

## EE247

### Lecture 4

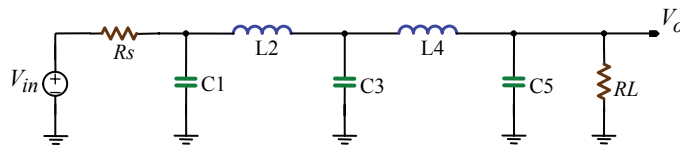
- Active ladder type filters
  - For simplicity, will start with all pole ladder type filters
    - Convert to integrator based form- example shown
  - Then will attend to high order ladder type filters incorporating zeros
    - Implement the same 7th order elliptic filter in the form of ladder RLC with zeros
      - Find level of sensitivity to component mismatch
      - Compare with cascade of biquads
    - Convert to integrator based form utilizing SFG techniques
  - Effect of integrator non-Idealities on filter frequency characteristics

## Summary Lecture 3

- Active Filters
  - Active biquads
    - Integrator-based filters
      - Signal flowgraph concept
      - First order integrator-based filter
      - Second order integrator-based filter & biquads
  - High order & high Q filters
    - Cascaded biquads & first order filters
      - Cascaded biquad sensitivity to component mismatch
    - Ladder type filters

## RLC Ladder Filters

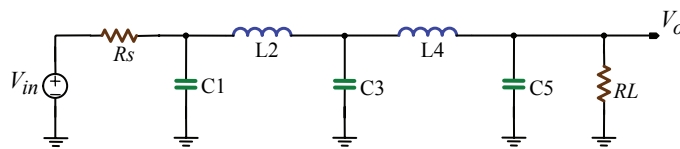
### Example: 5<sup>th</sup> Order Lowpass Filter



- Made of resistors, inductors, and capacitors
- Doubly terminated or singly terminated (with or w/o  $R_L$ )

*Doubly terminated LC ladder filters → Lowest sensitivity to component mismatch*

## LC Ladder Filters



- First step in the design process is to find values for  $L$ s and  $C$ s based on specifications:
  - Filter graphs & tables found in:
    - A. Zverev, *Handbook of filter synthesis*, Wiley, 1967.
    - A. B. Williams and F. J. Taylor, *Electronic filter design*, 3<sup>rd</sup> edition, McGraw-Hill, 1995.
  - CAD tools
    - Matlab,
    - Agilent ADS (includes Filter package → does the job of the tables)
    - Spice

## LC Ladder Filter Design Example

Design a LPF with maximally flat passband:

$$f_{-3dB} = 10\text{MHz}, f_{stop} = 20\text{MHz}$$

$$R_s > 27\text{dB @ } f_{stop}$$

- Maximally flat passband → Butterworth

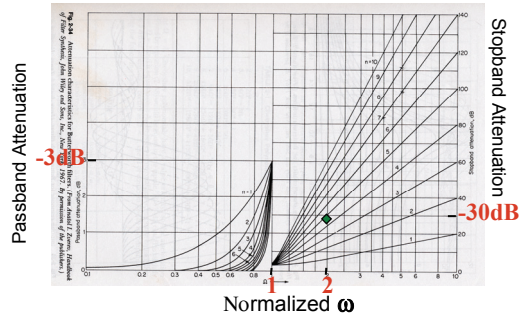
- Find minimum filter order
- Here standard graphs from filter books are used

$$f_{stop} / f_{-3dB} = 2$$

$$R_s > 27\text{dB}$$

Minimum Filter Order

⇒ 5th order Butterworth



From: Williams and Taylor, p. 2-37

## LC Ladder Filter Design Example

NORMALIZED FILTER DESIGN TABLES

11.3

Find values for L & C from Table: →

Note L & C values normalized to

$$\omega_{-3dB} = 1$$

Denormalization:

Multiply all  $L_{\text{Norm}}$ ,  $C_{\text{Norm}}$  by:

$$L_r = R / \omega_{-3dB}$$

$$C_r = 1 / (R \omega_{-3dB})$$

R is the value of the source and termination resistor (choose both 1Ω for now)

$$\text{Then: } L = L_r \times L_{\text{Norm}}$$

$$C = C_r \times C_{\text{Norm}}$$

TABLE 11-2 Butterworth LC Element Values (Continued)

n	$R_n$	$C_1$	$L_2$	$C_3$	$L_4$	$C_5$	$L_6$	$C_7$
5	1.0000	0.6180	1.6180	2.0000	1.6180	0.6180		
	0.9000	0.4416	1.0265	1.9095	1.7362	1.3867		
	0.8000	0.4698	0.8660	2.0605	1.5443	1.7380		
	0.7000	0.5173	0.7313	2.2849	1.3326	2.1083		
	0.6000	0.5860	0.6094	2.5998	1.1255	2.5524		
	0.5000	0.6857	0.4955	3.0510	0.9237	3.1331		
	0.4000	0.8378	0.3877	3.7357	0.7274	3.9648		
	0.3000	1.0537	0.2848	4.8835	0.5367	5.3073		
	0.2000	1.5677	0.1861	7.1849	0.3518	7.9345		
	0.1000	3.1522	0.0912	14.0945	0.1727	15.7105		
	Inf.	1.5451	1.6944	1.3820	0.8944	0.3090		
6	1.0000	0.5176	1.4142	1.9319	1.4142	0.5176		
	1.1111	0.2890	1.0403	1.3217	2.0539	1.7443	1.3347	
	1.2500	0.2445	1.1163	1.1237	2.2389	1.5498	1.6881	
	1.4286	0.2072	1.2263	0.9567	2.4991	1.3464	2.0618	
	1.6667	0.1732	1.4071	0.8011	2.8580	1.1431	2.5092	
	2.0000	0.1412	1.6531	0.6542	3.3687	0.9423	3.0938	
	2.5000	0.1108	2.0275	0.5159	4.1408	0.7450	3.9305	
	3.3333	0.0816	2.6599	0.3788	5.4325	0.5517	5.2804	
	5.0000	0.0535	3.9170	0.2484	8.0201	0.3628	7.9216	
	10.0000	0.0263	7.7053	0.1222	15.7855	0.1788	15.7875	
	Inf.	1.5529	1.7593	1.5529	1.2016	0.7579	0.2268	
7	1.0000	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450
	0.9000	0.2985	0.7111	1.4043	1.4891	2.1249	1.7268	1.2961
	0.8000	0.3215	0.6657	1.5174	1.2777	2.3338	1.5461	1.6520
	0.7000	0.3571	0.5154	1.6883	1.0910	2.6177	1.3498	2.0277
	0.6000	0.4075	0.4322	1.9284	0.9170	3.0050	1.1503	2.4771
	0.5000	0.4799	0.3536	2.2726	0.7512	3.5532	0.9515	3.0640
	0.4000	0.5899	0.2782	2.7950	0.5917	4.3799	0.7542	3.9037
	0.3000	0.7745	0.2055	3.6706	0.4373	5.7612	0.5690	5.2583
	0.2000	1.1448	0.1350	5.4267	0.2874	8.5263	0.3692	7.9079
	0.1000	2.2571	0.0665	10.7004	0.1417	16.8222	0.1823	15.7480
	Inf.	1.5576	1.7988	1.6588	1.3972	1.0550	0.6560	0.2225

From: Williams and Taylor, p. 11.3

## LC Ladder Filter Design Example

NORMALIZED FILTER DESIGN TABLES

11.3

Find values for L & C from Table:

Normalized values:

$$C1_{\text{Norm}} = C5_{\text{Norm}} = 0.618$$

$$C3_{\text{Norm}} = 2.0$$

$$L2_{\text{Norm}} = L4_{\text{Norm}} = 1.618$$

Denormalization:

$$\text{Since } \omega_{3dB} = 2\pi \times 10\text{MHz}$$

$$L_r = R/\omega_{3dB} = 15.9 \text{ nH}$$

$$C_r = 1/(R\omega_{3dB}) = 15.9 \text{ nF}$$

$$R = 1$$

$$\Rightarrow C1 = C5 = 9.836 \text{ nF}, C3 = 31.83 \text{ nF}$$

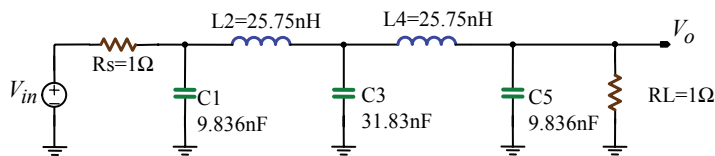
$$\Rightarrow L2 = L4 = 25.75 \text{ nH}$$

TABLE 11-2 Butterworth LC Element Values (Continued)

n	$R_k$	$G_1$	$L_2$	$C_3$	$L_4$	$C_5$	$L_6$	$C_7$
5	1.0000	0.6180	1.6180	2.0000	1.6180	0.6180		
	0.9000	0.4416	1.0265	1.9095	1.7562	1.3887		
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	0.7000	0.5173	0.7313	2.2849	1.3326	2.1083		
	0.6000	0.5860	0.6094	2.5998	1.1255	2.5524		
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	10.0000	0.0263	7.7053	0.1222	15.7855	0.1788	15.7375	
	Inf.	1.5529	1.7593	1.5529	1.2016	0.7579	0.2588	
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	Inf.	1.5576	1.7988	1.6588	1.3972	1.0550	0.6560	0.2225

From: Williams and Taylor, p. 11.3

## Last Lecture: Example: 5<sup>th</sup> Order Butterworth Filter



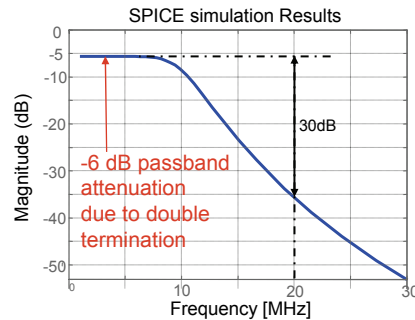
Specifications:

$$f_{-3dB} = 10\text{MHz},$$

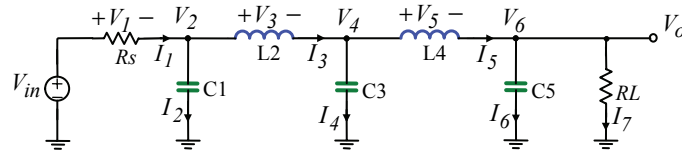
$$f_{\text{stop}} = 20\text{MHz}$$

$$R_s > 27\text{dB}$$

Used filter tables to obtain  
 $L_s$  &  $C_s$

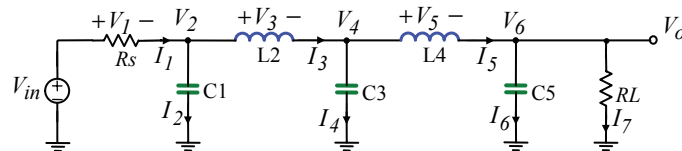


## Low-Pass RLC Ladder Filter Conversion to Integrator Based Active Filter



- To convert RLC ladder prototype to integrator based filter:
  - Use Signal Flowgraph technique
    - ✓ Name currents and voltages for all components
    - ✓ Use KCL & KVL to derive equations
    - ✓ Make sure reactive elements expressed as  $1/s$  term
      - $V(C) = f(I)$  &  $I(L) = f(V)$
    - ✓ Use state-space description to derive the SFG
    - ✓ Modify & simplify the SFG for implementation with integrators e.g. convert all current nodes to voltage

## Low-Pass RLC Ladder Filter Conversion to Integrator Based Active Filter



- Use KCL & KVL to derive equations:

$$\begin{aligned}
 V_1 &= V_{in} - V_2, & V_2 &= \frac{I_2}{sC_1}, & V_3 &= V_2 - V_4 \\
 V_4 &= \frac{I_4}{sC_3}, & V_5 &= V_4 - V_6, & V_6 &= \frac{I_6}{sC_5}, & V_o &= V_6 \\
 I_1 &= \frac{V_1}{R_s}, & I_2 &= I_1 - I_3, & I_3 &= \frac{V_3}{sL_2} \\
 I_4 &= I_3 - I_5, & I_5 &= \frac{V_5}{sL_4}, & I_6 &= I_5 - I_7, & I_7 &= \frac{V_6}{R_L}
 \end{aligned}$$

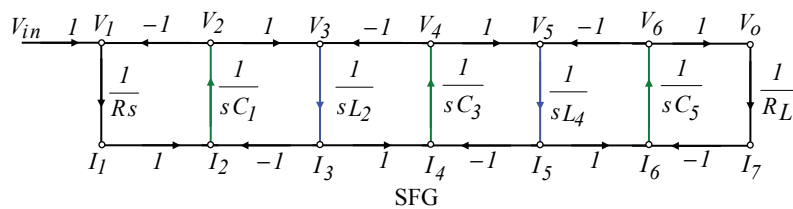
## Low-Pass RLC Ladder Filter Signal Flowgraph

$$V_1 = V_{in} - V_2, \quad V_2 = \frac{I_2}{sC_1}, \quad V_3 = V_2 - V_4$$

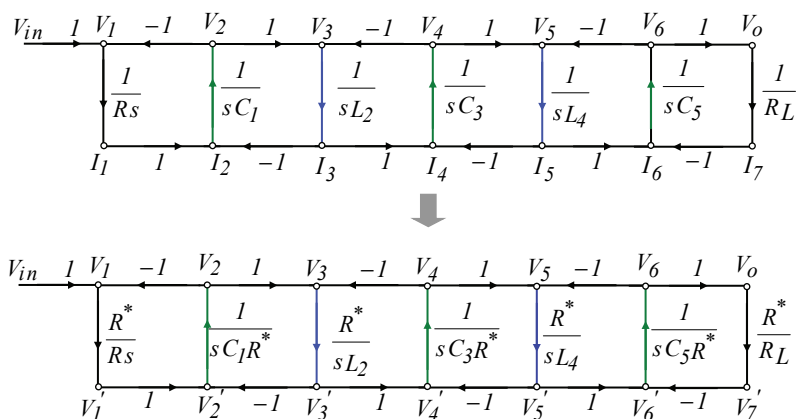
$$V_4 = \frac{I_4}{sC_3}, \quad V_5 = V_4 - V_6, \quad V_6 = \frac{I_6}{sC_5}, \quad V_o = V_6$$

$$I_1 = \frac{V_1}{Rs}, \quad I_2 = I_1 - I_3, \quad I_3 = \frac{V_3}{sL_2}$$

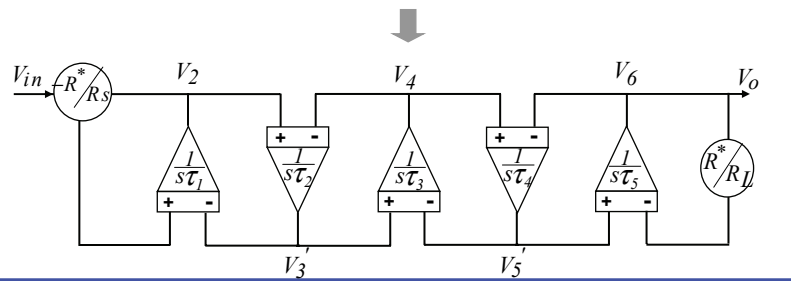
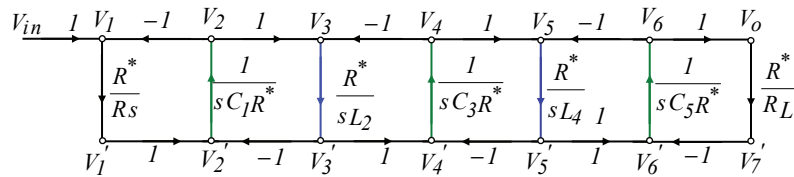
$$I_4 = I_3 - I_5, \quad I_5 = \frac{V_5}{sL_4}, \quad I_6 = I_5 - I_7, \quad I_7 = \frac{V_6}{RL}$$



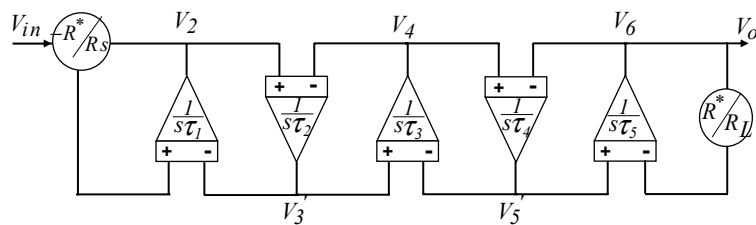
## Low-Pass RLC Ladder Filter Normalize



## Low-Pass RLC Ladder Filter Synthesize



## Low-Pass RLC Ladder Filter Integrator Based Implementation



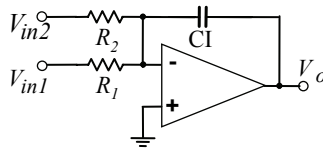
$$\tau_1 = C_1 \cdot R^* \quad , \quad \tau_2 = \frac{L_2}{R^*} = C_2 \cdot R^* \quad , \quad \tau_3 = C_3 \cdot R^* \quad , \quad \tau_4 = \frac{L_4}{R^*} = C_4 \cdot R^* \quad , \quad \tau_5 = C_5 \cdot R^*$$

Main building block: Integrator

Let us start to build the filter with RC& Opamp type integrator

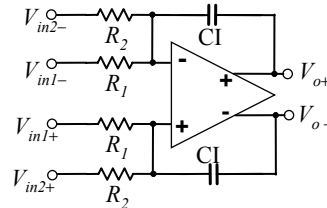
## Opamp-RC Integrator

### Single-Ended



$$V_o = -V_{in1} \times \frac{1}{sR_1C_I} - V_{in2} \times \frac{1}{sR_2C_I}$$

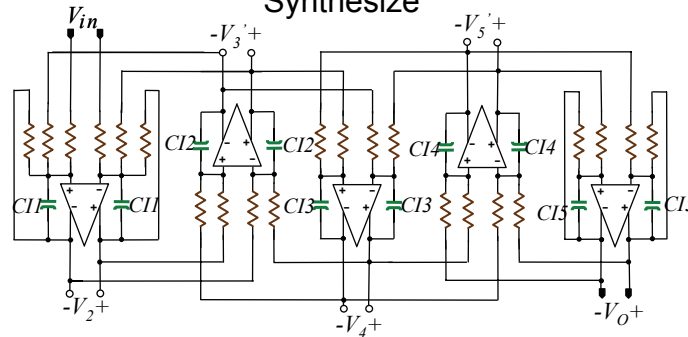
### Differential



$$V_{o+} - V_{o-} = (V_{in1+} - V_{in1-}) \times \frac{1}{sR_1C_I} + (V_{in2+} - V_{in2-}) \times \frac{1}{sR_2C_I}$$

Note: Implementation with single-ended integrator requires extra circuitry for sign inversion whereas in differential case both signal polarities are available

## Differential Integrator Based LP Ladder Filter Synthesize

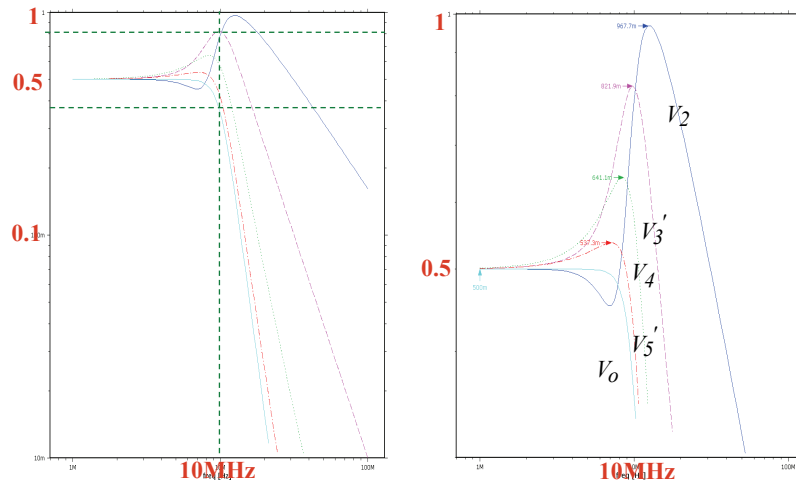


### • First iteration:

- All resistors are chosen =  $1\Omega$
- Values for  $\tau_x = R_x C_{I_x}$  found from RLC analysis
- Capacitors:  $CI1 = CI5 = 9.836nF$ ,  $CI2 = CI4 = 25.45nF$ ,  $CI3 = 31.83nF$



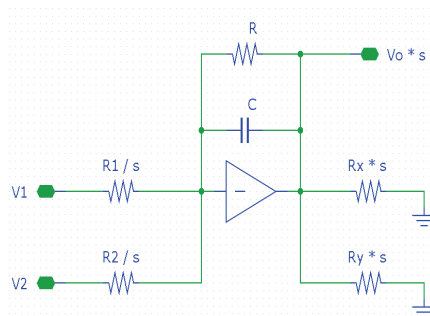
## Simulated Magnitude Response



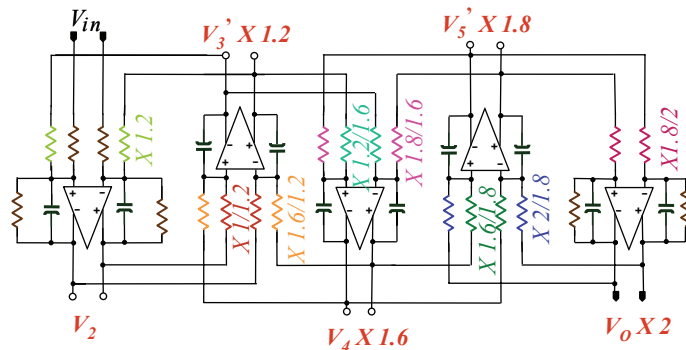
## Scale Node Voltages

To maximize dynamic range  
 → scale node voltages

Scale  $V_o$  by factor “s”



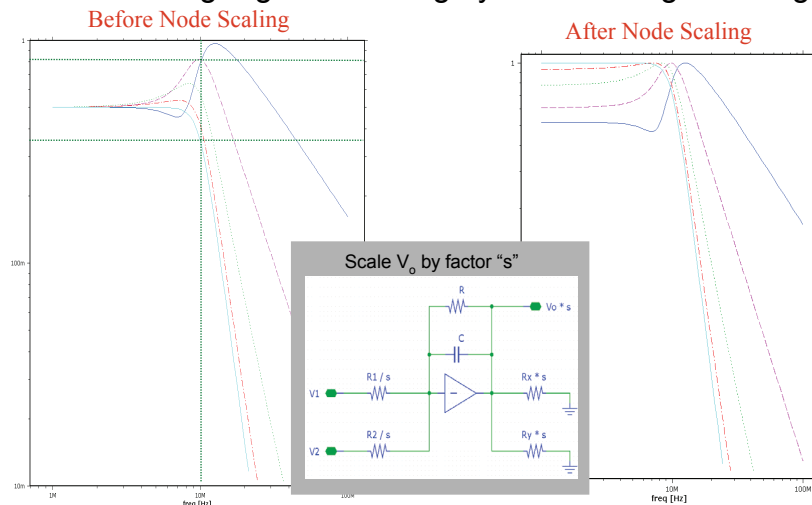
## Differential Integrator Based LP Ladder Filter Node Scaling



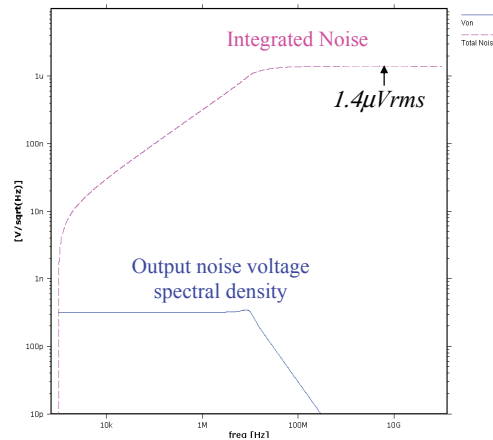
- Second iteration:

- Nodes scaled, note output node x2
- Resistor values scaled according to scaling of nodes
- Capacitors the same :  $C1=C5=9.836nF$ ,  $C2=C4=25.45nF$ ,  $C3=31.83nF$

## Maximizing Signal Handling by Node Voltage Scaling



# Filter Noise



Total noise @ the output:  
**1.4 μV rms**  
 (noiseless opamps)

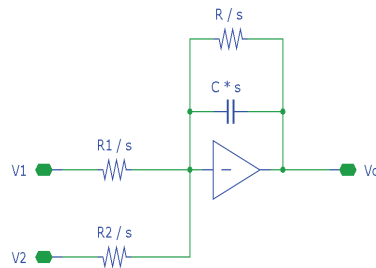
That's excellent, but:

- Capacitors too large for integration  
 → Unrealistically large Si area
- Resistors too small  
 → high power dissipation

Typical applications allow higher noise, assuming tolerable noise in the order of 140 μV rms ...

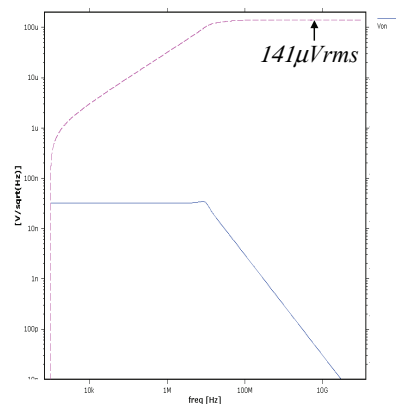
# Scale to Meet Noise Target

Scale capacitors and resistors to meet noise objective

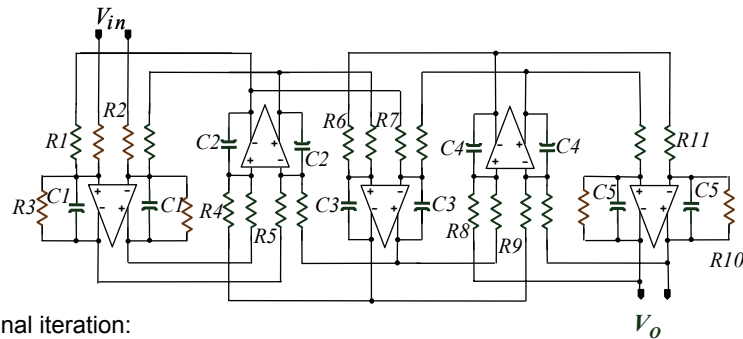


$$s = 10^{-4} \rightarrow (V_{n1}/V_{n2})^2$$

Noise after scaling: 141 μV rms (assuming noiseless opamps)



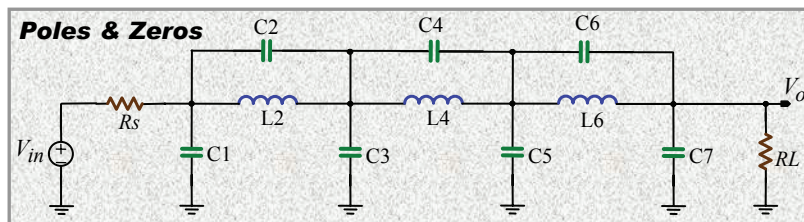
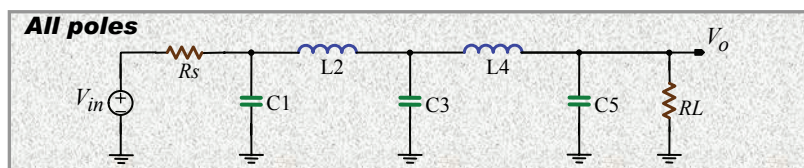
## Differential Integrator Based LP Ladder Filter Final Design



- Final iteration:

- Based on scaled nodes and noise considerations
- Capacitors:  $C1=C5=0.9836\text{pF}$ ,  $C2=C4=2.545\text{pF}$ ,  $C3=3.183\text{pF}$
- Resistors:  $R1=11.77\text{K}$ ,  $R2=9.677\text{K}$ ,  $R3=10\text{K}$ ,  $R4=12.82\text{K}$ ,  $R5=8.493\text{K}$ ,  $R6=11.93\text{K}$ ,  $R7=7.8\text{K}$ ,  $R8=10.75\text{K}$ ,  $R9=8.381\text{K}$ ,  $R10=10\text{K}$ ,  $R11=9.306\text{K}$

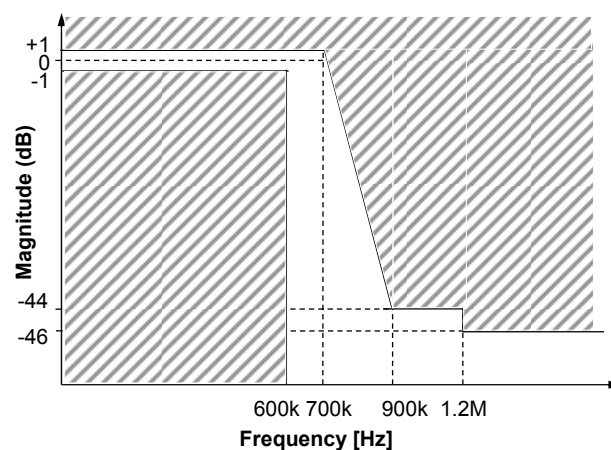
## RLC Ladder Filters Including Transmission Zeros



## RLC Ladder Filter Design Example

- Design a baseband filter for CDMA IS95 cellular phone receive path with the following specs.
  - Filter frequency mask shown on the next page
  - Allow enough margin for manufacturing variations
    - Assume overall tolerable pass-band magnitude variation of 1.8dB
    - Assume the -3dB frequency can vary by  $\pm 8\%$  due to manufacturing tolerances & circuit inaccuracies
  - Assume any phase impairment can be compensated in the digital domain
- \* Note this is the same example as for cascade of biquad while the specifications are given closer to a real product case

## RLC Ladder Filter Design Example CDMA IS95 Receive Filter Frequency Mask



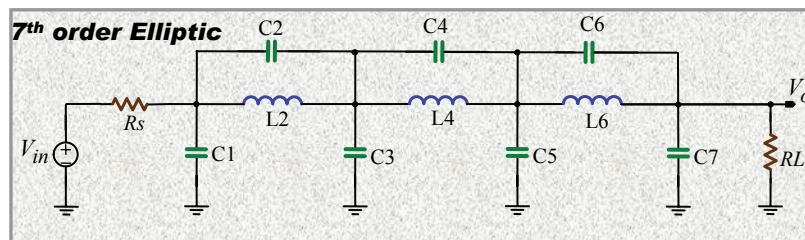
## RLC Ladder Filter Design

### Example: CDMA IS95 Receive Filter

- Since phase impairment can be corrected for, use filter type with max. roll-off slope/pole  
→ Filter type → Elliptic
- Design filter freq. response to fall well within the freq. mask
  - Allow margin for component variations & mismatches
- For the passband ripple, allow enough margin for ripple change due to component & temperature variations  
→ Design nominal passband ripple of 0.2dB
- For stopband rejection add a few dB margin  $44+5=49\text{dB}$
- Final design specifications:
  - $f_{\text{pass}} = 650 \text{ kHz}$   $R_{\text{pass}} = 0.2 \text{ dB}$
  - $f_{\text{stop}} = 750 \text{ kHz}$   $R_{\text{stop}} = 49 \text{ dB}$
- Use Matlab or ADS or filter tables to decide the min. order for the filter (same as cascaded biquad example)
  - 7<sup>th</sup> Order Elliptic

## RLC Low-Pass Ladder Filter Design

### Example: CDMA IS95 Receive Filter



- Use filter tables & charts to determine LC values

## RLC Ladder Filter Design

### Example: CDMA IS95 Receive Filter

- Specifications
  - fpass = 650 kHz                      Rpass = 0.2 dB
  - fstop = 750 kHz                     Rstop = 49 dB
- Use filter tables to determine LC values
  - Table from: A. Zverev, *Handbook of filter synthesis*, Wiley, 1967
  - Elliptic filters tabulated wrt “reflection coefficient  $\rho$ ”

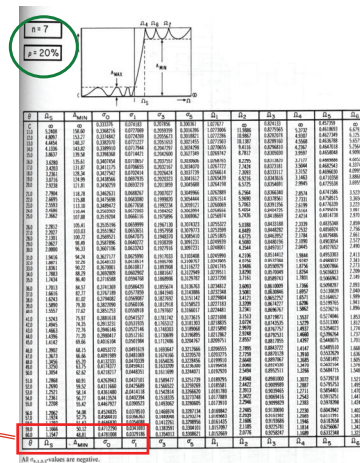
$$R_{pass} = -10 \times \log(1 - \rho^2)$$

- Since  $R_{pass}=0.2dB \rightarrow \rho=20\%$
- Use table accordingly

## RLC Ladder Filter Design

### Example: CDMA IS95 Receive Filter

- Table from Zverev book page #281 & 282:
- Since our spec. is  $A_{min}=44\text{dB}$  add 5dB margin & design for  $A_{min}=49\text{dB}$



• Table from Zverev  
page #281 & 282:

• Normalized  
component values:

C1=1.17677

C2=0.19393

L2=1.19467

C3=1.51134

C4=1.01098

L4=0.72398

C5=1.27776

C6=0.71211

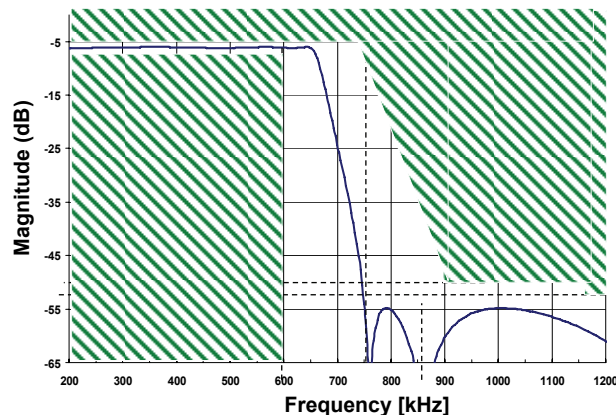
L6=0.80165

C7=0.83597

$K^2 = 1.0$									
$\beta$	$C_1$	$C_2$	$L_2$	$C_3$	$C_4$	$L_4$	$C_5$	$C_6$	$C_7$
1.0	1.335	0.00000	1.389	2.240	0.00000	1.515	2.240	0.00000	1.389
1.1	1.3364	0.00003	1.38116	2.21490	0.02330	1.48669	2.20558	0.01637	1.36266
1.2	1.33982	0.00099	1.36714	2.21011	0.02777	1.46125	2.19903	0.01952	1.36559
1.3	1.32892	0.00704	1.38102	2.20451	0.03264	1.47534	2.19192	0.02295	1.36161
1.4	1.32794	0.00818	1.37982	2.19929	0.03752	1.46997	2.18424	0.02567	1.35975
1.5	1.32690	0.00941	1.37861	2.19307	0.04242	1.46513	2.17601	0.02868	1.35900
1.6	1.32577	0.01072	1.37712	2.18683	0.04737	1.45484	2.16721	0.03199	1.34775
1.7	1.32457	0.01213	1.37564	2.17999	0.05237	1.44708	2.15786	0.03559	1.34249
1.8	1.32330	0.01362	1.37406	2.17273	0.05733	1.43886	2.14796	0.03941	1.33801
1.9	1.32194	0.01521	1.37238	2.16507	0.06234	1.43019	2.13750	0.04337	1.33190
2.0	1.32051	0.01689	1.37061	2.15700	0.06748	1.42107	2.12649	0.04757	1.32478
2.1	1.31900	0.01866	1.36874	2.14852	0.07277	1.41149	2.11493	0.05113	1.31827
2.2	1.31741	0.02054	1.36677	2.13964	0.07827	1.40147	2.10283	0.05512	1.31137
2.3	1.31574	0.02250	1.36470	2.13035	0.08394	1.39100	2.09018	0.05954	1.30418
2.4	1.31398	0.02457	1.36253	2.12066	0.08977	1.38009	2.07699	0.06437	1.29666
2.5	1.31215	0.02674	1.36026	2.11057	0.09577	1.36874	2.06327	0.06957	1.28887
2.6	1.31022	0.02901	1.35788	2.10000	0.10199	1.35695	2.04901	0.07519	1.28066
2.7	1.30822	0.03138	1.35540	2.08919	0.10848	1.34473	2.03422	0.08133	1.27218
2.8	1.30612	0.03386	1.35281	2.07790	0.11520	1.33207	2.01890	0.08797	1.26336
2.9	1.30394	0.03645	1.35012	2.06621	0.12216	1.31899	2.00305	0.09504	1.25423
3.0	1.30167	0.03914	1.34731	2.05413	0.12935	1.30549	1.98669	0.10254	1.24476
3.1	1.29930	0.04196	1.34439	2.04165	0.13683	1.29156	1.96980	0.11055	1.23497
3.2	1.29684	0.04488	1.34136	2.02878	0.14457	1.27722	1.95241	0.11905	1.22485
3.3	1.29429	0.04793	1.33821	2.01552	0.15256	1.26247	1.93450	0.12805	1.21440
3.4	1.29164	0.05109	1.33494	2.00187	0.16083	1.24730	1.91589	0.13757	1.20362
3.5	1.28889	0.05438	1.33155	1.98782	0.16937	1.23173	1.89717	0.14761	1.19259
3.6	1.28603	0.05780	1.32803	1.97339	0.17817	1.21576	1.87776	0.15823	1.18106
3.7	1.28307	0.06135	1.32439	1.95857	0.18720	1.19939	1.85766	0.16951	1.16908
3.8	1.28001	0.06504	1.32062	1.94336	0.19646	1.18263	1.83747	0.18151	1.15659
3.9	1.27683	0.06887	1.31671	1.92777	0.20598	1.16544	1.81659	0.19425	1.14359
4.0	1.27355	0.07284	1.31267	1.91179	0.21577	1.14795	1.79504	0.20766	1.13000
4.1	1.27014	0.07696	1.30849	1.89547	0.22580	1.13003	1.77342	0.22163	1.11679
4.2	1.26662	0.08123	1.30416	1.87867	0.23607	1.11174	1.75113	0.23617	1.10392
4.3	1.26297	0.08566	1.29969	1.86154	0.24659	1.09308	1.72827	0.25131	1.09026
4.4	1.25920	0.09026	1.29506	1.84403	0.25734	1.07406	1.70517	0.26707	1.07579
4.5	1.25529	0.09504	1.29027	1.82614	0.26836	1.05467	1.68151	0.28348	1.06056
4.6	1.25125	0.09999	1.28532	1.80786	0.27967	1.03493	1.65741	0.30055	1.04479
4.7	1.24707	0.10513	1.28020	1.78920	0.29126	1.01484	1.63287	0.31827	1.02837
4.8	1.24274	0.11046	1.27491	1.77015	0.30309	0.99439	1.60791	0.33661	1.01122
4.9	1.23826	0.11600	1.26943	1.75073	0.31516	0.97361	1.58252	0.35556	0.99331
5.0	1.23362	0.12175	1.26377	1.73092	0.32746	0.95260	1.55672	0.37511	0.97479
5.1	1.22882	0.12772	1.25791	1.71072	0.34000	0.93130	1.53051	0.39526	0.95569
5.2	1.22385	0.13394	1.25184	1.69014	0.35287	0.90977	1.50390	0.41600	0.93592
5.3	1.21869	0.14040	1.24556	1.66917	0.36606	0.88798	1.47690	0.43731	0.91540
5.4	1.21335	0.14712	1.23906	1.64782	0.37955	0.86597	1.44952	0.45916	0.89426
5.5	1.20781	0.15412	1.23233	1.62607	0.39332	0.84370	1.42177	0.48156	0.87252
5.6	1.20207	0.16141	1.22534	1.60392	0.40736	0.82116	1.39363	0.50449	0.85016
5.7	1.19610	0.16892	1.21810	1.58138	0.42166	0.79836	1.36518	0.52794	0.82721
5.8	1.18991	0.17666	1.21058	1.55844	0.43620	0.77530	1.33637	0.55189	0.80366
5.9	1.18342	0.18463	1.20278	1.53510	0.45098	0.75200	1.30720	0.57634	0.77951
6.0	1.17677	0.19293	1.19467	1.51134	0.46600	0.72838	1.27776	0.60127	0.75476
$\beta$	$C_1$	$C_2$	$L_2$	$C_3$	$C_4$	$L_4$	$C_5$	$C_6$	$C_7$

## RLC Filter Frequency Response

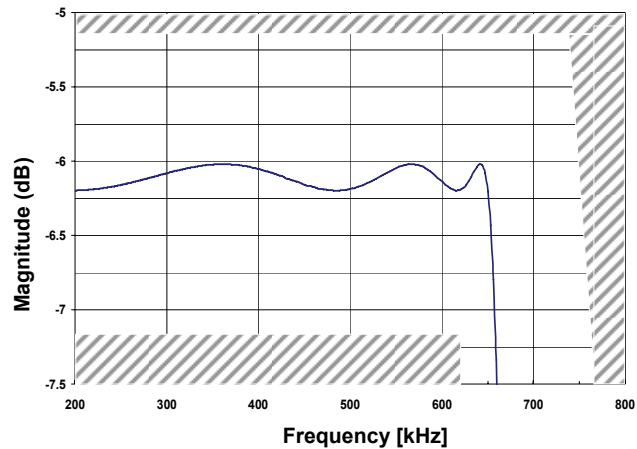
- Component values denormalized
- Frequency response simulated
- Frequency mask superimposed
- Frequency response well within spec.





## Frequency Response Passband Detail

- Passband well within spec.
- Make sure enough margin is allowed for variations due to process & temperature



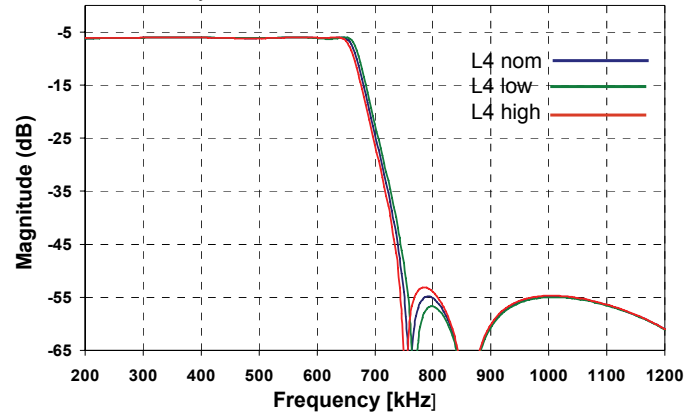
## RLC Ladder Filter Sensitivity

- The design has the same specifications as the previous example implemented with cascaded biquads
- To compare the sensitivity of RLC ladder versus cascaded-biquads:
  - Changed all  $L$ s &  $C$ s one by one by 2% in order to change the pole/zeros by 1% (similar test as for cascaded biquad)
  - Found frequency response  $\rightarrow$  most sensitive to  $L_4$  variations
  - Note that by varying  $L_4$  both poles & zeros are varied

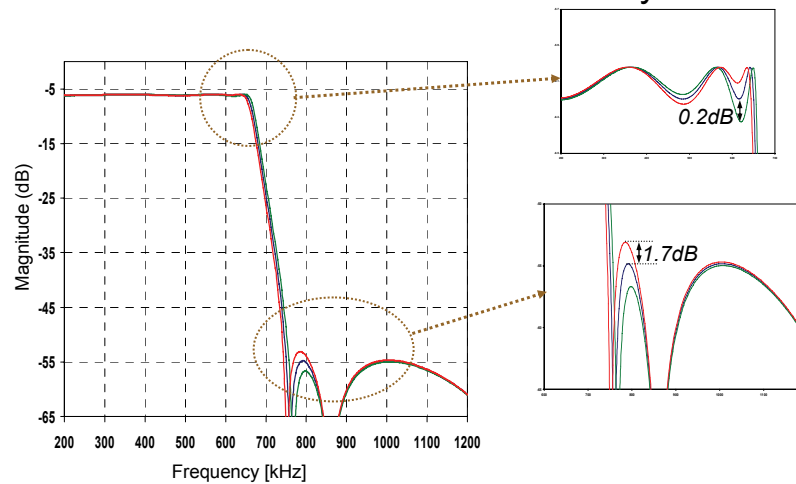
## RCL Ladder Filter Sensitivity

Component mismatch in RLC filter:

- Increase L4 from its nominal value by 2%
- Decrease L4 by 2%



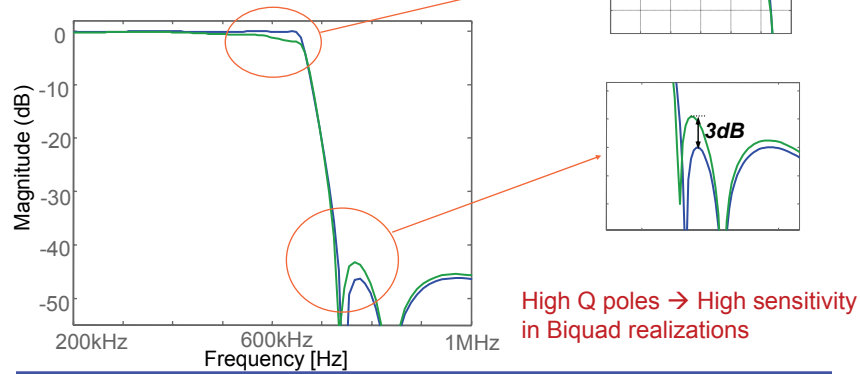
## RCL Ladder Filter Sensitivity



## Sensitivity of Cascade of Biquads

Component mismatch in Biquad 4 (highest Q pole):

- Increase  $\omega_{p4}$  by 1%
- Decrease  $\omega_{z4}$  by 1%



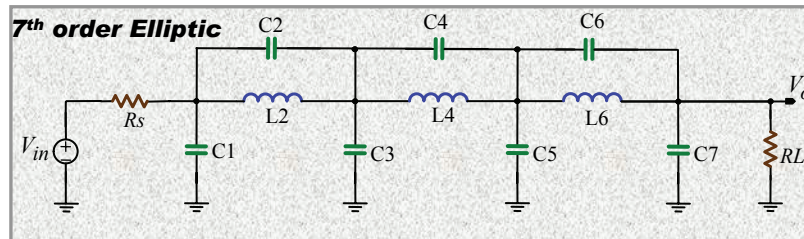
## Sensitivity Comparison for Cascaded-Biquads versus RLC Ladder

- 7<sup>th</sup> Order elliptic filter
  - 1% change in pole & zero pair

	<u>Cascaded Biquad</u>	<u>RLC Ladder</u>
Passband deviation	2.2dB (29%)	0.2dB (2%)
Stopband deviation	3dB (40%)	1.7dB (21%)

*Doubly terminated LC ladder filters  $\Rightarrow$  Significantly lower sensitivity compared to cascaded-biquads particularly within the passband*

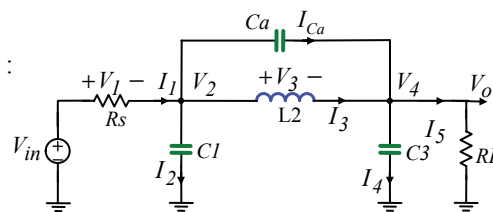
## RLC Ladder Filter Design Example: CDMA IS95 Receive Filter



- Previously learned to design integrator based ladder filters without transmission zeros
  - Question:
    - How do we implement the transmission zeros in the integrator-based version?
    - Preferred method → no extra power dissipation → no extra active elements

## Integrator Based Ladder Filters How Do to Implement Transmission zeros?

- Use KCL & KVL to derive :



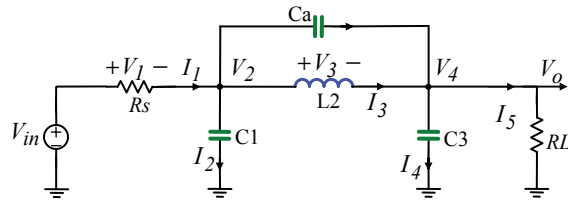
$$I_2 = I_1 - I_3 - I_{C_a}, \quad I_{C_a} = (V_2 - V_4)sC_a, \quad V_2 = \frac{I_2}{sC_1}$$

$$\text{Substituting for } I_2 \rightarrow V_2 = \frac{I_1 - I_3 - I_{C_a}}{sC_1}$$

$$\text{Substituting for } I_{C_a} \text{ and rearranging: } V_2 = \frac{I_1 - I_3}{s(C_1 + C_a)} + V_4 \times \frac{C_a}{C_1 + C_a}$$

## Integrator Based Ladder Filters

### How Do to Implement Transmission zeros?



- Use KCL & KVL to derive :

$$V_2 = \frac{I_1 - I_3}{s(C_1 + C_a)} + V_4 \times \frac{C_a}{C_1 + C_a}$$

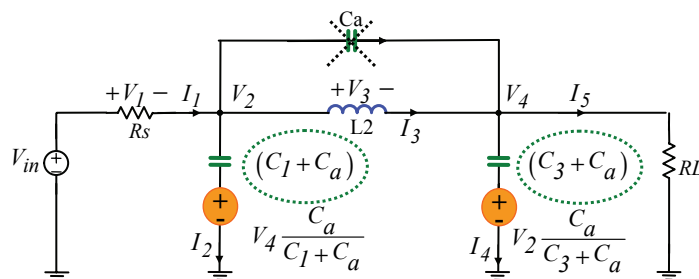
*Frequency independent constants  
Can be substituted by:*

$$V_4 = \frac{I_3 - I_5}{s(C_3 + C_a)} + V_2 \times \frac{C_a}{C_3 + C_a}$$

*Voltage-Controlled Voltage Source*

## Integrator Based Ladder Filters

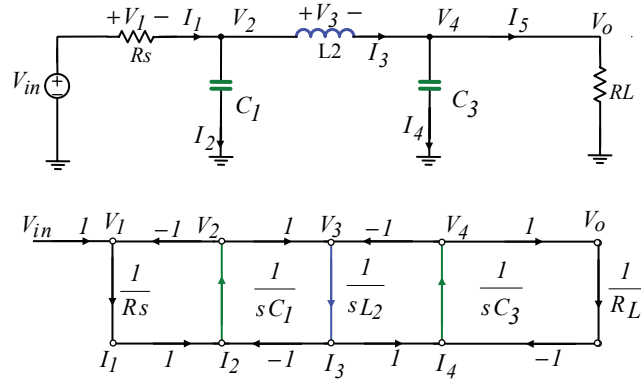
### Transmission zeros



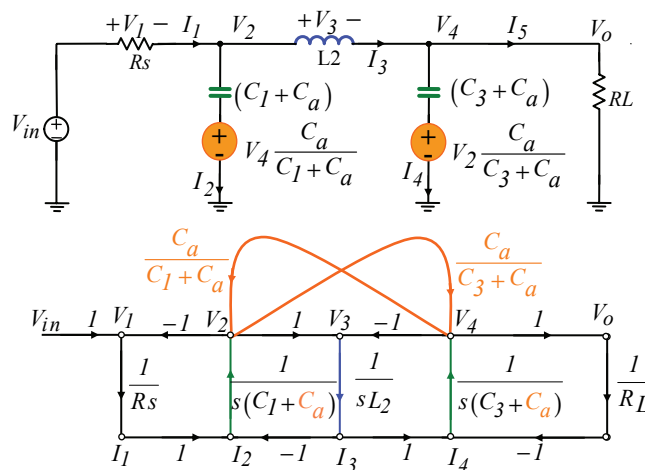
- Replace *shunt capacitors* with *voltage controlled voltage sources*:

$$\left. \begin{aligned} V_2 &= \frac{I_1 - I_3}{s(C_1 + C_a)} + V_4 \frac{C_a}{C_1 + C_a} \\ V_4 &= \frac{I_3 - I_5}{s(C_3 + C_a)} + V_2 \frac{C_a}{C_3 + C_a} \end{aligned} \right\} \text{Exact same expressions as with } C_a \text{ present}$$

### 3<sup>rd</sup> Order Lowpass Filter All Poles & No Zeros



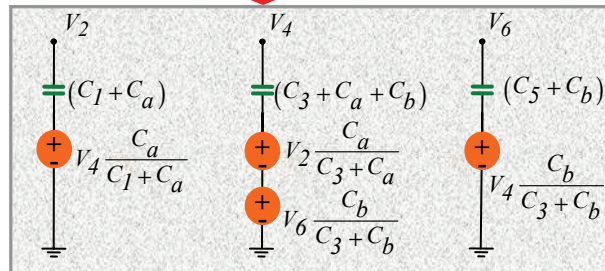
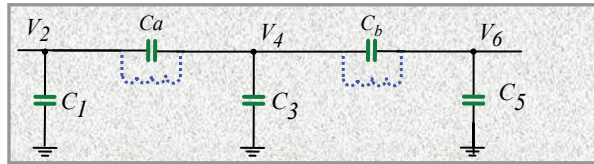
### Implementation of Zeros in Active Ladder Filters Without Use of Active Elements



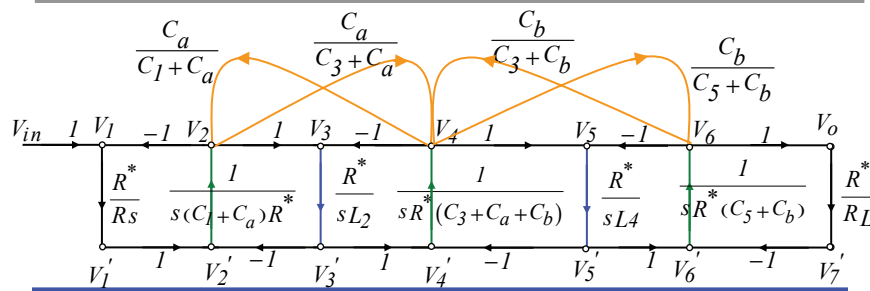
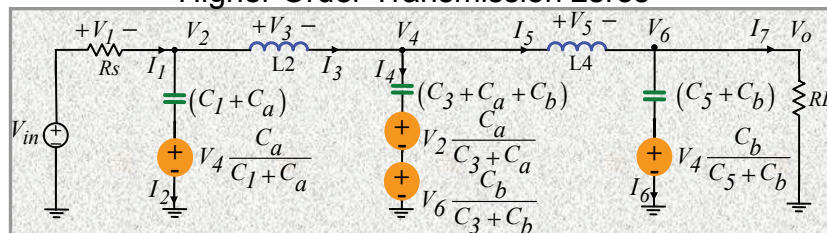
## Integrator Based Ladder Filters

### Higher Order Transmission zeros

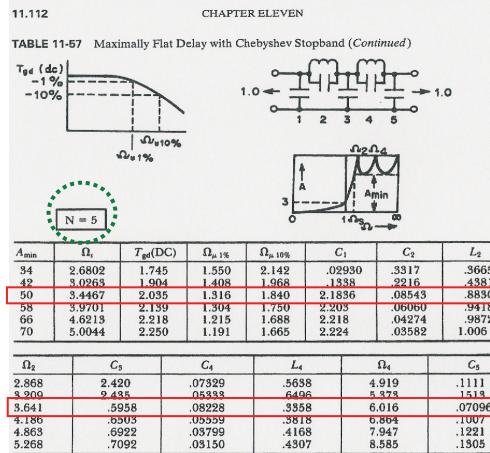
Convert zero generating Cs in C loops to voltage-controlled voltage sources



## Higher Order Transmission zeros



## Example: 5<sup>th</sup> Order Chebyshev II Filter



• 5<sup>th</sup> order Chebyshev II

• Table from: Williams & Taylor book, p. 11.112

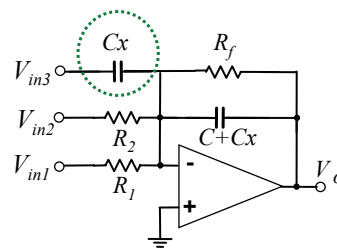
• 50dB stopband attenuation

•  $f_{-3dB} = 10\text{MHz}$

## Transmission Zero Generation Opamp-RC Integrator

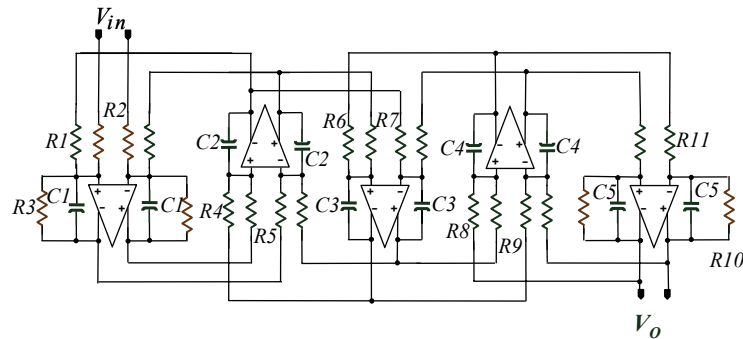
$$V_o = -\frac{1}{s(C+C_x)} \left[ \frac{V_{in1}}{R_1} + \frac{V_{in2}}{R_2} + \frac{V_o}{R_f} \right]$$

$$-V_{in3} \times \frac{C_x}{C+C_x}$$

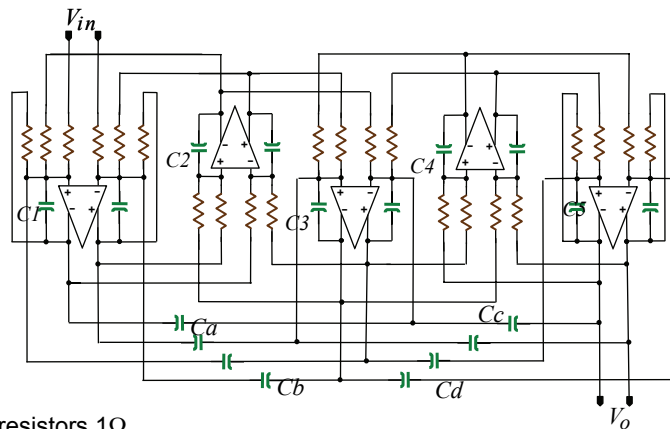




## Differential Integrator Based LP Ladder Filter Final Design 5<sup>th</sup> Order All-Pole



## Differential 5<sup>th</sup> Order Chebyshev Lowpass Filter

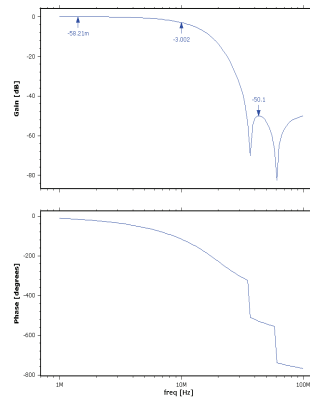


□ All resistors  $1\Omega$

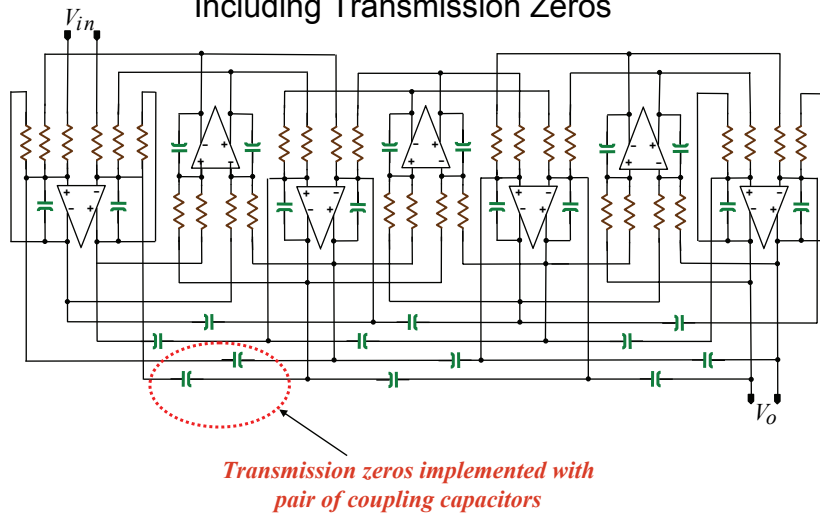
□ Capacitors:  $C1=36.11nF$ ,  $C2=14.05nF$ ,  $C3=12.15nF$ ,  $C4=5.344nF$ ,  $C5=2.439nF$

□ Coupling capacitors:  $C_a=1.36nF$ ,  $C_b=1.36nF$ ,  $C_c=1.31nF$ ,  $C_d=1.31nF$

## 5<sup>th</sup> Order Chebyshev II Filter Simulated Frequency Response



## 7<sup>th</sup> Order Differential Lowpass Filter Including Transmission Zeros



## Effect of Integrator Non-Idealities on Filter Frequency Characteristics

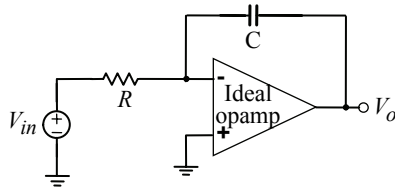
- In the passive filter design (RLC filters) section:
  - Reactive element (L & C) non-idealities → expressed in the form of Quality Factor (Q)
  - Filter impairments due to component non-idealities explained in terms of component Q
- In the context of active filter design (integrator-based filters)
  - Integrator non-idealities → Translates to the form of Quality Factor (Q)
  - Filter impairments due to integrator non-idealities explained in terms of integrator Q

## Effect of Integrator Non-Idealities on Filter Performance

- Ideal integrator characteristics
- Real integrator characteristics:
  - Effect of opamp finite DC gain
  - Effect of integrator non-dominant poles

## Effect of Integrator Non-Idealities on Filter Performance

### Ideal Integrator



Ideal Integrator:

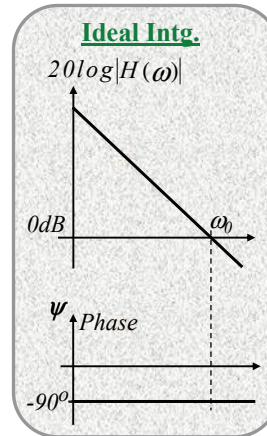
*DC gain =  $\infty$*

*Single pole @  $DC$*

*$\rightarrow$  no non-dominant poles*

$$H(s) = \frac{-\omega_0}{s}$$

$$\omega_0 = 1/RC$$



## Ideal Integrator Quality Factor

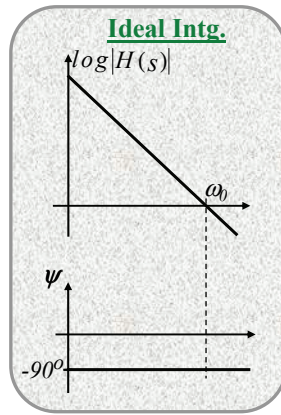
**Ideal intg. transfer function:**  $H(s) = \frac{-\omega_0}{s} = \frac{-\omega_0}{j\omega} = -\frac{1}{j\frac{\omega}{\omega_0}}$

Since component Q is defined as:: 
$$\begin{cases} H(j\omega) = \frac{1}{R(\omega) + jX(\omega)} \\ Q = \frac{X(\omega)}{R(\omega)} \end{cases}$$

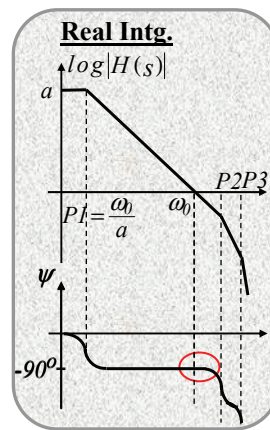
Then:

$$Q_{ideal}^{intg.} = \infty$$

## Real Integrator Opamp Related Non-Idealities

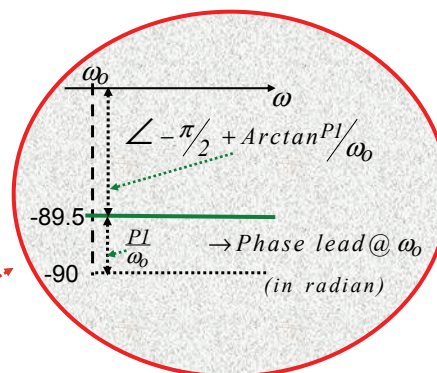
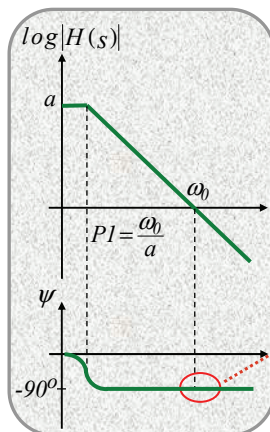


$$H(s) = \frac{-\omega_0}{s}$$



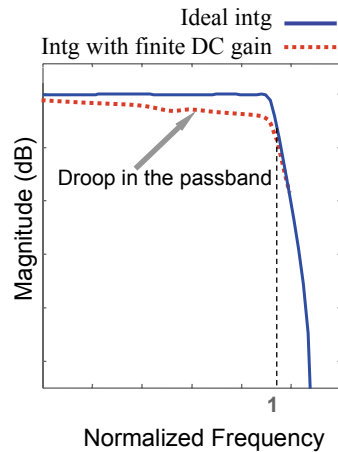
$$H(s) \approx \frac{-a}{\left(1 + s \frac{a}{\omega_0}\right) \left(1 + \frac{s}{p_2}\right) \left(1 + \frac{s}{p_3}\right) \dots}$$

## Effect of Integrator Finite DC Gain on Q



Example:  $a=100 \rightarrow P1/\omega_0 = 1/100$   
 $\rightarrow$  phase error  $\cong +0.5 \text{ degree}$

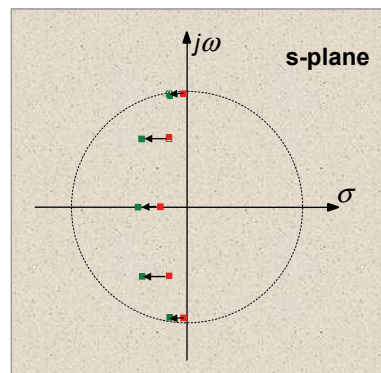
## Effect of Integrator Finite DC Gain on Q Example: Lowpass Filter



- Finite opamp DC gain
  - Phase lead @  $\omega_0$
  - Droop in the passband

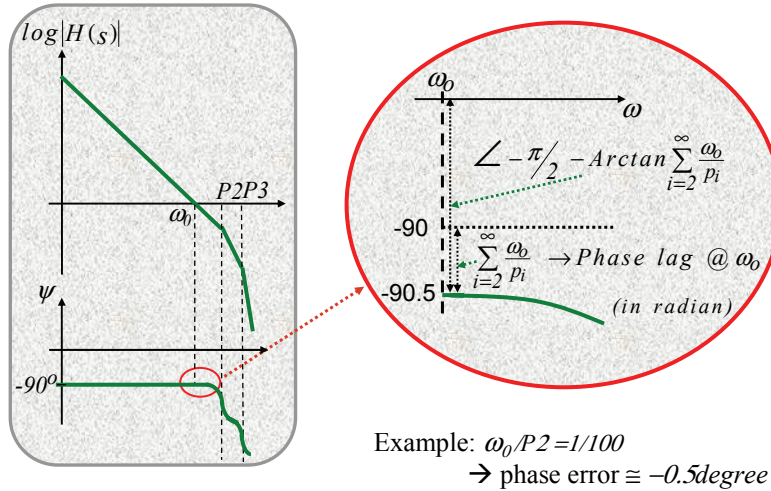
## Effect of Integrator Finite DC Gain on Q Example: Lowpass Filter

- Effect of opamp finite DC gain on filter singularities
- Pushes the ideal poles away from the  $j\omega$  axis
- Results in Q reduction of the poles and thus droop in the passband

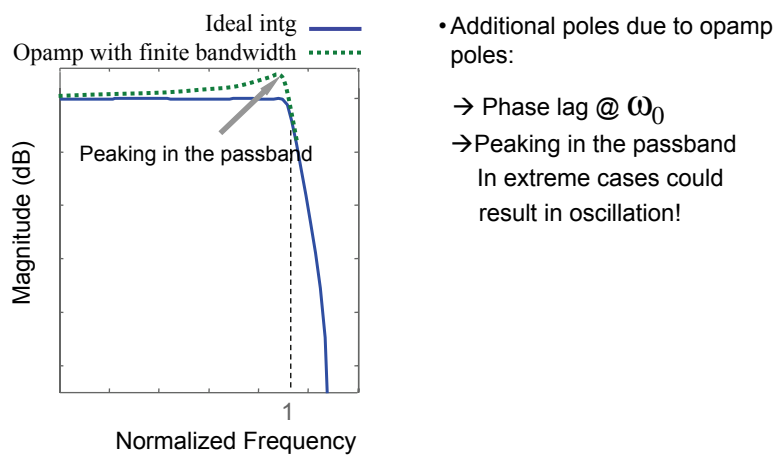


- Opamp with finite DC gain
- Ideal opamps & ideal filter pole locations

## Effect of Integrator Opamp Related Non-Dominant Poles

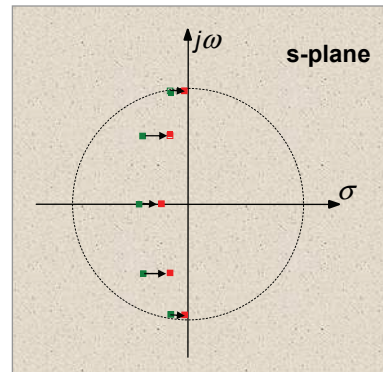


## Effect of Integrator Non-Dominant Poles Example: Lowpass Filter



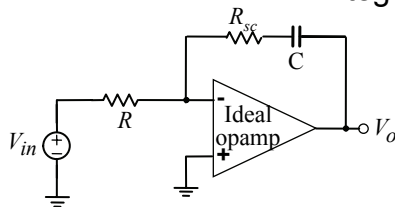
## Effect of Integrator Non-Dominant Poles Example: Lowpass Filter

- Effect of opamp finite bandwidth on filter singularities
- Pushes the ideal poles towards  $j\omega$  axis
- Results in Q enhancement of the poles and thus peaking in the passband



- Opamp with finite bandwidth
- Ideal opamps & ideal pole locations

## Effect of Integrating Capacitor Series Resistance on Integrator Q

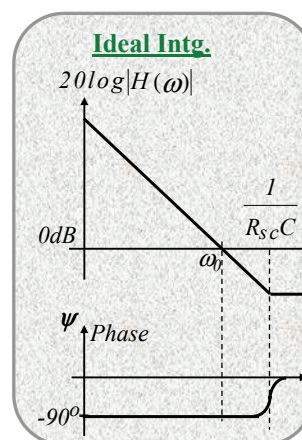


Finite  $R_{sc}$  adds LHP zero @  $\frac{1}{R_{sc}C}$

$$H(s) = \frac{-a_0(1 + R_{sc}Cs)}{s}$$

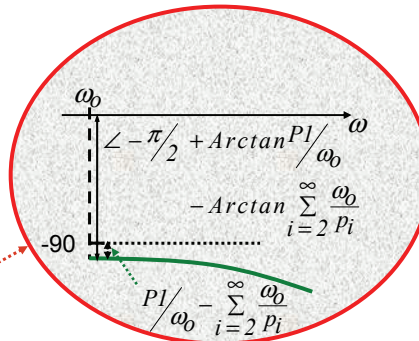
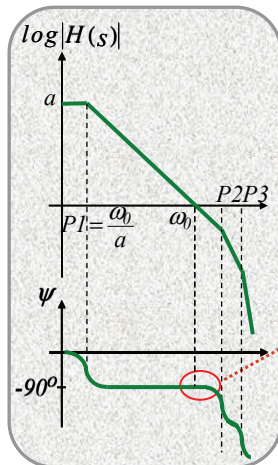
$$\rightarrow Q_{intg} \approx \frac{R}{R_{sc}}$$

Typically, opamp non-idealities dominate  $Q_{intg}$





## Effect of Integrator Non-Dominant Poles & Finite DC Gain on Q



Note that the two terms have different signs  
→ Can cancel each other's effect!

## Integrator Quality Factor

Real intg. transfer function: 
$$H(s) \approx \frac{-a}{\left(1 + s \frac{a}{\omega_0}\right) \left(1 + \frac{s}{p_2}\right) \left(1 + \frac{s}{p_3}\right) \dots}$$

Based on the definition of Q and assuming that:

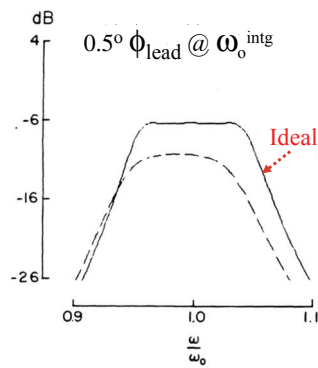
$$\frac{\omega_0}{p_{2,3,\dots}} \ll 1 \quad \& \quad a \gg 1$$

It can be shown that in the vicinity of unity-gain-frequency:

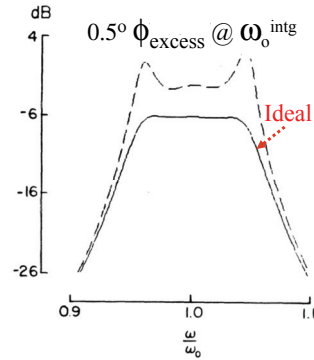
$$Q_{real}^{intg.} \approx \frac{1}{\frac{1}{a} - \omega_0 \sum_{i=2}^{\infty} \frac{1}{p_i}}$$

Phase lead @  $\omega_0$       Phase lag @  $\omega_0$

Example:  
Effect of Integrator Finite Q on Bandpass Filter Behavior

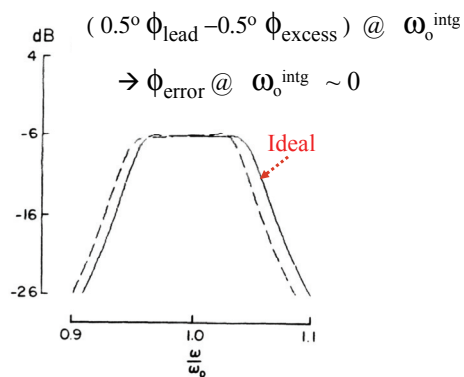


Integrator DC gain=100



Integrator P2 @  $100 \cdot \omega_0$

Example:  
Effect of Integrator Q on Filter Behavior



Integrator DC gain=100 & P2 @  $100 \cdot \omega_0$

## Summary

### Effect of Integrator Non-Idealities on Q

$$Q_{ideal}^{intg.} = \infty$$

$$Q_{real}^{intg.} \approx \frac{1}{\frac{1}{a} - \omega_0 \sum_{i=2}^{\infty} \frac{1}{p_i}}$$

- Amplifier finite DC gain reduces the overall Q in the same manner as series/parallel resistance associated with passive elements
- Amplifier poles located above integrator unity-gain frequency enhance the Q!
  - If non-dominant poles close to unity-gain freq. → Oscillation
- Depending on the location of unity-gain-frequency, the two terms can cancel each other out!
- Overall quality factor of the integrator has to be much higher compared to the filter's highest pole Q