High oversampling ratio desirable





1st Order Filter Transient Analysis



- ZOH: Emulates an ideal S/H → pick signal after settling (usually at end of clock phase)
- Adds delay and $\sin(x)/x$ shaping
- When in doubt, use a ZOH in periodic ac simulations

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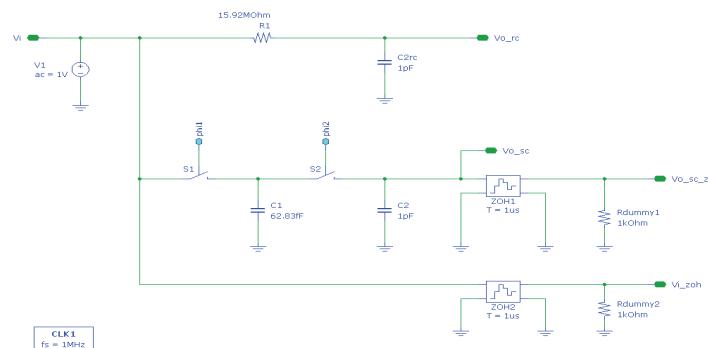
Periodic AC Analysis

1st Order RC / SC LPF

$f_s = 1\text{MHz}$
 $f_c = 50\text{kHz}$
 $f_x = 3.571\text{kHz}$

Periodic AC Analysis PAC1
log sweep from 1 to 31M (1001 steps)

Netlist
ahd_include "zoh.def"

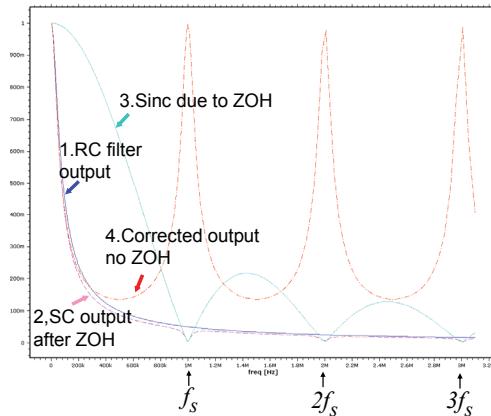


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1st Order Filter Magnitude Response



- 1. RC filter output
- 2. SC output after ZOH
- 3. Output after single ZOH
- 4. Output w/o effect of ZOH
 - (2) over (3)
 - Repeats filter shape around nf_s
 - Identical to RC for $f \ll f_s/2$

Periodic AC Analysis

- SPICE frequency analysis
 - ac linear, **time-invariant** circuits
 - pac linear, **time-variant** circuits
- SpectreRF statements


```
V1 ( Vi 0 ) vsource type=dc dc=0 mag=1
      pacmag=1
      PSS1 pss period=1u errpreset=conservative
      PAC1 pac start=1 stop=1M lin=1001
```
- Output
 - Divide results by $\text{sinc}(f/f_s)$ to correct for ZOH distortion

Spectre Circuit File

```
rc_pac
simulator lang=spectre
ahdl_include "zoh.def"

S1 ( Vi c1 phil 0 ) relay ropen=100G rclosed=1 vt1=-500m vt2=500m
S2 ( c1 Vo_sc phi2 0 ) relay ropen=100G rclosed=1 vt1=-500m vt2=500m
C1 ( c1 0 ) capacitor c=314.159f
C2 ( Vo_sc 0 ) capacitor c=1p
R1 ( Vi Vo_rc ) resistor r=3.1831M
C2rc ( Vo_rc 0 ) capacitor c=1p
CLK1_Vphil ( phil 0 ) vsource type=pulse val0=-1 val1=1 period=1u
width=450n delay=50n rise=10n fall=10n
CLK1_Vphi2 ( phi2 0 ) vsource type=pulse val0=-1 val1=1 period=1u
width=450n delay=550n rise=10n fall=10n
V1 ( Vi 0 ) vsource type=dc dc=0 mag=1 pacmag=1
PSS1 pss period=1u errpreset=conservative
PAC1 pac start=1 stop=3.1M log=1001
ZOH1 ( Vo_sc_zoh 0 Vo_sc 0 ) zoh period=1u delay=500n aperture=1n tc=10p
ZOH2 ( Vi_zoh 0 Vi 0 ) zoh period=1u delay=0 aperture=1n tc=10p
```

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ZOH Circuit File

```
// Copy from the SpectreRF Primer
module zoh (Pout, Nout, Pin, Nin) (period,
delay, aperture, tc)
node [V,I] Pin, Nin, Pout, Nout;
parameter real period=1 from (0:inf);
parameter real delay=0 from [0:inf];
parameter real aperture=1/100 from (0:inf);
parameter real tcs=1/500 from (0:inf);
{
integer n; real start, stop;
node [V,I] hold;
analog {
// determine the point when aperture begins
n = ($time() - delay + aperture) / period
+ 0.5;
start = n*period + delay - aperture;
$break_point(start);

// determine the time when aperture ends
n = ($time() - delay) / period + 0.5;
stop = n*period + delay;
$break_point(stop);
}

// Implement switch with effective series
// resistence of 1 Ohm
if ( ($time() > start) && ($time() <=
stop) )
I(hold) <- V(hold) - V(Pin, Nin);
else
I(hold) <- 1.0e-12 * (V(hold) - V(Pin,
Nin));

// Implement capacitor with an effective
// capacitance of tc
I(hold) <- tc * dot(V(hold));

// Buffer output
V(Pout, Nout) <- V(hold);

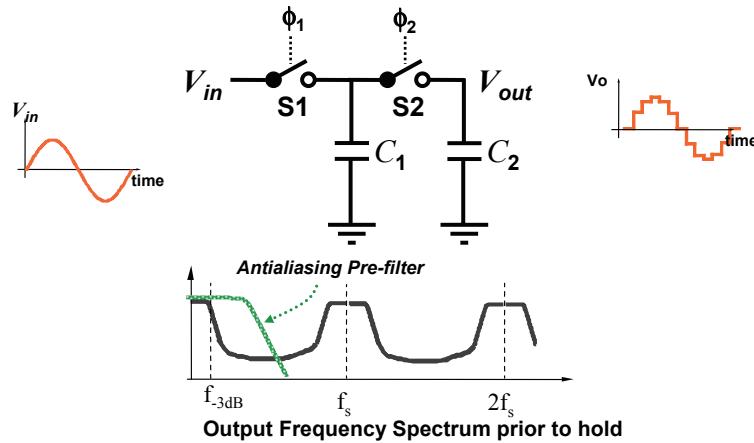
// Control time step tightly during
// aperture and loosely otherwise
if (($time() >= start) && ($time() <=
stop))
$bound_step(tc);
else
$bound_step(period/5);
}
}
```

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First Order S.C. Filter

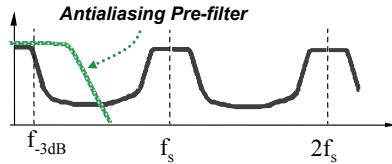


Switched-Capacitor Filters → problem with aliasing

Sampled-Data Systems (Filters) Anti-aliasing Requirements

- Frequency response repeats at $f_s, 2f_s, 3f_s, \dots$
- High frequency signals close to $f_s, 2f_s, \dots$ folds back into passband of interest (aliasing)
- In most cases must pre-filter input to a sampled-data systems (filter) to attenuate signal at:
 $f > f_s/2$ (Nyquist → $f_{max} < f_s/2$)
- Usually, anti-aliasing filter → included on-chip as continuous-time filter with relaxed specs. (no tuning)

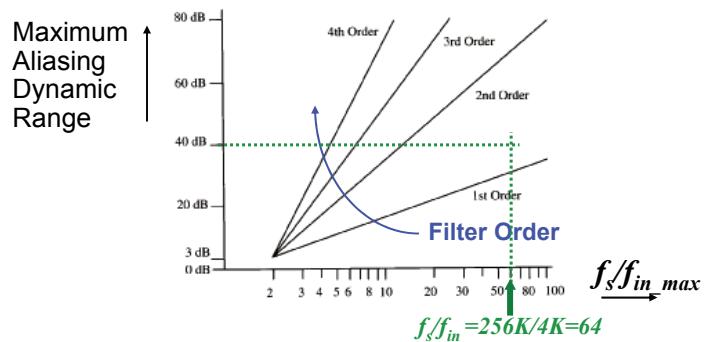
Example : Anti-Aliasing Filter Requirements



- Voice-band CODEC S.C. filter $f_{-3dB} = 4\text{kHz}$ & $f_s = 256\text{kHz}$
- Anti-aliasing filter requirements:
 - Need at least 40dB attenuation of all out-of-band signals which can alias inband
 - Incur no phase-error from 0 to 4kHz (signal band of interest)
 - Gain error due to anti-aliasing filter $\rightarrow 0$ to 4kHz $< 0.05\text{dB}$
 - Allow $\pm 30\%$ variation for anti-aliasing filter corner frequency (no tuning)

Need to find minimum required filter order

Oversampling Ratio versus Anti-Aliasing Filter Order



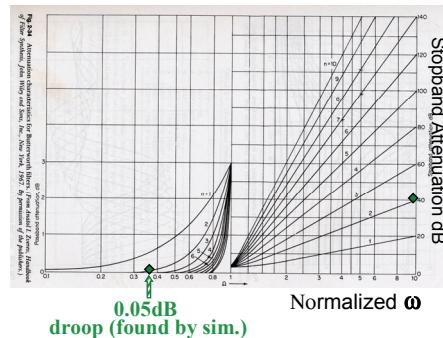
* Assumption \rightarrow anti-aliasing filter is Butterworth type

\rightarrow 2nd order Butterworth

\rightarrow Need to find minimum anti-aliasing filter corner frequency for mag. droop $< 0.05\text{dB}$

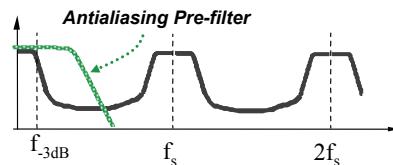
Example : Anti-Aliasing Filter Specifications

- Normalized frequency for 0.05dB droop: need perform passband simulation \rightarrow normalized $\omega=0.34 \rightarrow 4\text{kHz}/0.34=12\text{kHz}$
- Set anti-aliasing filter corner frequency for minimum corner frequency 12kHz \rightarrow Find nominal corner frequency: $12\text{kHz}/0.7=17.1\text{kHz}$
- Check if attenuation requirement is satisfied for max. filter bandwidth $\rightarrow 17.1 \times 1.3=22.28\text{kHz}$
- Find $(f_s f_{sig})/f_{-3dB}^{\max}$ $\rightarrow 252/22.2=11.35 \rightarrow$ make sure at least 40dB attenuation
- Check phase-error within 4kHz bandwidth for min. filter bandwidth via simulation



From: Williams and Taylor, p. 2-37

Example : Anti-Aliasing Filter



- Voice-band S.C. filter $f_{-3dB}=4\text{kHz}$ & $f_s=256\text{kHz}$
- Anti-aliasing filter requirements:
 - Need 40dB attenuation at clock freq.
 - Incur no phase-error from 0 to 4kHz
 - Gain error 0 to 4kHz $< 0.05\text{dB}$
 - Allow +/-30% variation for anti-aliasing corner frequency (no tuning)

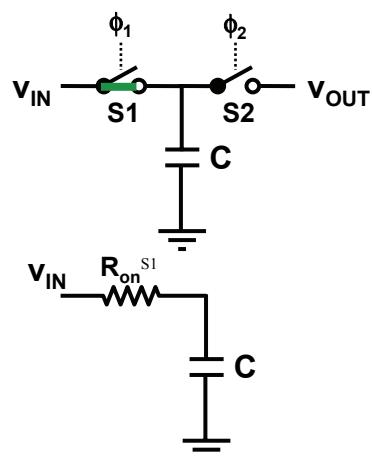
\rightarrow 2-pole Butterworth LPF with nominal corner freq. of 17kHz & no tuning (min.=12kHz & max.=22kHz corner frequency)

Summary

- Sampling theorem $\rightarrow f_s > 2f_{max_Signal}$
- Signals at frequencies $nf_S \pm f_{sig}$ fold back down to desired signal band, f_{sig}
 - This is called aliasing & usually mandates use of anti-aliasing pre-filters
- Oversampling helps reduce required order for anti-aliasing filter
- S/H function shapes the frequency response with $\sin x/x$ shape
 - Need to pay attention to droop in passband due to $\sin x/x$
- If the above requirements are not met, CT signals can NOT be recovered from sampled-data networks without loss of information

Switched-Capacitor Network Noise

- During ϕ_1 high: Resistance of switch S1 (R_{on}^{S1}) produces a noise voltage on C with variance kT/C (lecture 1- first order filter noise)



- The corresponding noise charge is:

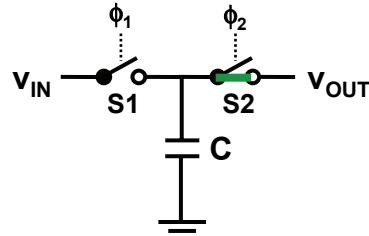
$$Q^2 = C^2 V^2 = C^2 \cdot kT/C = kTC$$

- ϕ_1 low: S1 open → This charge is sampled on C

Switched-Capacitor Noise

- During ϕ_2 high: Resistance of switch S2 contributes to an uncorrelated noise charge on C at the end of ϕ_2 : with variance kT/C
- Mean-squared noise charge transferred from v_{IN} to v_{OUT} per sample period is:

$$Q^2 = 2kTC$$



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Switched-Capacitor Noise

- The mean-squared noise current due to S1 and S2's kT/C noise is :

$$\text{Since } i = \frac{Q}{t} \text{ then } \bar{i^2} = (Qf_s)^2 = 2k_B T C f_s^2$$

- This noise is approximately white and distributed between 0 and $f_s/2$ (noise spectra \rightarrow single sided by convention)
The spectral density of the noise is found:

$$\frac{\bar{i^2}}{\Delta f} = \frac{2k_B T C f_s^2}{f_s / 2} = 4k_B T C f_s$$

$$\text{Since } R_{EQ} = \frac{1}{f_s C} \text{ then:}$$

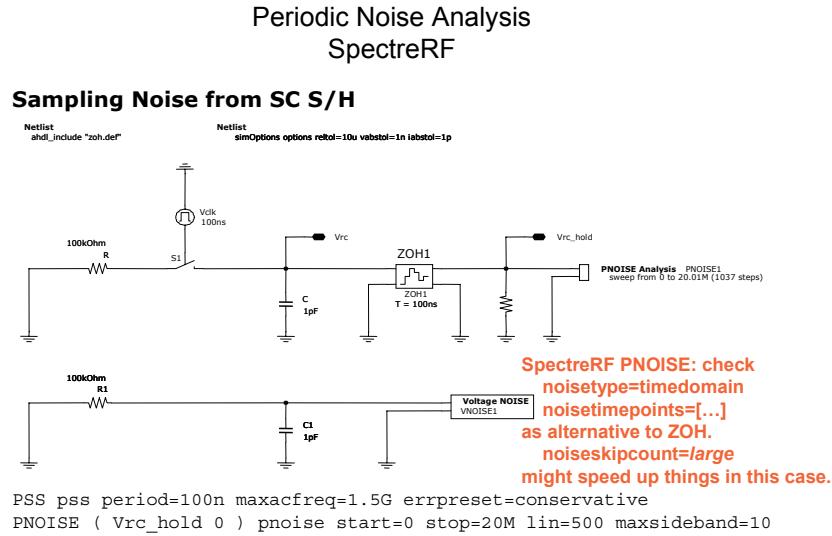
$$\frac{\bar{i^2}}{\Delta f} = \frac{4k_B T}{R_{EQ}}$$

→ S.C. resistor noise = a physical resistor noise with same value!

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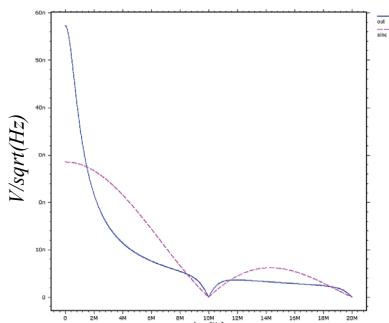


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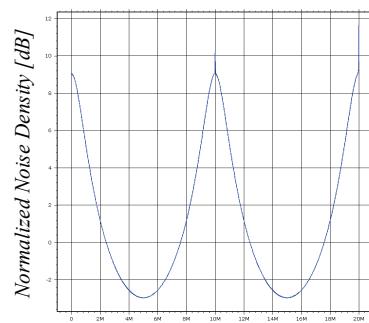
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Sampled Noise Spectrum



Spectral density of sampled noise including $\sin x/x$ effect



Noise spectral density with $\sin x/x$ effect taken out

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Total Noise

