

EE247

Lecture 5

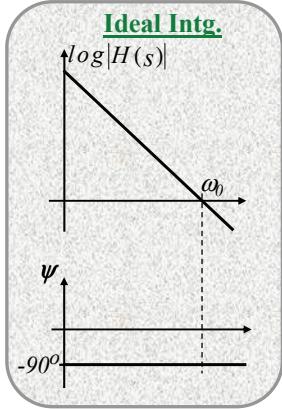
- Filters

- Effect of integrator non-idealities on filter behavior
 - Integrator quality factor and its influence on filter frequency characteristics (review for last lecture)
 - Filter dynamic range limitations due to limited integrator linearity
 - Measures of linearity: Harmonic distortion, intermodulation distortion, intercept point
 - Effect of integrator component variations and mismatch on filter response
- Various integrator topologies utilized in monolithic filters
 - Resistor + C based filters
 - Transconductance (gm) + C based filters
 - Switched-capacitor filters
- Continuous-time filter considerations
 - Facts about monolithic R_s , g_{ms} , & C_s and its effect on integrated filter characteristics

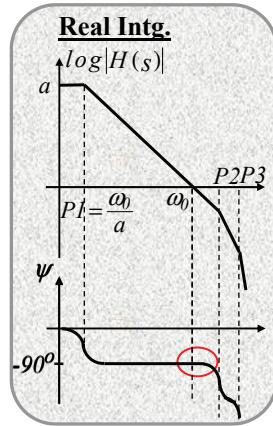
Summary of Lecture 4

- Ladder type RLC filters converted to integrator based active filters
 - All pole ladder type filters
 - Convert RLC ladder filters to integrator based form
 - Example: 5th order Butterworth filter
 - High order ladder type filters incorporating zeros
 - 7th order elliptic filter in the form of ladder RLC with zeros
 - Sensitivity to component mismatch
 - Compare with cascade of biquads
- *Doubly terminated LC ladder filters* ⇒ *Lowest sensitivity to component variations*
- Convert to integrator based form utilizing SFG techniques
- Example: Differential high order filter implementation
- Effect of integrator non-idealities on continuous-time filter behavior
 - Effect of integrator finite DC gain & non-dominant poles on filter frequency response

Real Integrator Non-Idealities



$$H(s) = \frac{-\omega_b}{s}$$



$$H(s) \approx \frac{-a}{(1+s\frac{a}{\omega_b})(1+\frac{s}{p_2})(1+\frac{s}{p_3})...}$$

Summary

Effect of Integrator Non-Idealities on Q

$$Q_{ideal}^{intg.} = \infty$$

$$Q_{real}^{intg.} \approx \frac{I}{\frac{1}{a} - \omega_b \sum_{i=2}^{\infty} \frac{1}{p_i}}$$

- Amplifier DC gain reduces the overall Q in the same manner as series/parallel resistance associated with passive elements
- Amplifier poles located above integrator unity-gain frequency enhance the Q!
 - If non-dominant poles close to unity-gain freq. → Oscillation
- Depending on the location of unity-gain-frequency, the two terms can cancel each other out!
- Overall quality factor of the integrator has to be much higher compared to the filter's highest pole Q

Effect of Integrator Non-Linearities on Overall Integrator-Based Filter Performance

- Dynamic range of a filter is determined by the ratio of maximum signal output with acceptable performance over total noise
- Maximum signal handling capability of a filter is determined by the non-linearities associated with its building blocks
- Integrator linearity function of opamp/R/C (or any other component used to build the integrator) linearity-
- Linearity specifications for active filters typically given in terms of :
 - Maximum allowable harmonic distortion @ the output
 - Maximum tolerable intermodulation distortion
 - Intercept points & compression point referred to output or input

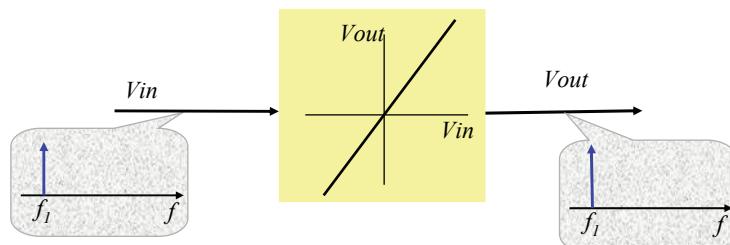
Component Linearity versus Overall Filter Performance 1- Ideal Components

Ideal DC transfer characteristics:

Perfectly linear output versus input transfer function with no clipping

$V_{out} = \alpha V_{in}$ for $-\infty \leq V_{in} \leq \infty$

If $V_{in} = A \sin(\omega t)$ $\rightarrow V_{out} = \alpha A \sin(\omega t)$



Component Linearity versus Overall Filter Performance

2- Semi-Ideal Components

Semi-ideal DC transfer characteristics:

Perfectly linear output versus input transfer function with clipping

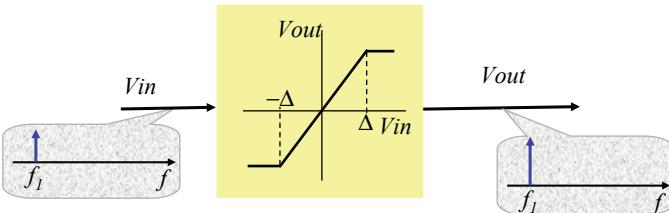
$$V_{out} = \alpha V_{in} \text{ for } -\Delta \leq V_{in} \leq +\Delta$$

$$V_{out} = -\Delta \alpha \text{ for } V_{in} \leq -\Delta$$

$$V_{out} = \Delta \alpha \text{ for } V_{in} \geq +\Delta$$

$$\text{If } V_{in} = A \sin(\omega t) \rightarrow V_{out} = \alpha A \sin(\omega t) \text{ for } -\Delta \leq V_{in} \leq +\Delta$$

Clipped & distorted otherwise



Effect of Component Non-Linearity on Overall Filter Linearity

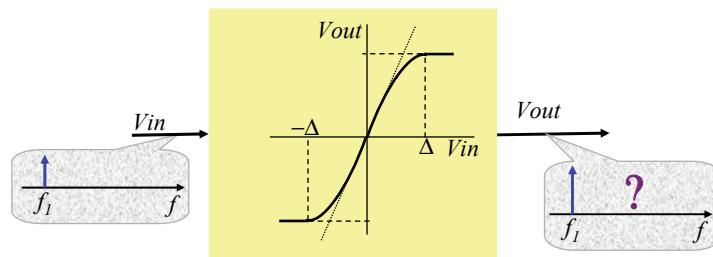
Real Components including Non-Linearity

Real DC transfer characteristics: Both soft non-linearities & hard (clipping)

$$V_{out} = \alpha_1 V_{in} + \alpha_2 V_{in}^2 + \alpha_3 V_{in}^3 + \dots \text{ for } -\Delta \leq V_{in} \leq \Delta$$

Clipped otherwise

$$\text{If } V_{in} = A \sin(\omega t)$$



Effect of Component Non-Linearity on Overall Filter Linearity Real Components including Non-Linearities

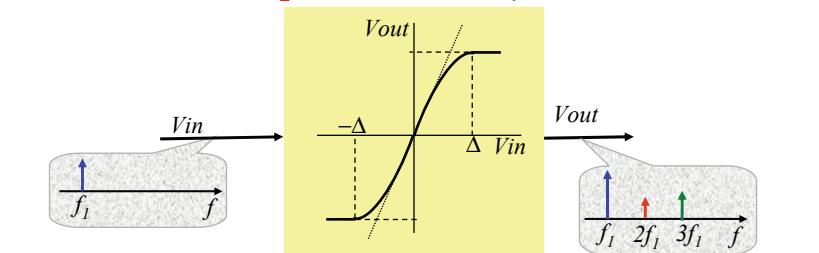
Typical real circuit DC transfer characteristics:

$$V_{out} = \alpha_1 V_{in} + \alpha_2 V_{in}^2 + \alpha_3 V_{in}^3 + \dots \quad \text{If } V_{in} = A \sin(\omega_l t) \text{ & } A < \Delta$$

Then:

$$\rightarrow V_{out} = \alpha_1 A \sin(\omega_l t) + \alpha_2 A^2 \sin(\omega_l t)^2 + \alpha_3 A^3 \sin(\omega_l t)^3 + \dots$$

$$\text{or } V_{out} = \alpha_1 A \sin(\omega_l t) + \frac{\alpha_2 A^2}{2} (1 - \cos(2\omega_l t)) + \frac{\alpha_3 A^3}{4} (3 \sin(\omega_l t) - \sin(3\omega_l t)) + \dots$$



Effect of Component Non-Linearity on Overall Filter Linearity Harmonic Distortion

$$\begin{aligned} V_{out} = & \alpha_1 A \sin(\omega t) + \frac{\alpha_2 A^2}{2} (1 - \cos(2\omega t)) \\ & + \frac{\alpha_3 A^3}{4} (3 \sin(\omega t) - \sin(3\omega t)) + \dots \end{aligned}$$

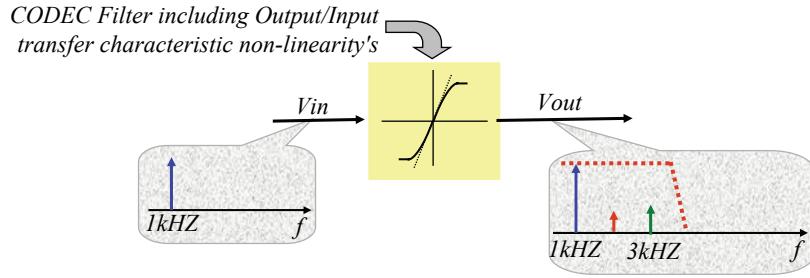
$$HD2 = \frac{\text{amplitude } 2^{\text{nd}} \text{ harmonic distortion component}}{\text{amplitude fundamental}}$$

$$HD3 = \frac{\text{amplitude } 3^{\text{rd}} \text{ harmonic distortion component}}{\text{amplitude fundamental}}$$

$\rightarrow HD2 = \frac{1}{2} \times \frac{\alpha_2}{\alpha_1} A, \quad HD3 = \frac{1}{4} \times \frac{\alpha_3}{\alpha_1} A^2$

Example: Significance of Filter Harmonic Distortion in Voice-Band CODECs

- Voice-band CODEC filter (CODEC stands for coder-decoder, telephone circuitry includes CODECs with extensive amount of integrated active filters)
- Specifications includes limits associated with maximum allowable harmonic distortion at the output (< typically < 1% \rightarrow -40dB)



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Lecture 5: Integrator-Based Filters

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Effect of Component Non-Linearity on Overall Filter Linearity Intermodulation Distortion

DC transfer characteristics including nonlinear terms, input 2 sinusoidal waveforms:

$$V_{out} = \alpha_1 V_{in} + \alpha_2 V_{in}^2 + \alpha_3 V_{in}^3 + \dots$$

$$\text{If } V_{in} = A_1 \sin(\omega_1 t) + A_2 \sin(\omega_2 t)$$

Then V_{out} will have the following components:

$$\alpha_1 V_{in} \rightarrow \alpha_1 A_1 \sin(\omega_1 t) + \alpha_1 A_2 \sin(\omega_2 t)$$

$$\alpha_2 V_{in}^2 \rightarrow \alpha_2 A_1^2 \sin^2(\omega_1 t) + \alpha_2 A_2^2 \sin^2(\omega_2 t) + 2\alpha_2 A_1 A_2 \sin(\omega_1 t) \sin(\omega_2 t) + \dots$$

$$\rightarrow \frac{\alpha_2 A_1^2}{2} (1 - \cos(2\omega_1 t)) + \frac{\alpha_2 A_2^2}{2} (1 - \cos(2\omega_2 t))$$

$$+ \alpha_2 A_1 A_2 [\cos((\omega_1 - \omega_2)t) - \cos((\omega_1 + \omega_2)t)]$$

$$\alpha_3 V_{in}^3 \rightarrow +\alpha_3 A_1^3 \sin^3(\omega_1 t) + \alpha_3 A_2^3 \sin^3(\omega_2 t)$$

$$+ 3\alpha_3 A_1^2 A_2 \sin(\omega_1 t)^2 \sin(\omega_2 t) + 3\alpha_3 A_2^2 A_1 \sin(\omega_2 t)^2 \sin(\omega_1 t)$$

$$+\underbrace{\frac{3\alpha_3 A_1^2 A_2}{4} [\sin(2\omega_1 + \omega_2)t - \sin(2\omega_1 - \omega_2)t]}$$

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Lecture 5: Integrator-Based Filters

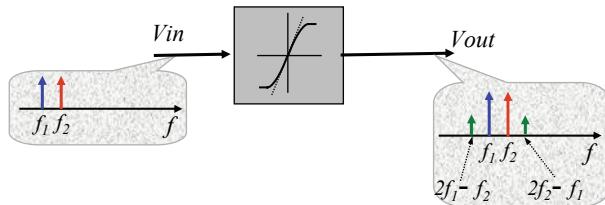
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Effect of Component Non-Linearity on Overall Filter Linearity Intermodulation Distortion

Real DC transfer characteristics, input 2 sin waves:

$$V_{out} = \alpha_1 V_{in} + \alpha_2 V_{in}^2 + \alpha_3 V_{in}^3 + \dots$$

$$\text{If } V_{in} = A_1 \sin(\omega_1 t) + A_2 \sin(\omega_2 t)$$



For f_1 & f_2 close in frequency → Components associated with $(2f_1-f_2)$ & $(2f_2-f_1)$ are the closest to the fundamental signals on the frequency axis and thus most harmful

Effect of Component Non-Linearity on Overall Filter Linearity Intermodulation Distortion

Intermodulation distortion is measured in terms of IM2 and IM3:

Typically for input two sinusoids with equal amplitude ($A_1 = A_2 = A$)

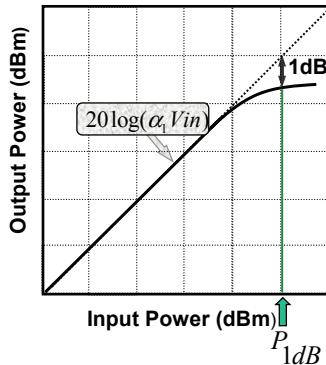
$$IM2 = \frac{\text{amplitude 2}^{\text{nd}} \text{ IM component}}{\text{amplitude fundamental}}$$

$$IM3 = \frac{\text{amplitude 3}^{\text{rd}} \text{ IM component}}{\text{amplitude fundamental}}$$

$$IM2 = \frac{\alpha_2}{\alpha_1} A + \dots \quad IM3 = \frac{3}{4} \frac{\alpha_3}{\alpha_1} A^2 + \frac{25}{8} \frac{\alpha_5}{\alpha_1} A^4 + \dots$$

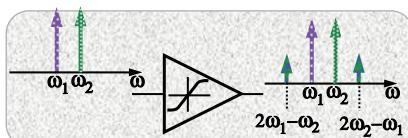
Wireless Communications Measure of Linearity

1dB Compression Point



$$V_{out} = \alpha_1 V_{in} + \alpha_2 V_{in}^2 + \alpha_3 V_{in}^3 + \dots$$

Wireless Communications Measure of Linearity Third Order Intercept Point



$$V_{out} = \alpha_1 V_{in} + \alpha_2 V_{in}^2 + \alpha_3 V_{in}^3 + \dots$$

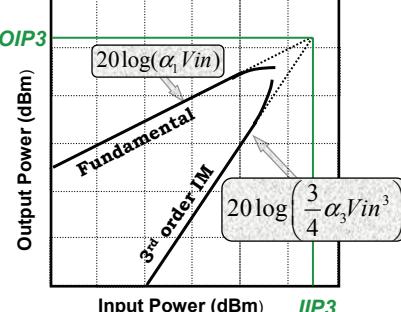
$$IM_3 = \frac{3rd}{1st}$$

$$\begin{aligned} &= \frac{3}{4} \frac{\alpha_3}{\alpha_1} V_{in}^2 + \frac{25}{8} \frac{\alpha_5}{\alpha_1} V_{in}^4 + \dots \\ &= 1 @ IIP3 \end{aligned}$$

Typically:

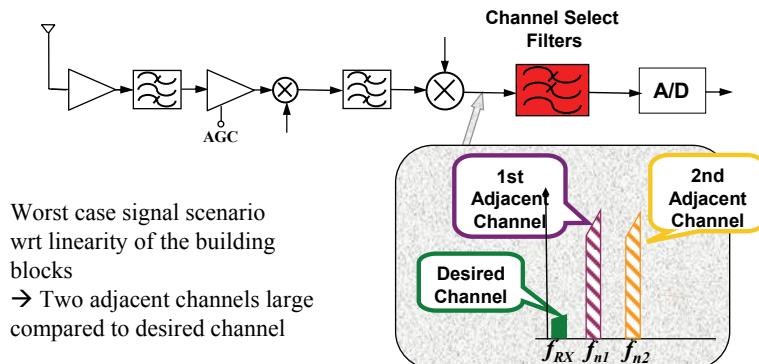
$$IIP_3 - P_{1dB} = 9.6dB$$

Most common measure of linearity for wireless circuits:
→ OIP3 & IIP3, Third order output/input intercept point

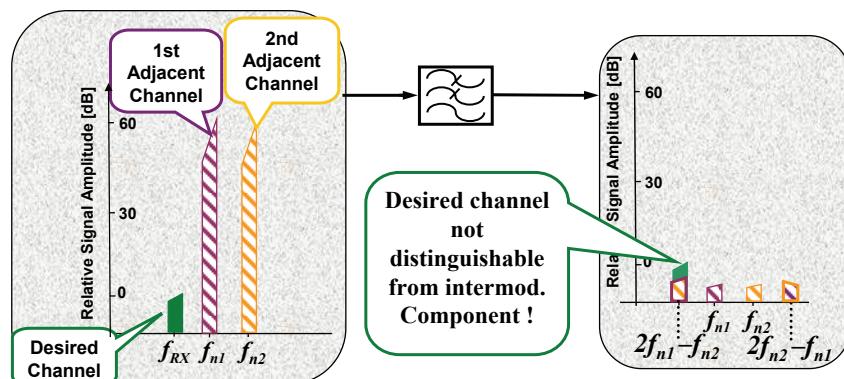


Example: Significance of Filter Intermodulation Distortion in Wireless Systems

- Typical wireless receiver architecture

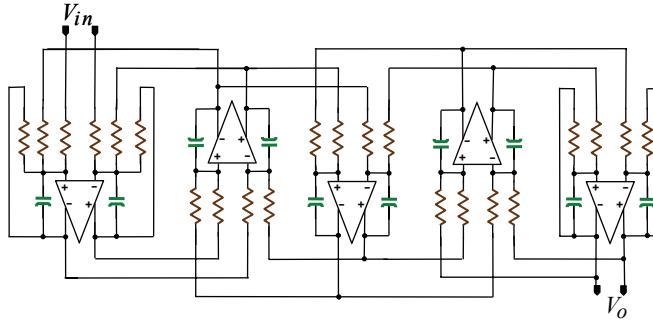


Example: Significance of Filter Intermodulation Distortion in Wireless Systems



- Adjacent channels can be as much as 60dB higher compared to the desired RX signal!
- Notice that in this example, 3rd order intermodulation component associated with the two adjacent channel, falls on the desired channel signal!

Filter Linearity

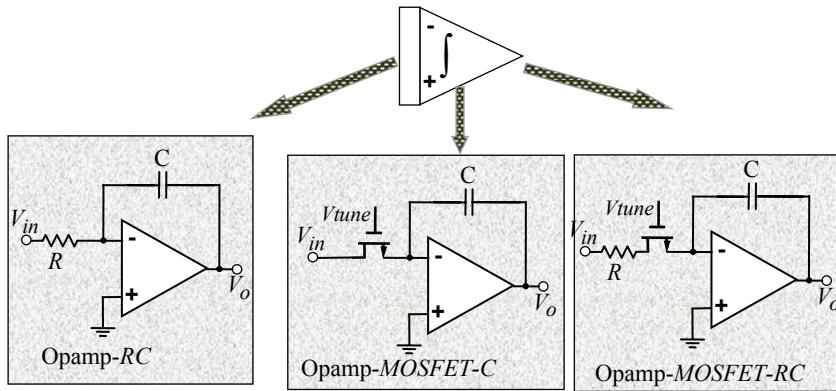


- Maximum signal handling capability is usually determined by the specifications wrt harmonic distortion and /or intermodulation distortion
Distortion in a filter is a function of linearity of the components
- Example: In the above circuit linearity of the filter is mainly a function of linearity of the **opamp** voltage transfer characteristics

Various Types of Integrator Based Filter

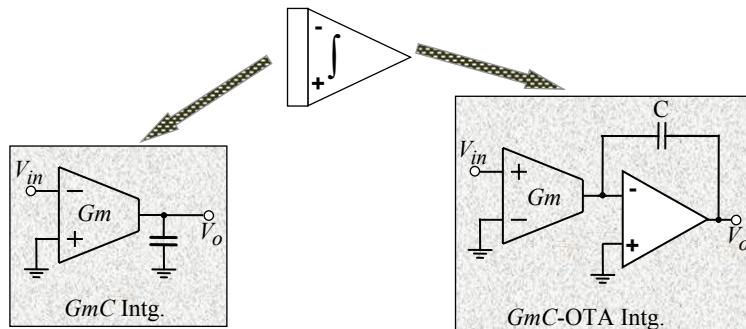
- Continuous Time
 - Resistive element based
 - Opamp-RC
 - Opamp-MOSFET-C
 - Opamp-MOSFET-RC
 - Transconductance (G_m) based
 - G_m -C
 - Opamp- G_m -C
- Sampled Data
 - Switched-capacitor Integrator

Continuous-Time Resistive Element Type Integrators
Opamp-RC & Opamp-MOSFET-C & Opamp-MOSFET-RC



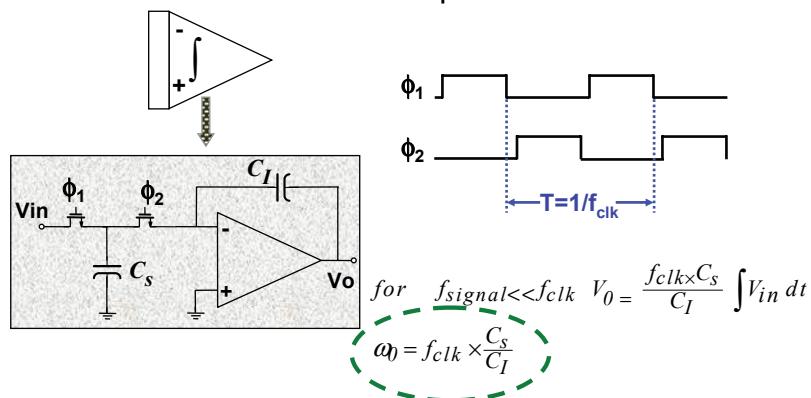
$$\text{Ideal transfer function: } \frac{V_o}{V_{in}} = \frac{-\omega_b}{s} \text{ where } \omega_b = \frac{1}{R_{eq}C}$$

Continuous-Time Transconductance Type Integrator
Gm-C & Opamp-Gm-C



$$\text{Ideal transfer function: } \frac{V_o}{V_{in}} = \frac{-\omega_b}{s} \text{ where } \omega_b = \frac{G_m}{C}$$

Integrator Implementation Switched-Capacitor



Main advantage: Critical frequency function of ratio of caps & clock freq.
 → Critical filter frequencies (e.g. LPF -3dB freq.) very accurate

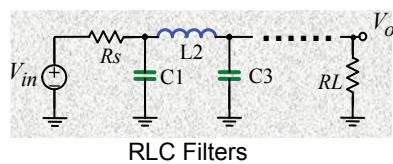
Few Facts About Monolithic Rs & Cs & Gms

- Monolithic continuous-time filter critical frequency set by RxC or C/Gm
 - Absolute value of integrated Rs & Cs & Gms are quite variable
 - Rs vary due to doping and etching non-uniformities
 - Could vary by as much as $\sim +20$ to 40% due to process & temperature variations
 - Cs vary because of oxide thickness variations and etching inaccuracies
 - Could vary $\sim +10$ to 15%
 - Gms typically function of mobility, oxide thickness, current, device geometry ...
 - Could vary $> \sim +40\%$ or more with process & temp. & supply voltage
- Continuous-time filter critical frequency could vary by over $\pm 50\%$

Few Facts About Monolithic Rs & Cs

- While absolute value of monolithic R_s & C_s and g_{ms} are quite variable, with special attention paid to layout, C & R & g_{ms} quite well-matched
 - Ratios very accurate and stable over processing, temperature, and time
- With special attention to layout (e.g. interleaving, use of dummy devices, common-centroid geometries...):
 - Capacitor mismatch $\ll 0.1\%$
 - Resistor mismatch $< 0.1\%$
 - Gm mismatch $< 0.5\%$

Impact of Component Variations on Filter Characteristics



Facts about RLC filters

- ω_{-3dB} determined by absolute value of L_s & C_s
- Shape of filter depends on ratios of normalized L & C

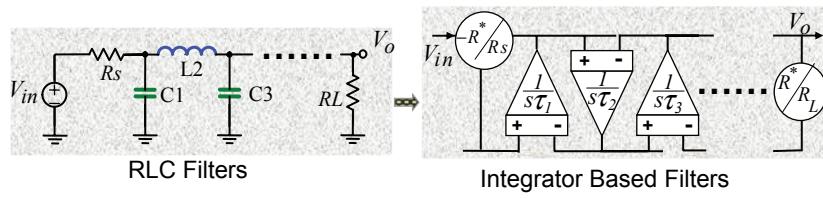
$$C_I^{RLC} = C_r \times C_I^{Norm} = \frac{C_I^{Norm}}{R \times \omega_{-3dB}}$$

$$L_2^{RLC} = L_r \times L_2^{Norm} = \frac{L_2^{Norm} \times R^*}{\omega_{-3dB}}$$

Effect of Monolithic R & C Variations on Filter Characteristics

- Filter shape (whether Elliptic with 0.1dB Rpass or Butterworth..etc) is a function of *ratio* of normalized *Ls & Cs* in RLC filters
- Critical frequency (e.g. ω_{-3dB}) function of *absolute value* of *Ls x Cs*
- Absolute value of integrated *Rs & Cs & Gms* are quite variable
- *Ratios* very accurate and stable over time and temperature
→ What is the effect of on-chip component variations on monolithic filter frequency characteristics?

Impact of Process Variations on Filter Characteristics

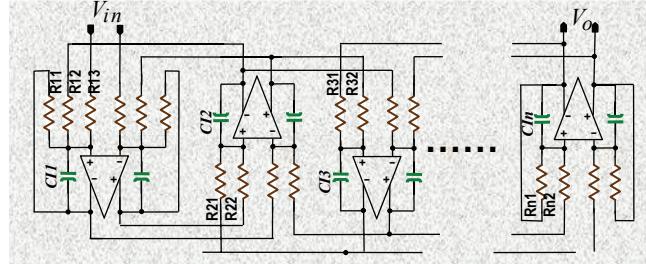


$$\tau_I = C_I^{RLC} \cdot R^* = \frac{C_I^{Norm}}{\omega_{-3dB}}$$

$$\tau_2 = \frac{L_2^{RLC}}{R^*} = \frac{L_2^{Norm}}{\omega_{-3dB}}$$

$$\frac{\tau_1}{\tau_2} = \frac{C_I^{Norm}}{L_2^{Norm}}$$

Impact of Process Variations on Filter Characteristics

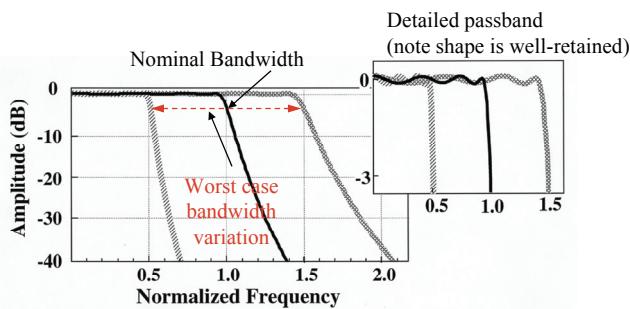


$$\begin{aligned}\tau_1^{int\,g} &= C_{I1} \cdot R_1 = \frac{C_1^{Norm}}{\omega_{-3dB}} \\ \tau_2^{int\,g} &= C_{I2} \cdot R_2 = \frac{L_2^{Norm}}{\omega_{-3dB}} \\ \frac{\tau_1^{int\,g}}{\tau_2^{int\,g}} &= \frac{C_{I1} \cdot R_1}{C_{I2} \cdot R_2} = \frac{C_1^{Norm}}{L_2^{Norm}}\end{aligned}$$

}

Variation in absolute value of integrated
Rs & Cs → change in critical freq. (ω_{-3dB})
Since Ratios of Rs & Cs very accurate
→ Continuous-time monolithic filters retain their
shape due to good component matching even
with variability in absolute component values

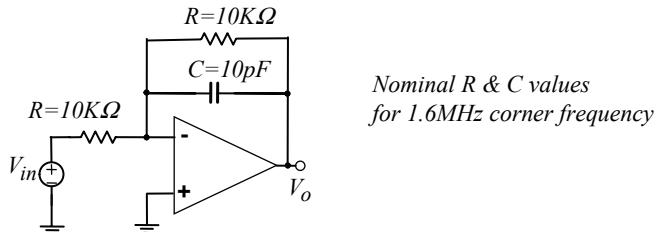
Example: LPF Worst Case Corner Frequency Variations



- While absolute value of on-chip RC (gm-C) time-constants could vary by as much as 100% (process & temp.)
- With proper precautions, excellent component matching can be achieved:
 - Well-preserved relative amplitude & phase vs freq. characteristics
 - Need to only adjust (tune) continuous-time filter critical frequencies

Tunable Opamp-RC Filters Example

- 1st order Opamp-RC filter is designed to have a corner frequency of 1.6MHz
- Assuming process variations of:
 - C varies by +/-10%
 - R varies by +/-25%
- Build the filter in such a way that the corner frequency can be adjusted post-manufacturing.



Filter Corner Frequency Variations

- Assuming expected process variations of:
 - Maximum C variations by +/-10%
 $C_{nom}=10\text{pF} \rightarrow C_{min}=9\text{pF}, C_{max}=11\text{pF}$
 - Maximum R variations by +/-25%
 $R_{nom}=10\text{K} \rightarrow R_{min}=7.5\text{K}, R_{max}=12.5\text{K}$
 - Corner frequency ranges from
 $\rightarrow 2.357\text{MHz}$ to 1.157MHz
- Corner frequency varies by +48% & -27%

Variable Resistor or Capacitor

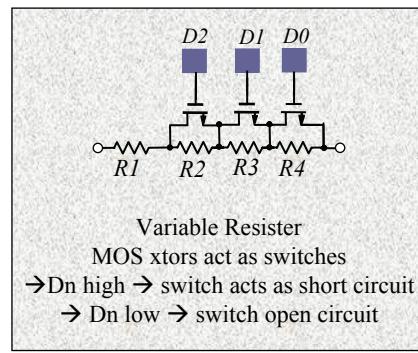
- In order to make provisions for filter to be tunable either R or C should be adjustable (this example → adjustable R)
- Monolithic Rs can only be made adjustable in discrete steps (not continuous)

$$\frac{R_{nom}^{max}}{R_{nom}} = \frac{f_{max}}{f_{nom}} = 1.48$$

$$\rightarrow R_{nom}^{max} = 14.8k\Omega$$

$$\frac{R_{nom}^{min}}{R_{nom}} = \frac{f_{min}}{f_{nom}} = 0.72$$

$$\rightarrow R_{nom}^{min} = 7.2k\Omega$$



Tunable Resistor

- Maximum C variations by +10% → C_{min}=9pF, C_{max}=11pF
- Maximum R variations by +25% → R_{min}=7.5K, R_{max}=12.5K
→ Corner frequency varies by +48% & -27%
- Assuming control signal has n = 3bit (0 or 1) for adjustment → R₂=2R₃=4R₄

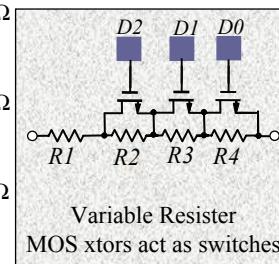
$$R_1 = R_{nom}^{min} = 7.2k\Omega$$

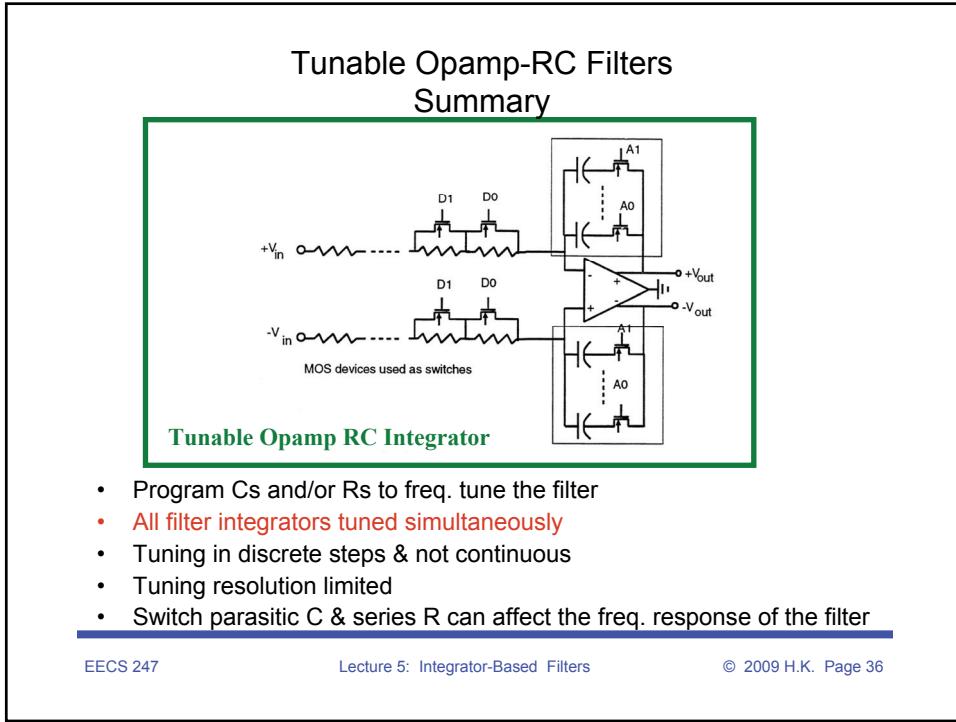
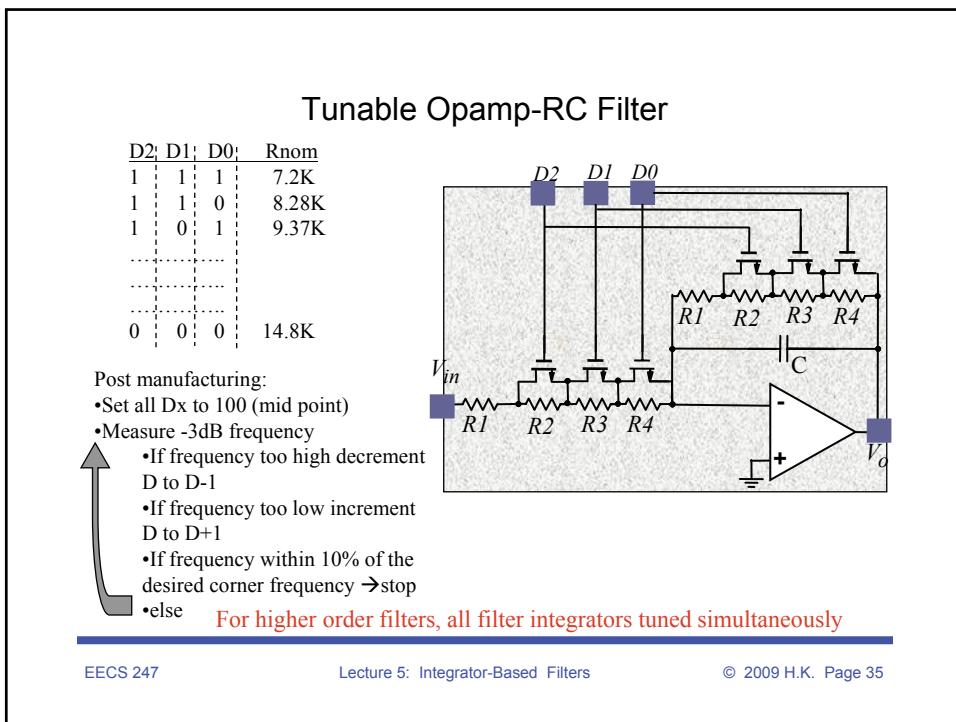
$$R_2 = (R_{nom}^{max} - R_{nom}^{min}) \times \frac{2^{n-1}}{2^n - 1} = (14.8k - 7.2k) \frac{4}{7} = 4.34k\Omega$$

$$R_3 = (R_{nom}^{max} - R_{nom}^{min}) \times \frac{2^{n-2}}{2^n - 1} = (14.8k - 7.2k) \frac{2}{7} = 2.17k\Omega$$

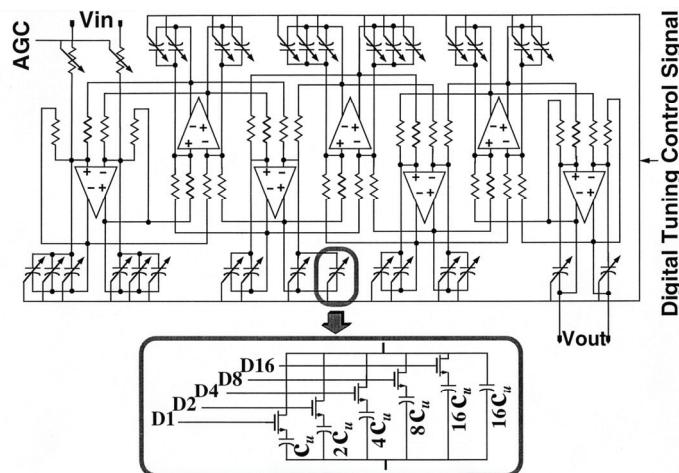
$$R_4 = (R_{nom}^{max} - R_{nom}^{min}) \times \frac{2^{n-3}}{2^n - 1} = (14.8k - 7.2k) \frac{1}{7} = 1.08k\Omega$$

Tuning resolution ≈ 1.08k/10k ≈ 10%





Example: Tunable Low-Pass Opamp-RC Filter Adjustable Capacitors



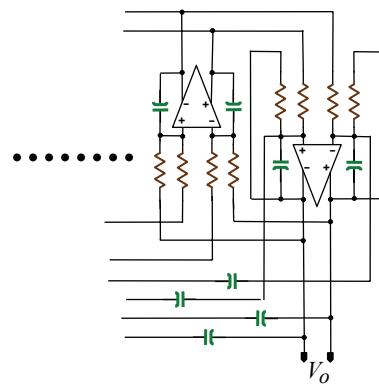
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Lecture 5: Integrator-Based Filters

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Opamp RC Filters

- Advantages
 - Since resistors are quite linear, linearity only a function of opamp linearity
→ good linearity
- Disadvantages
 - Opamps have to drive resistive load, low output impedance is required
→ High power consumption
 - Continuous tuning not possible—tuning only in discrete steps
 - Tuning requires programmable Rs and/or Cs



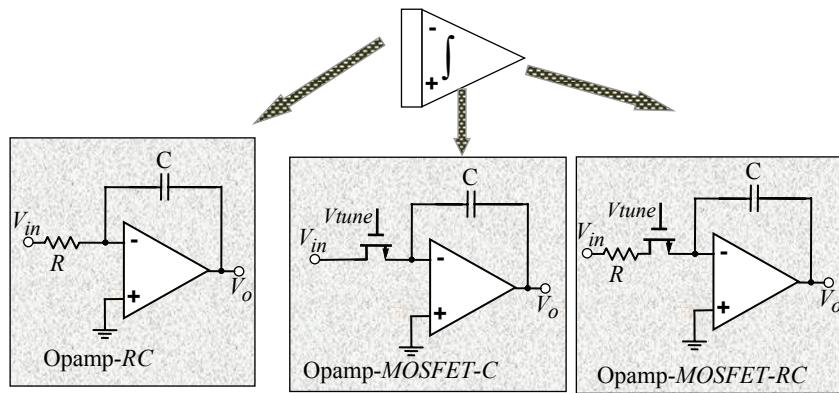
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Lecture 5: Integrator-Based Filters

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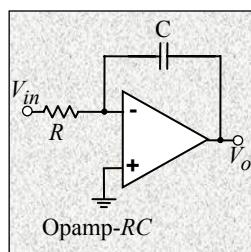
Integrator Implementation

Opamp-RC & Opamp-MOSFET-C & Opamp-MOSFET-RC



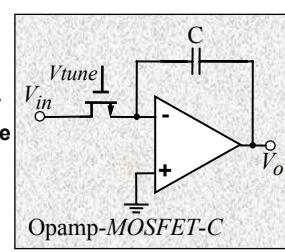
$$\frac{V_o}{V_{in}} = \frac{-\omega_b}{s} \quad \text{where} \quad \omega_b = \frac{1}{R_{eq}C}$$

Use of MOSFETs as Variable Resistors

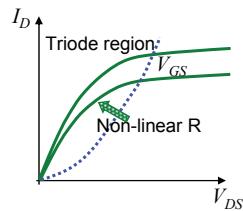


R replaced by MOSFET
Operating in triode mode

→ Continuously
variable resistor:



MOSFET IV characteristic:

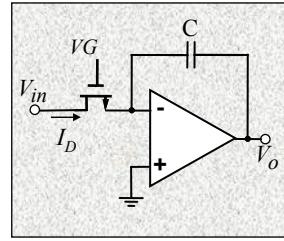


Opamp MOSFET-C Integrator Single-Ended Integrator

$$I_D = \mu C_{ox} \frac{W}{L} \left[(V_{gs} - V_{th}) V_{ds} - \frac{V_{ds}^2}{2} \right]$$

$$I_D = \mu C_{ox} \frac{W}{L} \left[(V_{gs} - V_{th}) V_i - \frac{V_i^2}{2} \right]$$

$$G = \frac{\partial I_D}{\partial V_i} = \mu C_{ox} \frac{W}{L} (V_{gs} - V_{th} - V_i)$$



→ Tunable by varying V_G :

By varying V_G effective admittance is tuned
→ Tunable integrator time constant

Problem: Single-ended MOSFET-C Integrator → Effective R non-linear
Note that the non-linearity is mainly 2nd order type

Use of MOSFETs as Resistors Differential Integrator

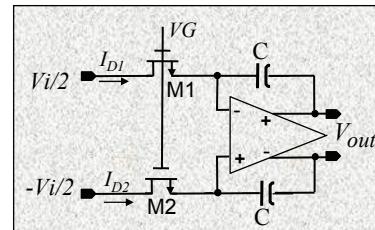
$$I_D = \mu C_{ox} \frac{W}{L} \left(V_{gs} - V_{th} - \frac{V_{ds}}{2} \right) V_{ds}$$

$$I_{DI} = \mu C_{ox} \frac{W}{L} \left(V_{gs} - V_{th} - \frac{V_i}{4} \right) \frac{V_i}{2}$$

$$I_{D2} = -\mu C_{ox} \frac{W}{L} \left(V_{gs} - V_{th} + \frac{V_i}{4} \right) \frac{V_i}{2}$$

$$I_{DI} - I_{D2} = \mu C_{ox} \frac{W}{L} (V_{gs} - V_{th}) V_i$$

$$G = \frac{\partial (I_{DI} - I_{D2})}{\partial V_i} = \mu C_{ox} \frac{W}{L} (V_{gs} - V_{th})$$



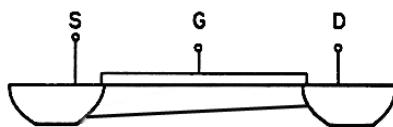
Opamp-MOSFET-C

- Non-linear term is of even order & cancelled!
- Admittance independent of V_i

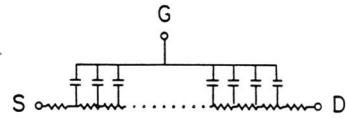
Problem: Threshold voltage dependence

Use of MOSFET as Resistor Issues

MOS xtor operating in triode region
Cross section view



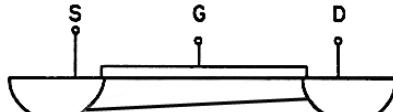
Distributed channel resistance & gate capacitance



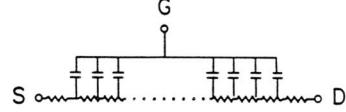
- Distributed nature of gate capacitance & channel resistance results in infinite no. of high-frequency poles:
 - Excess phase @ the unity-gain frequency of the integrator
 - Enhanced integrator Q
 - Enhanced filter Q,
 - Peaking in the filter passband

Use of MOSFET as Resistor Issues

MOS xtor operating in triode region
Cross section view



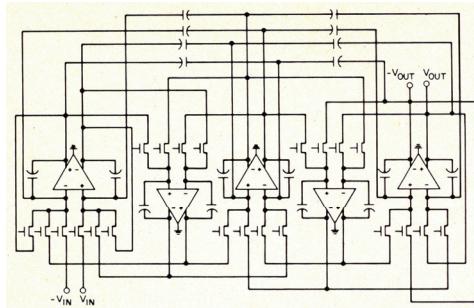
Distributed channel resistance & gate capacitance



- Tradeoffs affecting the choice of device channel length:
 - Filter performance mandates well-matched MOSFETs → long channel devices desirable
 - Excess phase increases with L^2 → Q enhancement and potential for oscillation!
 - Tradeoff between device matching and integrator Q
 - This type of filter limited to low frequencies

Example: Opamp MOSFET-C Filter

- Suitable for low frequency applications
- Issues with linearity
- Linearity achieved ~40-50dB
- Needs tuning

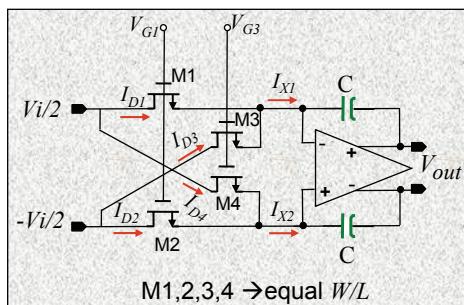


5th Order Elliptic MOSFET-C LPF
with 4kHz Bandwidth

Ref: Y. Tsividis, M.Banu, and J. Khouri, "Continuous-Time MOSFET-C Filters in VLSI", *IEEE Journal of Solid State Circuits* Vol. SC-21, No.1 Feb. 1986, pp. 15-30

Improved MOSFET-C Integrator

$$\begin{aligned}
 I_D &= \mu C_{ox} \frac{W}{L} \left(V_{gs} - V_{th} - \frac{V_{ds}}{2} \right) V_{ds} \\
 I_{D1} &= \mu C_{ox} \frac{W}{L} \left(V_{gs1} - V_{th} - \frac{V_i}{4} \right) \frac{V_i}{2} \\
 I_{D3} &= -\mu C_{ox} \frac{W}{L} \left(V_{gs3} - V_{th} + \frac{V_i}{4} \right) \frac{V_i}{2} \\
 I_{X1} &= I_{D1} + I_{D3} \\
 &= \mu C_{ox} \frac{W}{L} \left(V_{gs1} - V_{gs3} - \frac{V_i}{2} \right) \frac{V_i}{2} \\
 I_{X2} &= \mu C_{ox} \frac{W}{L} \left(V_{gs3} - V_{gs1} - \frac{V_i}{2} \right) \frac{V_i}{2} \\
 I_{X1} - I_{X2} &= \mu C_{ox} \frac{W}{L} (V_{gs1} - V_{gs3}) V_i \\
 G &= \frac{\partial (I_{X1} - I_{X2})}{\partial V_i} = \mu C_{ox} \frac{W}{L} (V_{gs1} - V_{gs3})
 \end{aligned}$$

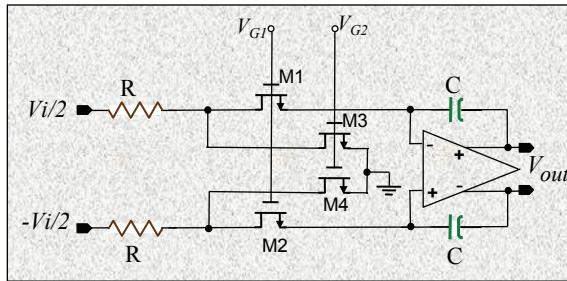


No threshold voltage dependence

Linearity achieved in the order of 50-70dB

Ref: Z. Czarnul, "Modification of the Banu-Tsividis Continuous-Time Integrator Structure," *IEEE Transactions on Circuits and Systems*, Vol. CAS-33, No. 7, pp. 714-716, July 1986.

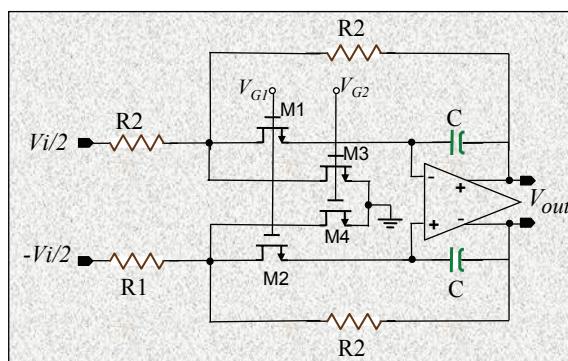
R-MOSFET-C Integrator



- Improvement over MOSFET-C by adding resistor in series with MOSFET
- Voltage drop primarily across fixed resistor \rightarrow small MOSFET V_{ds} \rightarrow improved linearity & reduced tuning range
- Generally low frequency applications

Ref: U-K Moon, and B-S Song, "Design of a Low-Distortion 22-kHz Fifth Order Bessel Filter," *IEEE Journal of Solid State Circuits*, Vol. 28, No. 12, pp. 1254-1264, Dec. 1993.

R-MOSFET-C Lossy Integrator

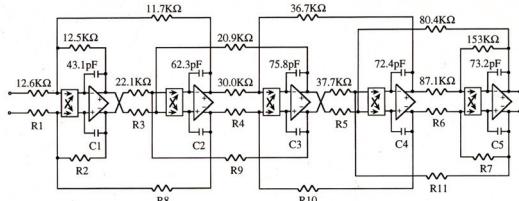


Negative feedback around the non-linear MOSFETs improves linearity but compromises frequency response accuracy

Ref: U-K Moon, and B-S Song, "Design of a Low-Distortion 22-kHz Fifth Order Bessel Filter," *IEEE Journal of Solid State Circuits*, Vol. 28, No. 12, pp. 1254-1264, Dec. 1993.

Example:

Opamp MOSFET-RC Filter



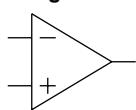
**5th Order Bessel MOSFET-RC LPF 22kHz bandwidth
THD \rightarrow 90dB for 4Vp-p , 2kHz input signal**

- Suitable for low frequency, low Q applications
- Significant improvement in linearity compared to MOSFET-C
- Needs tuning

Ref: U-K Moon, and B-S Song, "Design of a Low-Distortion 22-kHz Fifth Order Bessel Filter," *IEEE Journal of Solid State Circuits*, Vol. 28, No. 12, pp. 1254-1264, Dec. 1993.

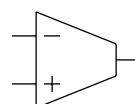
Operational Amplifiers (Opamps) versus Operational Transconductance Amplifiers (OTA)

Opamp
Voltage controlled
voltage source



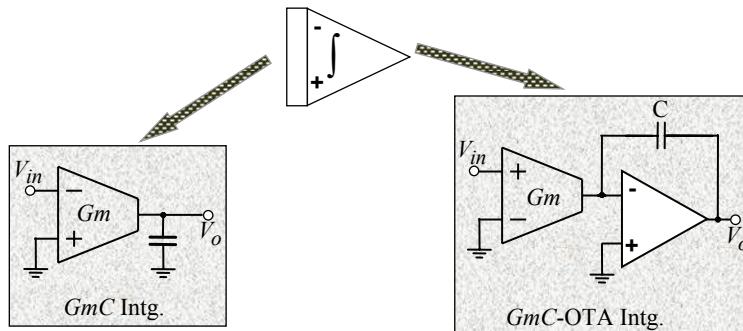
- Output in the form of voltage
- Low output impedance
- Can drive R-loads
- Good for RC filters,
OK for SC filters
- Extra buffer adds complexity,
power dissipation

OTA
Voltage controlled
current source



- Output in the form of current
- High output impedance
- In the context of filter design called *gm-cells*
- Cannot drive R-loads
- Good for SC & gm-C filters
- Typically, less complex compared to opamp \rightarrow higher freq. potential
- Typically lower power

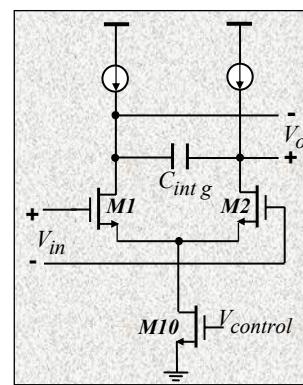
Integrator Implementation Transconductance-C & Opamp-Transconductance-C



$$\frac{V_o}{V_{in}} = \frac{-\omega_b}{s} \quad \text{where} \quad \omega_b = \frac{G_m}{C}$$

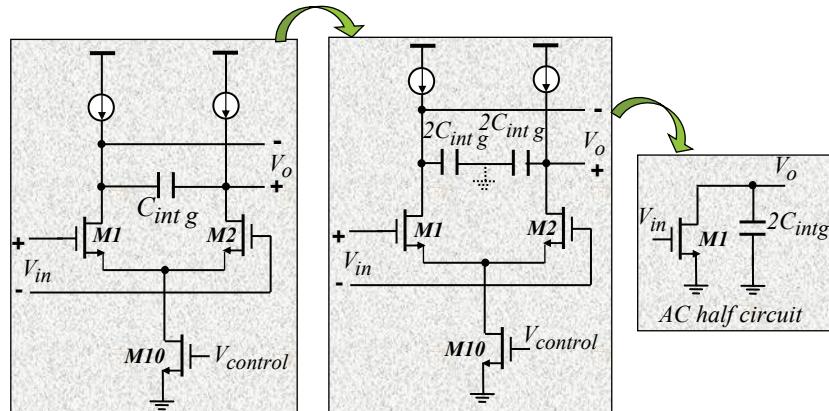
Gm-C Filters Simplest Form of CMOS Gm-C Integrator

- Transconductance element formed by the source-coupled pair $M1$ & $M2$
- All MOSFETs operating in saturation region
- Current in $M1$ & $M2$ can be varied by changing $V_{control}$
→ Transconductance of $M1$ & $M2$ varied through $V_{control}$



Ref: H. Khorramabadi and P.R. Gray, "High Frequency CMOS continuous-time filters," IEEE Journal of Solid-State Circuits, Vol.-SC-19, No. 6, pp.939-948, Dec. 1984.

Simplest Form of CMOS Gm-C Integrator AC Half Circuit



Gm-C Filters Simplest Form of CMOS Gm-C Integrator

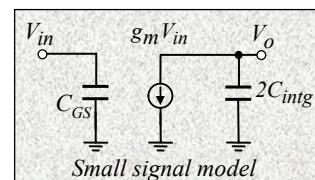
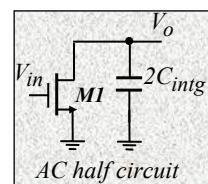
- Use ac half circuit & small signal model to derive transfer function:

$$V_o = -g_m^{M1,2} \times V_{in} \times 2C_{intg}s$$

$$\frac{V_o}{V_{in}} = -\frac{g_m^{M1,2}}{2C_{intg}s}$$

$$\frac{V_o}{V_{in}} = \frac{-\omega_0}{s}$$

$$\rightarrow \omega_0 = \frac{g_m^{M1,2}}{2 \times C_{intg}}$$



Gm-C Filters

Simplest Form of CMOS Gm-C Integrator

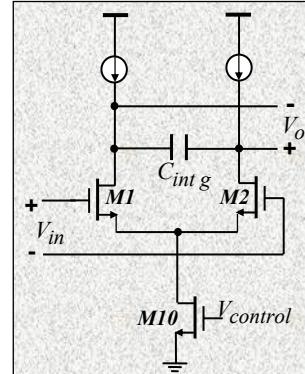
- MOSFET in saturation region:

$$I_d = \frac{\mu C_{ox}}{2} \frac{W}{L} (V_{gs} - V_{th})^2$$

- Gm is given by:

$$\begin{aligned} g_m^{M1 \& M2} &= \frac{\partial I_d}{\partial V_{gs}} = \mu C_{ox} \frac{W}{L} (V_{gs} - V_{th}) \\ &= 2 \frac{I_d}{(V_{gs} - V_{th})} \\ &= 2 \left(\frac{1}{2} \mu C_{ox} \frac{W}{L} I_d \right)^{1/2} \end{aligned}$$

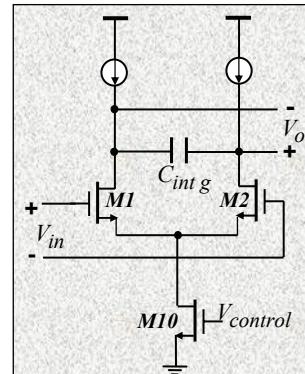
*Id varied via Vcontrol
→ gm tunable via Vcontrol*



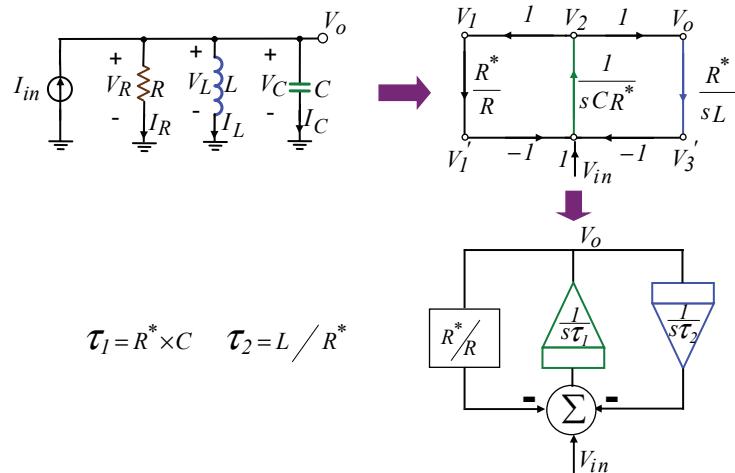
Gm-C Filters

2nd Order Gm-C Filter

- Use the Gm-cell to build a 2nd order bandpass filter



2nd Order Bandpass Filter



2nd Order Integrator-Based Bandpass Filter

$$\frac{V_{BP}}{V_{in}} = \frac{\tau_2 s}{\tau_1 \tau_2 s^2 + \beta \tau_2 s + 1}$$

$$\tau_1 = R^* \times C \quad \tau_2 = L / R^*$$

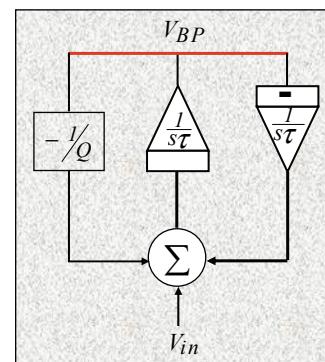
$$\beta = R^* / R$$

$$\omega_0 = 1 / \sqrt{\tau_1 \tau_2} = 1 / \sqrt{L C}$$

$$Q = 1 / \beta \times \sqrt{\tau_1 / \tau_2}$$

From matching point of view desirable:

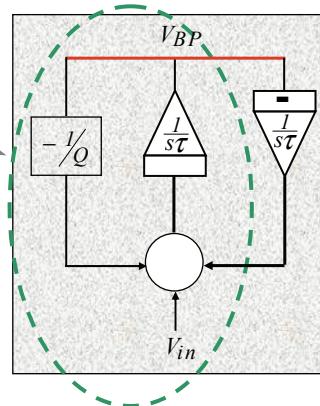
$$\tau_1 = \tau_2 = \tau = \frac{1}{\omega_0} \rightarrow Q = R / R^*$$



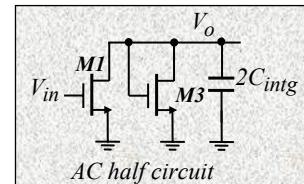
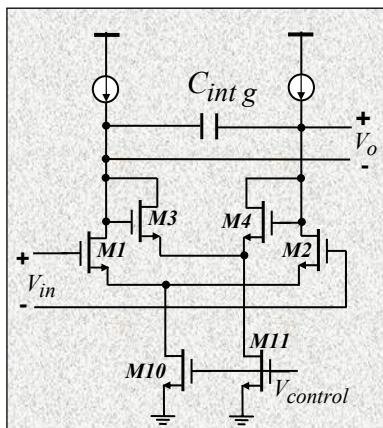
2nd Order Integrator-Based Bandpass Filter

First implement this part
With transfer function:

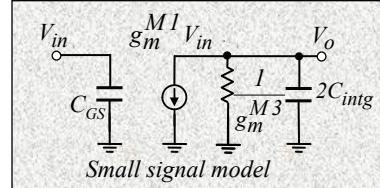
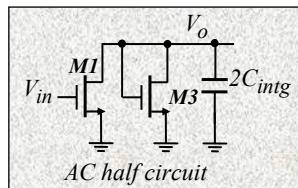
$$\frac{V_0}{V_{in}} = \frac{-1}{\frac{s}{\omega_0} + \frac{1}{Q}}$$



Terminated Gm-C Integrator



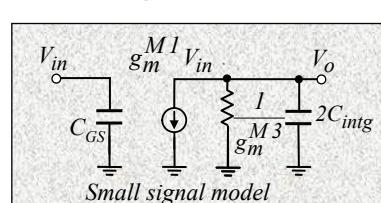
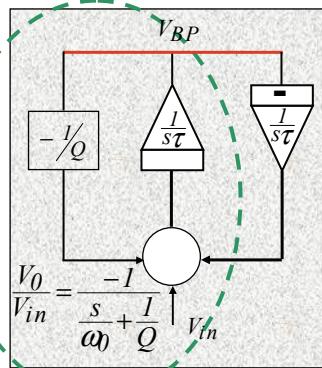
Terminated Gm-C Integrator



$$\frac{V_o}{V_{in}} = \frac{-1}{s \frac{2C_{intg}}{g_m^{MI}} + \frac{g_m^{M3}}{g_m^{MI}}}$$

$$\text{Compare to: } \frac{V_o}{V_{in}} = \frac{-1}{\frac{s}{\omega_0} + \frac{I}{Q}}$$

Terminated Gm-C Integrator



$$\rightarrow \omega_0 = \frac{g_m^{M1}}{2C_{intg}} \quad \& \quad Q = \frac{g_m^{M1}}{g_m^{M3}}$$

Question: How to define Q accurately?

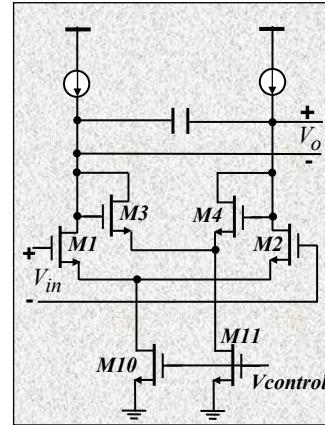
Terminated Gm-C Integrator

$$g_m^{M1} = 2 \left(\frac{1}{2} \mu C_{ox} \frac{W_{M1}}{L_{M1}} I_d^{M1} \right)^{1/2}$$

$$g_m^{M3} = 2 \left(\frac{1}{2} \mu C_{ox} \frac{W_{M3}}{L_{M3}} I_d^{M3} \right)^{1/2}$$

Let us assume equal channel lengths for M1, M3 then:

$$\frac{g_m^{M1}}{g_m^{M3}} = \left(\frac{I_d^{M1}}{I_d^{M3}} \times \frac{W_{M1}}{W_{M3}} \right)^{1/2}$$



Terminated Gm-C Integrator

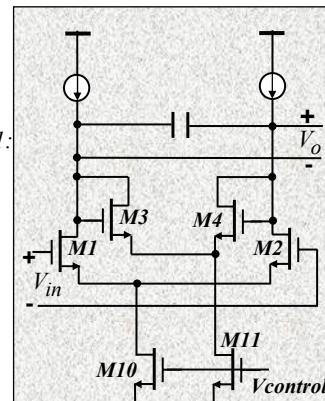
Note that:

$$\frac{I_d^{M1}}{I_d^{M3}} = \frac{I_d^{M10}}{I_d^{M11}}$$

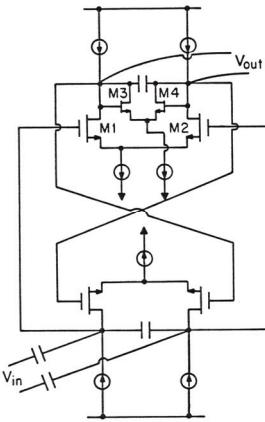
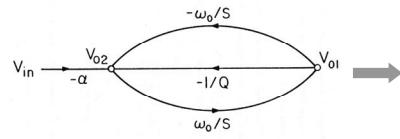
Assuming equal channel lengths for M10, M11:

$$\frac{I_d^{M10}}{I_d^{M11}} = \frac{W_{M10}}{W_{M11}}$$

$$\rightarrow \frac{g_m^{M1}}{g_m^{M3}} = \left(\frac{W_{M10}}{W_{M11}} \times \frac{W_{M1}}{W_{M3}} \right)^{1/2}$$



2nd Order Gm-C Filter



- Simple design
- Tunable
- Q function of device ratios:

$$Q = \frac{g_{m1,2}}{g_{m3,4}}$$