

# EE247

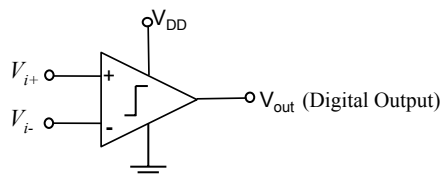
## Lecture 20

### ADC Converters

#### Comparator design

- Single-stage open-loop amplifier
- Cascade of open-loop amplifiers
- Problem associated with DC offset
  - Cascaded output series cancellation
  - Input series cancellation
  - Offset cancellation through additional input pair plus offset storage capacitors
- Latched comparators
- Comparator examples

### Voltage Comparators



Play an important role in majority of ADCs

Function: Compare the instantaneous value of two analog signals & generate a digital output voltage based on the sign of the difference:

$$\text{If } V_{i+} - V_{i-} > 0 \rightarrow V_{out} = "1"$$

$$\text{If } V_{i+} - V_{i-} < 0 \rightarrow V_{out} = "0"$$

# Voltage Comparator Architectures

## Comparator architectures:

- High gain amplifier with differential analog input & single-ended large swing output
  - Output swing has to be compatible with driving digital logic circuits
  - Open-loop amplification → no frequency compensation required
  - Precise gain not required
- Latched comparators; in response to a strobe (clock edge), input stage disabled & digital output stored in a latch till next strobe
  - Two options for implementation :
    - Latch-only comparator
    - Low-gain preamplifier + high-sensitivity latch
- Sampled-data comparators
  - T/H input
  - Offset cancellation

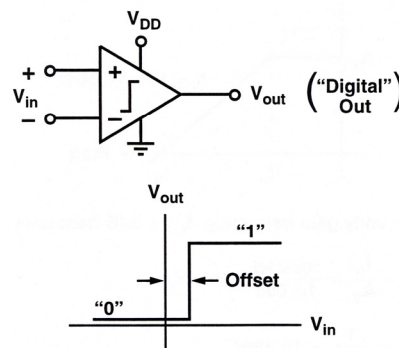
## Comparator Built with High-Gain Amplifier

Amplify  $V_{in(min)}$  to  $V_{DD}$   
 →  $V_{in(min)}$  determined by ADC resolution

Example: 12-bit ADC with:  
 -  $V_{FS} = 1.5V \rightarrow 1LSB = 0.36mV$   
 -  $V_{DD} = 1.8V$

→ For 1.8V output & 0.5LSB precision:

$$A_v^{Min} = \frac{1.8V}{0.18mV} \approx 10,000$$



# Comparators

## 1-Single-Stage Amplification

- Amplifier maximum Gain-Bandwidth product ( $f_u$ ) for a given technology, typically a function of maximum device  $f_t$

$$f_u = \text{unity-gain frequency, } f_o = \text{-3dB frequency} \quad f_o = \frac{f_u}{A_v}$$

Example:  $f_u = 10\text{GHz}$  &  $A_v = 10,000$

$$f_o = \frac{10\text{GHz}}{10,000} \approx 1\text{MHz}$$

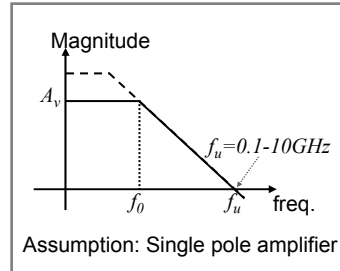
$$\tau_{\text{settling}} = \frac{1}{2\pi f_o} = 0.16\mu\text{sec}$$

Allow a few  $\tau$  for output to settle

$$f_{\text{Clock}}^{\text{Max.}} \rightarrow \frac{1}{5\tau_{\text{settling}}} \approx 1.26\text{MHz}$$

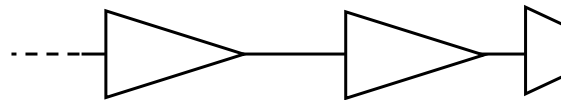
Too slow for majority of applications!

→ Try cascade of lower gain stages to broaden frequency of operation

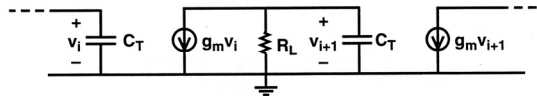


# Comparators

## 2- Cascade of Open Loop Amplifiers



The stages identical → small-signal model for the cascades:



One stage:

$$|A_v(0)| = g_m R_L$$

$$\omega_o = \text{-3dB frequency} = \frac{1}{R_L C_T}$$

$$\omega_u = \text{-unity gain frequency} = G \times \text{BW} = \frac{g_m}{C_T}$$

$$\therefore \omega_o = \frac{\omega_u}{|A_v(0)|}$$

## Open Loop Cascade of Amplifiers

For an N-stage cascade:

$$A_T(j\omega) = [A_V(j\omega)]^N = \frac{[A_V(0)]^N}{\left(1 + j\frac{\omega}{\omega_0}\right)^N}$$

Define

$\omega_{oN} \equiv$  -3dB frequency of the N-stage cascade

Then

$$|A_T(j\omega_{oN})| = \frac{[A_V(0)]^N}{\sqrt{2}}$$

and

$$\omega_{oN} = \omega_0 \sqrt{2^{1/N} - 1} = \frac{\omega_u}{[A_V(0)]} \sqrt{2^{1/N} - 1}$$

$\therefore$  For a specified  $|A_T(0)|$

$$|A_V(0)| = |A_T(0)|^{1/N}$$

$$\Rightarrow \omega_{oN} = \frac{\omega_u}{|A_T(0)|^{1/N}} \sqrt{2^{1/N} - 1}$$

Thus,

$$\begin{aligned} \frac{\omega_{oN}}{\omega_{o1}} &= \left[ \frac{\omega_u}{|A_T(0)|^{1/N}} \sqrt{2^{1/N} - 1} \right] \left/ \left[ \frac{\omega_u}{|A_T(0)|} \right] \right. \\ &= |A_T(0)|^{\left(\frac{N-1}{N}\right)} \sqrt{2^{1/N} - 1} \end{aligned}$$

Example:  $N=4, A_T=10000 \rightarrow \omega_{oN}=430\omega_{o1}$

## Open Loop Cascade of Amplifiers

For  $|A_T(\text{DC})|=10,000$

N	$\omega_{oN}/\omega_{o1}$	$ A_V(0) $
1	1	10,000
2	64	100
3	236	21.5
4	435	10
5	611	6.3
10	1067	2.5
20	1185	1.6

Example:

$$N=3, f_u=10\text{GHz} \ \& \ |A_T(0)|=10000$$

$$f_{oN} = \frac{10\text{GHz}}{(10,000)^{1/3}} \sqrt{2^{1/3}-1} \approx 237\text{MHz}$$

$$\tau_{\text{settling}} = \frac{1}{2\pi f_o} = 0.7\text{nsec}$$

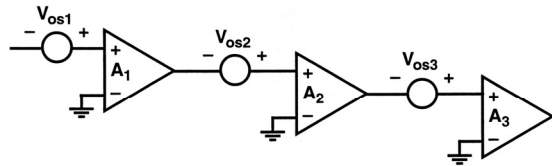
Allow a few  $\tau$  for output to settle

$$f_{\text{Clock}}^{\text{Max}} \rightarrow \frac{1}{5\tau_{\text{settling}}} \approx 290\text{MHz}$$

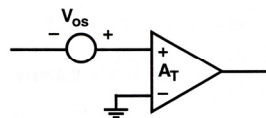
$f_{\text{max}}$  improved from 1.26MHz to 290MHz  $\rightarrow X236$

## Open Loop Cascade of Amplifiers Offset Voltage

- From offset point of view: high gain/stage is preferred



- Choice of # of stage  
→ bandwidth vs offset tradeoff

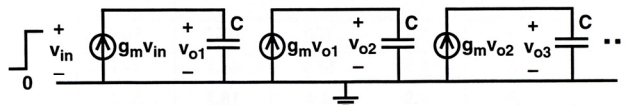


$$A_T = A_1 \cdot A_2 \cdot A_3$$

$$\text{Input-referred offset } \rightarrow V_{os} = V_{os1} + \frac{V_{os2}}{A_1} + \frac{V_{os3}}{A_1 \cdot A_2}$$

## Open Loop Cascade of Amplifiers Step Response

- Assuming linear behavior (not slew limited)



$$v_{o1} = \frac{1}{C} \int_0^t g_m v_{in} dt = \frac{g_m}{C} v_{in} t$$

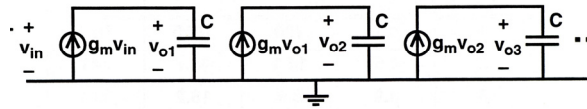
$$v_{o2} = \frac{1}{C} \int_0^t g_m v_{o1} dt = \frac{g_m}{C} \int_0^t \frac{g_m}{C} v_{in} t dt = \frac{1}{2} \left( \frac{g_m}{C} \right)^2 v_{in} t^2$$

$$v_{o3} = \frac{1}{C} \int_0^t g_m v_{o2} dt = \frac{g_m}{C} \int_0^t \left[ \frac{1}{2} \left( \frac{g_m}{C} \right)^2 v_{in} t^2 \right] dt$$

$$= \frac{1}{3} \left( \frac{g_m}{C} \right)^3 v_{in} t^3$$

# Open Loop Cascade of Amplifiers Step Response

• Assuming linear behavior



**N Stages**

$$v_{oN} = \left(\frac{g_m}{C}\right)^N \left(\frac{t^N}{N!}\right) v_{in}$$

For the output to reach a specified  $v_{out}$  (i.e.,  $v_{oN} = v_{out}$ ) the delay is

$$\tau_D = \left(\frac{C}{g_m}\right) \left[ (N!) \left(\frac{v_{out}}{v_{in}}\right) \right]^{1/N}$$

# Open Loop Cascade of Amplifiers Delay/(C/g<sub>m</sub>)

$$\tau_D = \left(\frac{C}{g_m}\right) \left[ (N!) \left(\frac{V_{out}}{V_{in}}\right) \right]^{1/N}$$

- Minimum total delay broad function of N
- Relationship between # of stages resulting in minimize delay ( $N_{opt}$ ) and gain ( $V_{out}/V_{in}$ ) approximately:

$$N_{opt} \approx 1 + \log_2 A_T \quad \text{for } A < 1000$$

$$N_{opt} \approx 1.2 \ln A_T \quad \text{for } A \geq 1000$$

		Delay/(C/g <sub>m</sub> )			
		10	100	1000	10K
$V_{out}/V_{in}$	N				
1	1	10	100	1000	10K
2	2	4.5	14.1	44.7	141
3	3	3.9	8.4	18.2	39.1
4	4	3.9	7.0	12.4	22.1
5	5	4.1	6.5	10.4	16.4
6	6	4.4	6.4	9.5	13.9
7	7	4.7	6.5	9.1	12.6
8	8	5.0	6.7	8.9	11.9
9	9	5.4	6.9	8.9	11.5
10	10	5.7	7.2	9.0	11.4
11	11	6.1	7.5	9.2	11.3
12	12	6.4	7.8	9.4	11.4
20	20	9.3	10.5	11.7	13.2

Ref: J.T. Wu, et al., "A 100-MHz pipelined CMOS comparator" *IEEE Journal of Solid-State Circuits*, vol. 23, pp. 1379 - 1385, December 1988.

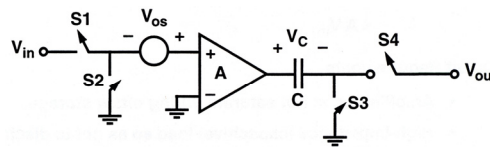
## Offset Cancellation

- In sampled-data cascade of amplifiers  $V_{os}$  can be cancelled  
→ Store on ac-coupling caps in series with amp stages
- Offset associated with a specific amp can be cancelled by storing it in series with either the input or the output of that stage
- Offset can be cancelled by adding a pair of auxiliary inputs to the amplifier and storing the offset on capacitors connected to the aux. inputs during offset cancellation phase

Ref: J.T. Wu, et al., "A 100-MHz pipelined CMOS comparator" *IEEE Journal of Solid-State Circuits*, vol. 23, pp. 1379 - 1385, December 1988.

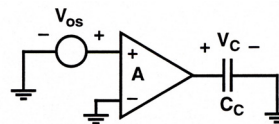
## Offset Cancellation Output Series Cancellation

- Amp modeled as ideal  
+  $V_{os}$  (input referred)



### 1- Store offset:

- S1, S4 → open
  - S2, S3 → closed
- $V_C = A \cdot V_{os}$



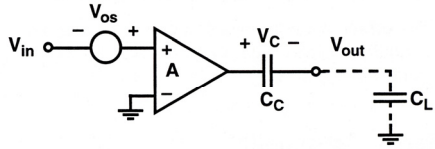
$$V_C = A \cdot V_{os}$$

Ref: J.T. Wu, et al., "A 100-MHz pipelined CMOS comparator" *IEEE Journal of Solid-State Circuits*, vol. 23, pp. 1379 - 1385, December 1988.

# Offset Cancellation Output Series Cancellation

**2- Amplify:**

- S2, S3 → open
- S1, S4 → closed
- $V_C = A \cdot V_{OS}$

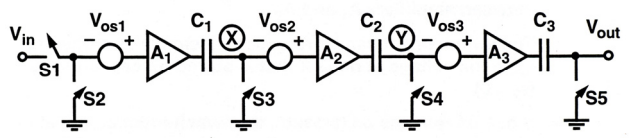


**Circuit requirements:**

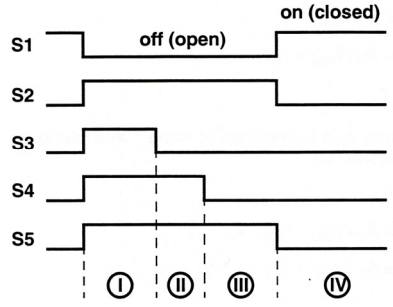
- Amp not saturate during offset storage
- High-impedance (C) load →  $C_c$  not discharged
- $C_c \gg C_L$  to avoid attenuation
- $C_c \gg C_{switch}$  avoid excessive offset due to charge injection

$$\begin{aligned}
 V_{out} &= A \cdot (V_{in} + V_{os}) - V_C \\
 &= A \cdot (V_{in} + V_{os}) - A \cdot V_{os} \\
 &= A \cdot V_{in}
 \end{aligned}$$

# Offset Cancellation Cascaded Output Series Cancellation

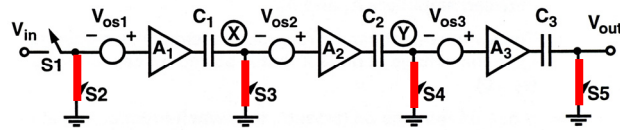


Note: Offset storage capacitors in series with the amplifier outputs





## Offset Cancellation Cascaded Output Series Cancellation

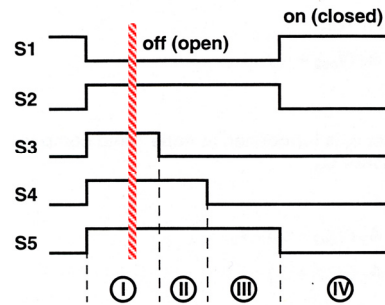


1- S1 → open, S2,3,4,5 closed

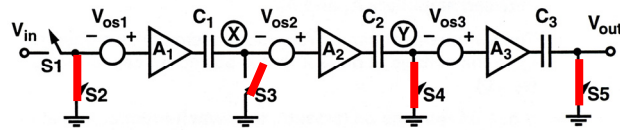
$$V_{C1} = A_1 \times V_{os1}$$

$$V_{C2} = A_2 \times V_{os2}$$

$$V_{C3} = A_1 \times V_{os3}$$



## Offset Cancellation Cascaded Output Series Cancellation



2- S3 → open first

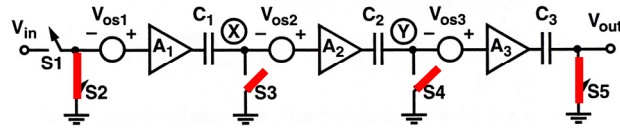
- Feedthrough from S3 → offset on X
- Switch offset,  $\epsilon_3$  induced on node X
- Since S4 remains closed, offset associated with  $\epsilon_3$  → stored on C2

$$V_X = \epsilon_3$$

$$V_{C1} = A_1 \times V_{os1} - \epsilon_3$$

$$V_{C2} = A_2 \times (V_{os2} + \epsilon_3)$$

## Offset Cancellation Cascaded Output Series Cancellation



3- S4 → open

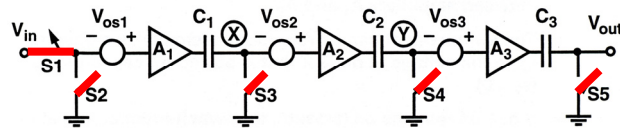
- Feedthrough from S4 → offset on Y
- Switch offset,  $\epsilon_4$  induces error on node Y
- Since S5 remains closed, offset associated with  $\epsilon_4$  → stored on C3

$$V_Y = \epsilon_4$$

$$V_{C2} = A_2 \times (V_{os2} + \epsilon_3) - \epsilon_4$$

$$V_{C3} = A_3 \times (V_{os3} + \epsilon_4)$$

## Offset Cancellation Cascaded Output Series Cancellation



4- S2 → open, S5 → open, S1 → closed

- S2 open & S1 closed → since input connected to low impedance source charge injection not of major concern
- Switch offset,  $\epsilon_5$  introduced due to S5 opening

$$V_X = A_1 \times (V_{in} + V_{os1}) - V_{C1}$$

$$= A_1 \times (V_{in} + V_{os1}) - (A_1 \cdot V_{os1} - \epsilon_3)$$

$$= A_1 \cdot V_{in} + \epsilon_3$$

## Offset Cancellation Cascaded Output Series Cancellation

$$\begin{aligned} V_y &= A_2 x (V_x + V_{os2}) - V_{C2} \\ &= A_2 x (A_1 V_{in} + \epsilon_3 + V_{os2}) - [A_2 \cdot (V_{os2} + \epsilon_3) - \epsilon_4] \\ &= A_1 \cdot A_2 \cdot V_{in} + \epsilon_4 \end{aligned}$$

$$\begin{aligned} V_{out} &= A_3 x (V_y + V_{os3}) - V_{C3} \\ &= A_3 \cdot (A_2 x A_1 V_{in} + \epsilon_4 + V_{os3}) - [A_3 \cdot (V_{os3} + \epsilon_4) - \epsilon_5] \\ &= A_1 \cdot A_2 \cdot A_3 \cdot V_{in} + \epsilon_5 \end{aligned}$$

## Offset Cancellation Cascaded Output Series Cancellation

$$\begin{aligned} V_{out} &= A_1 \cdot A_2 \cdot A_3 \cdot (V_{in} + \epsilon_5 / A_1 \cdot A_2 \cdot A_3) \\ \text{Input-Referred Offset} &= \epsilon_5 / A_1 \cdot A_2 \cdot A_3 \end{aligned}$$

Example:

3-stage open-loop differential amplifier with series offset cancellation + output amplifier (see Ref.)

$$A_{\text{Total}}(\text{DC}) = 2 \times 10^6 = 126\text{dB}$$

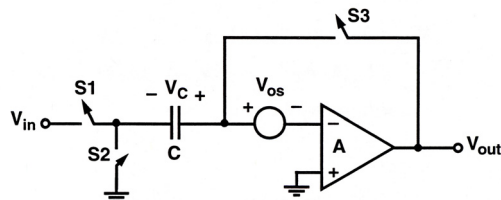
$$\text{Input-referred offset} < 5\mu\text{V}$$

Ref: R. Poujois and J. Borel, "A low drift fully integrated MOSFET operational amplifier," *IEEE Journal of Solid-State Circuits*, vol. 13, pp. 499 - 503, August 1978.

## Offset Cancellation Output Series Cancellation

- Advantages:
  - Almost complete cancellation
  - Closed-loop stability not required
- Disadvantages:
  - Gain per stage must be small
  - Offset storage C in the signal path → could slow down overall performance

## Offset Cancellation Input Series Cancellation

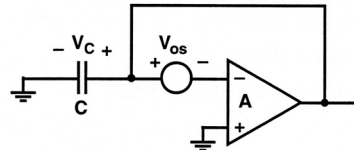


Ref: R. Poujois and J. Borel, "A low drift fully integrated MOSFET operational amplifier," *IEEE Journal of Solid-State Circuits*, vol. 13, pp. 499 - 503, August 1978.

## Offset Cancellation Input Series Cancellation

### 1- Store offset

S1 = 0 (off)  
S2, S3 = 1 (conducting)



Note: Mandates closed-loop stability

$$V_C = -A(V_C - V_{os})$$

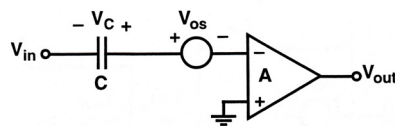
$$= \left(\frac{A}{A+1}\right)V_{os}$$

Ref: R. Poujois and J. Borel, "A low drift fully integrated MOSFET operational amplifier," *IEEE Journal of Solid-State Circuits*, vol. 13, pp. 499 - 503, August 1978.

## Offset Cancellation Input Series Cancellation

### 2- Amplify

S2, S3 → open  
S1 → closed



$$V_{out} = -A(V_{in} + V_C - V_{os}) = -A\left[V_{in} + V_{os}\left(\frac{A}{A+1} - 1\right)\right]$$

$$\therefore V_{out} = -A\left(V_{in} - \frac{V_{os}}{A+1}\right)$$

and

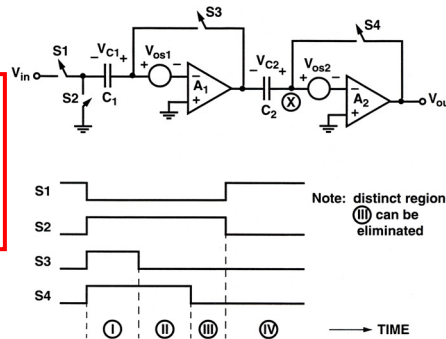
$$\text{Input-Referred Offset} = \frac{V_{os}}{A+1}$$

Example: A=4  
→ Input-referred offset =  $V_{os}/5$

## Offset Cancellation Cascaded Input Series Cancellation

$$V_{out} = A_1 A_2 \left[ V_{in} + \frac{V_{os2}}{A_1(A_2 + 1)} - \frac{\epsilon_2}{A_1} \right]$$

$$\text{Input-Referred Offset} = \frac{V_{os2}}{A_1(A_2 + 1)} - \frac{\epsilon_2}{A_1}$$



$\epsilon_2$  → charge injection associated with opening of S4

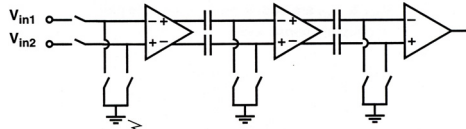
## Offset Cancellation Input Series Cancellation

- Advantages:
  - In applications such as C-array successive approximation ADCs can use C-array to store offset
- Disadvantages:
  - Cancellation not complete
  - Requires closed loop stability
  - Offset storage C in the signal path- could slow down overall performance

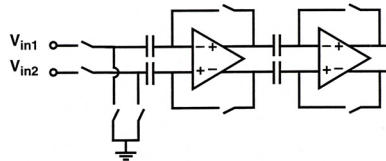
# CMOS Comparators Cascade of Gain Stages

Fully differential gain stages  $\rightarrow$  1<sup>st</sup> order cancellation of switch feedthrough offset

1- Output series offset cancellation

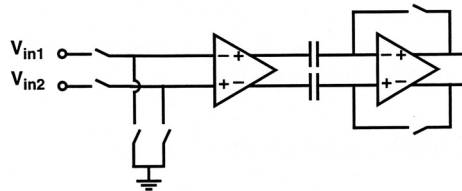


2- Input series offset cancellation



# CMOS Comparators Cascade of Gain Stages

3-Combined input & output series offset cancellation

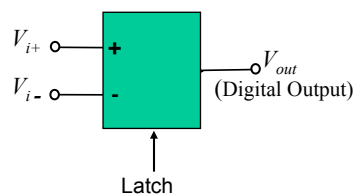


$V_{os1}$  &  $V_{os2}$  are both stored on a single pair of coupling capacitors

# Offset Cancellation

- Cancel offset by additional pair of inputs + offset storage Cs + an extra clock phase for offset storage (Lecture 18 slide 46 -48)

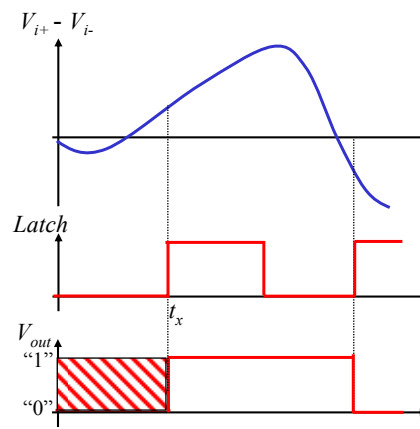
# Latched Comparators



Compares two input voltages at time  $t_x$  & generates a digital output:

$$\text{If } V_{i+} - V_{i-} > 0 \rightarrow V_{out} = \text{"1"}$$

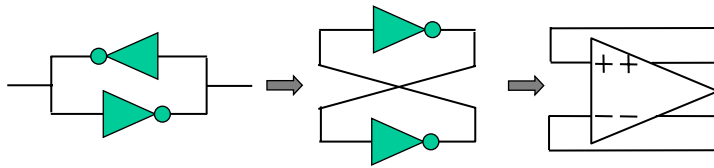
$$\text{If } V_{i+} - V_{i-} < 0 \rightarrow V_{out} = \text{"0"}$$





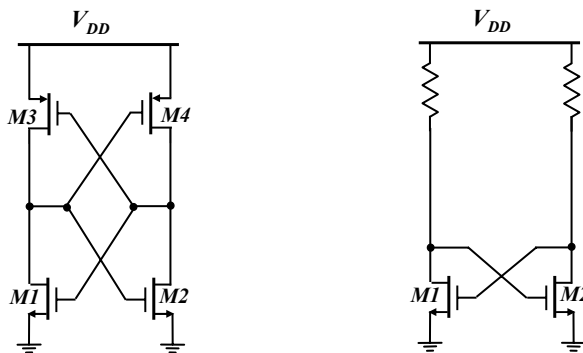
# CMOS Latched Comparators

Comparator amplification need not be linear  
→ can use a latch → regeneration



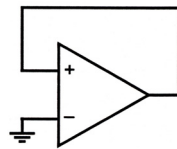
Latch → Amplification + positive feedback

# Simplest Form of CMOS Latch



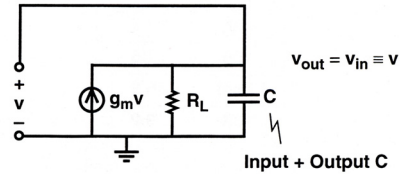
## CMOS Latched Comparators Small Signal Model

Latch can be modeled as a:  
→ Single-pole amp + positive feedback



$$V_{out} = V_{in}$$

Small signal ac half circuit



## CMOS Latched Comparator Latch Delay

$$g_m V = \frac{V}{R_L} + C \frac{dV}{dt}$$

$$\frac{g_m}{C} \left( 1 - \frac{1}{g_m R_L} \right) V = \frac{dV}{dt} \quad \frac{g_m}{C} \left( 1 - \frac{1}{g_m R_L} \right) dt = \frac{dV}{V}$$

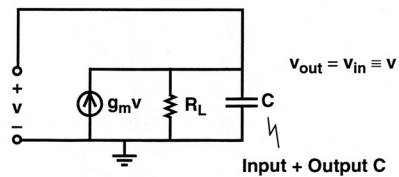
$$\text{Integrating both sides: } \frac{g_m}{C} \left( 1 - \frac{1}{g_m R_L} \right) \int_{t_1}^{t_2} dt = \int_{V_1}^{V_2} \frac{1}{V} dV \quad \left( \int_b^a \frac{1}{x} dx = \ln x \Big|_b^a = \ln a - \ln b = \ln \frac{a}{b} \right)$$

Latch Delay:

$$t_D = t_2 - t_1 = \frac{C}{g_m} \left( \frac{1}{1 - \frac{1}{g_m R_L}} \right) \ln \left( \frac{V_2}{V_1} \right)$$

For  $g_m R_L \gg 1$

$$t_D \approx \frac{C}{g_m} \ln \left( \frac{V_2}{V_1} \right)$$



## CMOS Latched Comparators

Normalized Latch Delay

$$t_D \approx \frac{C}{g_m} \ln \left( \frac{V_2}{V_1} \right)$$

$$\frac{V_2}{V_1} \rightarrow \text{Latch Gain} = A_L$$

$$\rightarrow t_D \approx \frac{C}{g_m} \ln A_L$$

$$\tau_D(\text{3-stage amp}) = 18.2(C/g_m)$$

$A_L$	$\frac{t_D}{C/g_m}$
10	2.3
100	4.6
1000	6.9
10K	9.2

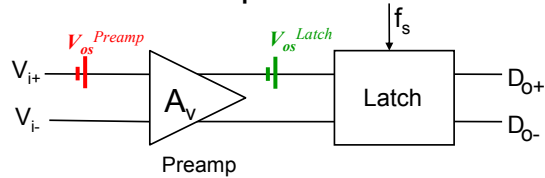
Compared to a 3-stage open-loop cascade of amps for equal overall gain of 1000

→ Latch faster by about x3

## Latch-Only Comparator

- Much faster compared to cascade of open-loop amplifiers
- Main problem associated with latch-only comparator topology:
  - High input-referred offset voltage (as high as 100mV!)
  - Solution:
    - Use preamplifier to amplify the signal and reduce overall input-referred offset

## Pre-Amplifier + Latch Overall Input-Referred Offset



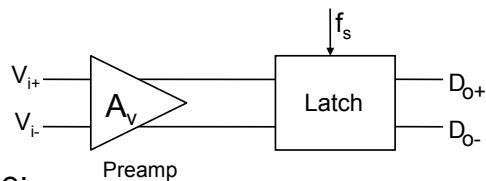
Latch offset attenuated by preamp gain when referred to preamp input.  
Assuming the two offset sources are uncorrelated:

$$\sigma_{\text{Input-Referred\_Offset}} = \sqrt{\sigma_{V_{os\_Preamp}}^2 + \frac{1}{A_{Preamp}^2} \sigma_{V_{os\_Latch}}^2}$$

Example:  $\sigma_{V_{os\_Preamp}} = 4mV$  &  $\sigma_{V_{os\_Latch}} = 50mV$  &  $A_{Preamp} = 10$

$$\sigma_{\text{Input-Referred\_Offset}} = \sqrt{4^2 + \frac{1}{10^2} 50^2} = 6.4mV$$

## Pre-Amplifier Tradeoffs



- Example:
  - Latch offset 50 to 100mV
  - Preamp DC gain 10X
  - Preamp input-referred latch offset 50 to 100mV
  - Input-referred preamplifier offset 2 to 10mV
  - Overall input-referred offset 5.5 to 14mV

→ Addition of preamp reduces the latch input-referred offset reduced by ~7 to 9X → ~allows extra 3-bit resolution for ADC!

## Comparator Preamplifier Gain-Speed Tradeoffs

- Amplifier maximum Gain-Bandwidth product ( $f_u$ ) for a given technology, typically a function of maximum device  $f_t$

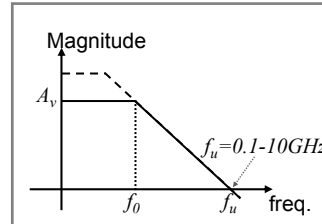
$f_u$  = unity gain frequency,  $f_0$  =  $-3dB$  frequency &  $\tau_0$  = settling time

$$f_0 = \frac{f_u}{A_{\text{preamp}}}$$

For example assuming preamp has a gain of 10:

$$f_0 = \frac{f_u}{A_{\text{preamp}}} = \frac{1\text{GHz}}{10} = 100\text{MHz}$$

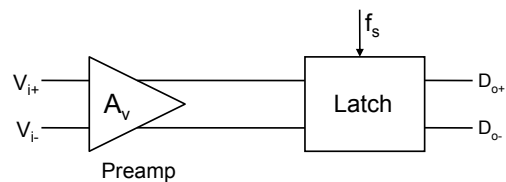
$$\tau_0 = \frac{1}{2\pi f_0} = \frac{A_{\text{preamp}}}{2\pi f_u} = 1.6\text{nsec}$$



- Tradeoff:

- To reduce the effect of latch offset  $\rightarrow$  high preamp gain desirable
- Fast comparator  $\rightarrow$  low preamp gain
- $\rightarrow$  Choice of preamp gain: compromise speed v.s. input-referred latch offset

## Latched Comparator

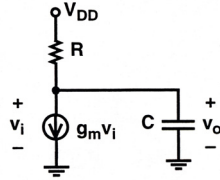


Important features:

- Maximum clock rate  $f_s \rightarrow$  settling time, slew rate, small signal bandwidth
- Resolution  $\rightarrow$  gain, offset
- Overdrive recovery
- Input capacitance (and linearity of input capacitance!)
- Power dissipation
- Input common-mode range and CMR
- Kickback noise
- ...

# Comparator Overdrive Recovery

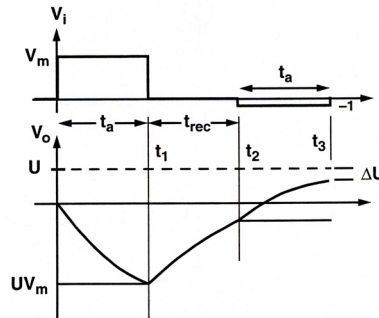
Linear model for a single-pole amplifier:



$U \rightarrow$  amplification after time  $t_a$

During reset amplifier settles exponentially to its zero input condition with  $\tau_0=RC$

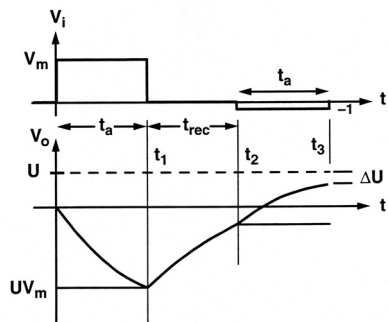
Assume  $V_m \rightarrow$  maximum input normalized to  $1/2lsb (=1)$



Example: Worst case input/output waveforms

Previous input  $\rightarrow$  max. possible e.g. VFS  
Current input  $\rightarrow$  min. input-referred signal (0.5LSB)

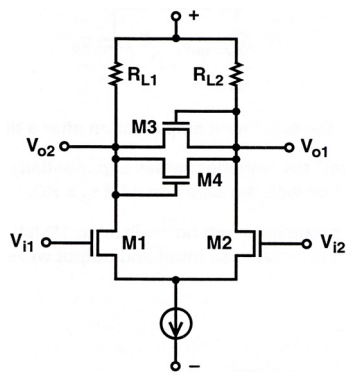
# Comparators Overdrive Recovery



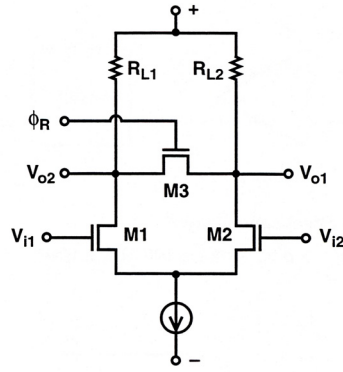
Example: Worst case input/output waveforms

- If recovery time is not long enough to allow output to discharge (recover) from previous state- then it may not be able to resolve the current input  $\rightarrow$  error
- To minimize this effect:
  1. Passive clamp
  2. Active restore
  3. Low gain/stage

## Comparators Overdrive Recovery Limiting Output Voltage



Clamp  
Adds parasitic capacitance



Active Restore  
After outputs are latched by following stage  
→ Activate  $\phi_R$  & equalize output nodes

## CMOS Preamp + Latched Comparator Delay in Response

Latch delay previously found:

$$\tau_D \approx \frac{C}{g_m} \ln\left(\frac{V_2}{V_1}\right)$$

Assuming gain of  $A_v$  for the preamplifier then  $V_1 = A_v \times V_{in}$

$$\tau_D \approx \frac{C}{g_m} \ln\left(\frac{V_0}{A_v V_{in}}\right)$$

## Latched Comparator Including Preamplifier Example

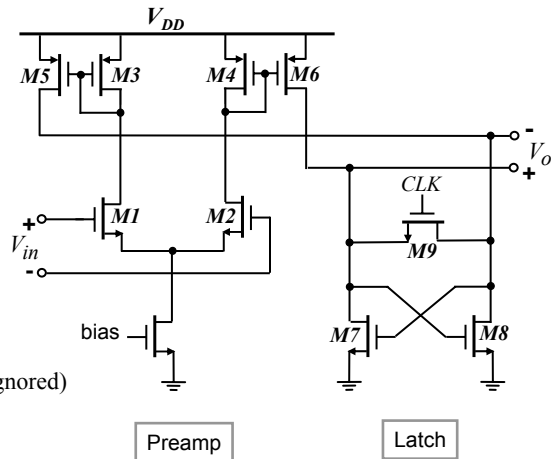
Preamplifier gain:

$$A_v = \frac{g_m^{M1}}{g_m^{M3}} = \frac{(V_{GS}^{M3} - V_{th}^{M3})}{(V_{GS}^{M1} - V_{th}^{M1})}$$

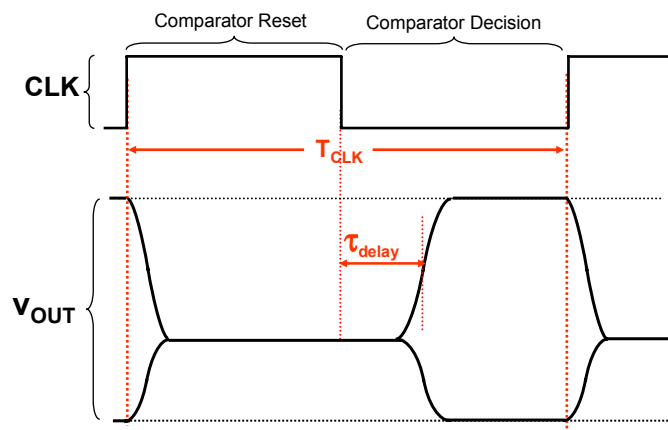
Comparator delay:

(for simplicity, preamp delay ignored)

$$\tau_D \approx \frac{C}{g_m} \ln \left( \frac{V_0}{A_v V_{in}} \right)$$

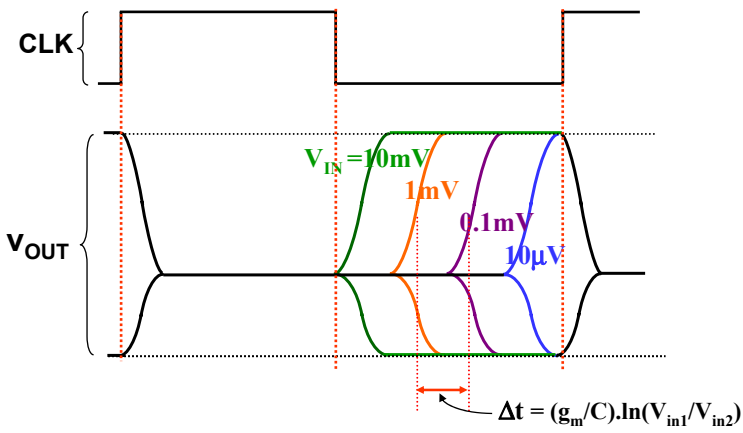


## Comparator Dynamic Behavior

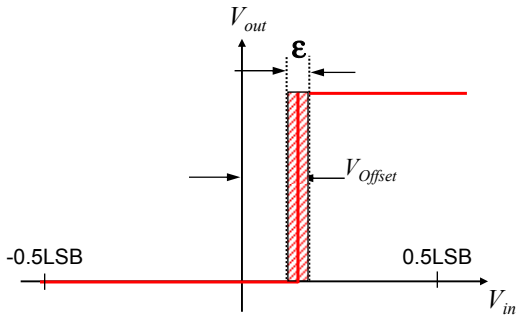




# Comparator Resolution

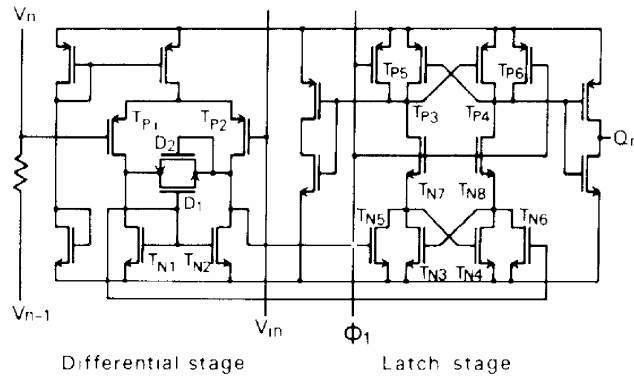


# Comparator Voltage Transfer Function Non-Idealities



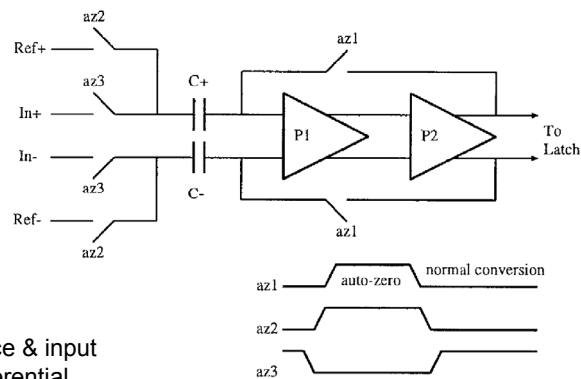
- $V_{Offset}$  → Comparator offset voltage
- $\epsilon$  → Meta-Stable region (output ambiguous)

# CMOS Comparator Example



- Flash ADC: 8bits,  $\pm 1/2$ LSB INL @  $f_s=15\text{MHz}$  ( $V_{ref}=3.8\text{V}$ ,  $\text{LSB}\sim 15\text{mV}$ )
  - No offset cancellation
- Ref: A. Yukawa, "A CMOS 8-Bit High-Speed A/D Converter IC," JSSC June 1985, pp. 775-9

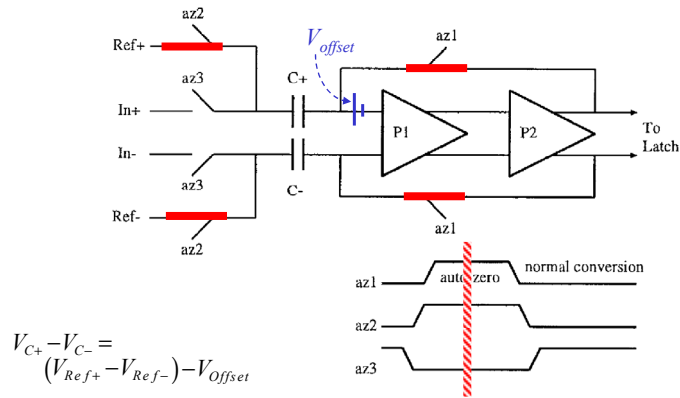
# Comparator with Auto-Zero



Note:  
Reference & input  
both differential

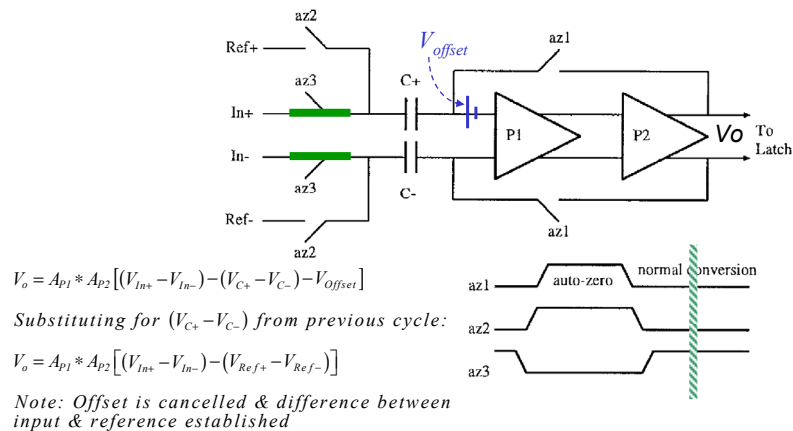
Ref: I. Mehr and L. Singer, "A 500-Msample/s, 6-Bit Nyquist-Rate ADC for Disk-Drive Read-Channel Applications," JSSC July 1999, pp. 912-20.

## Flash ADC Comparator with Auto-Zero



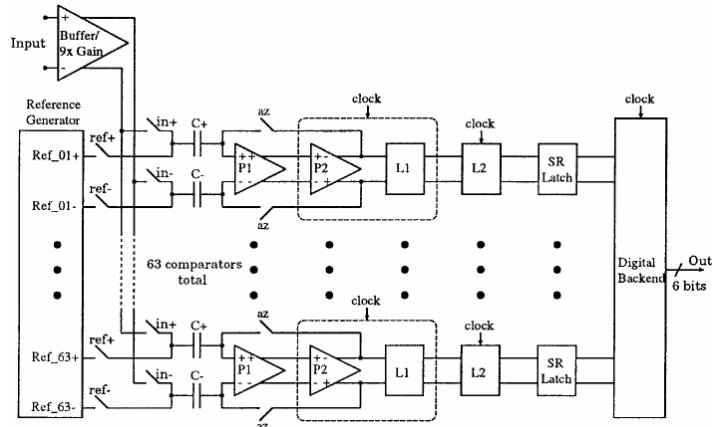
Ref: I. Mehr and D. Dalton, "A 500-Msample/s, 6-Bit Nyquist-Rate ADC for Disk-Drive Read-Channel Applications," JSSC July 1999, pp. 912-20.

## Flash ADC Comparator with Auto-Zero



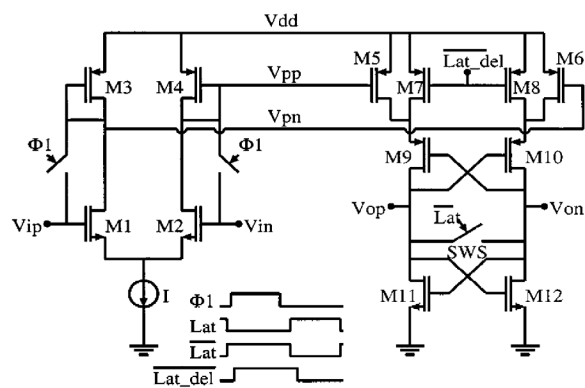
Ref: I. Mehr and D. Dalton, "A 500-Msample/s, 6-Bit Nyquist-Rate ADC for Disk-Drive Read-Channel Applications," JSSC July 1999, pp. 912-20.

## Flash ADC Using Comparator with Auto-Zero



Ref: I. Mehr and D. Dalton, "A 500-Msample/s, 6-Bit Nyquist-Rate ADC for Disk-Drive Read-Channel Applications," JSSC July 1999, pp. 912-20.

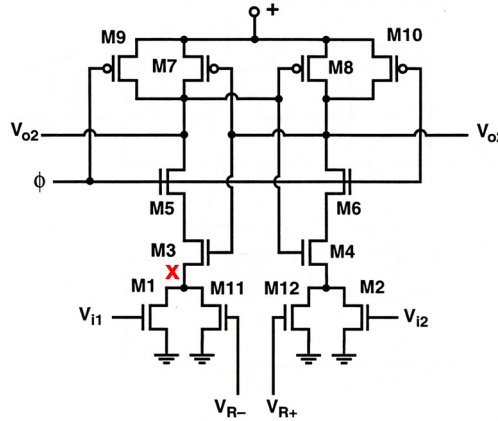
## Auto-Zero Implementation



Ref: I. Mehr and L. Singer, "A 55-mW, 10-bit, 40-Msample/s Nyquist-Rate CMOS ADC," JSSC March 2000, pp. 318-25

## Comparator Example

- Variation on Yukawa latch used w/o preamp
- Good for low resolution ADCs (in this case 1.5bit/stage for a pipeline)
- Note: M1, M2, M11, M12 operate in triode mode
- M11 & M12 added to vary comparator threshold
- Conductance at node X is sum of  $G_{M1}$  &  $G_{M11}$



Ref: T. B. Cho and P. R. Gray, "A 10 b, 20 Msample/s, 35 mW pipeline A/D converter," *IEEE Journal of Solid-State Circuits*, vol. 30, pp. 166 - 172, March 1995

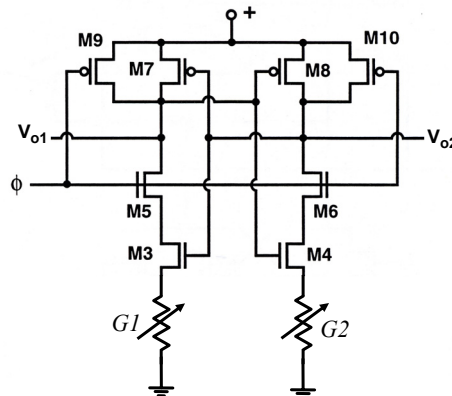
## Comparator Example (continued)

- M1, M2, M11, M12 operate in triode mode with all having equal L
- Conductance of input devices:
 
$$G_1 = \frac{\mu C_{ox}}{L} \times [W_1(V_{I1} - V_{th}) + W_{11}(V_{R-} - V_{th})]$$

$$G_2 = \frac{\mu C_{ox}}{L} \times [W_2(V_{I2} - V_{th}) + W_{11}(V_{R+} - V_{th})]$$

$$\rightarrow \Delta G = \frac{\mu C_{ox} W_1}{L} \times \left[ (V_{I1} - V_{I2}) - \frac{W_{11}}{W_1} (V_{R+} - V_{R-}) \right]$$
- To 1st order, for  $W_1 = W_2$  &  $W_{11} = W_{12}$ 

$$V_{th}^{latch} = W_{11} / W_1 \times V_R$$
 where  $V_R = V_{R+} - V_{R-}$ .
   
 $\rightarrow V_R$  fixed  $W_{11}, I_2$  varied from comparator to comparator  $\rightarrow$  Eliminates need for resistive divider



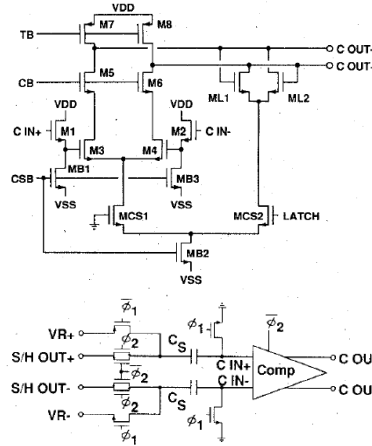
Ref: T. B. Cho and P. R. Gray, "A 10 b, 20 Msample/s, 35 mW pipeline A/D converter," *IEEE Journal of Solid-State Circuits*, vol. 30, pp. 166 - 172, March 1995

## Comparator Example

- Used in a pipelined ADC with digital correction  
→ no offset cancellation required

Differential reference & input

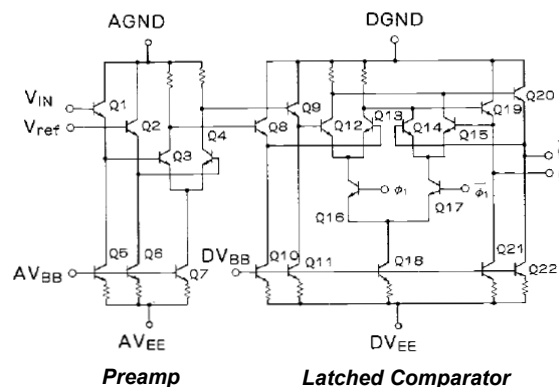
- M7, M8 operate in triode region
- Preamp gain ~10
- Input buffers suppress kick-back
- $\phi_1$  high →  $C_s$  charged to VR &  $\phi_{2B}$  is also high → current diverted to latch → comparator output in hold mode
- $\phi_2$  high →  $C_s$  connected to S/Hout & comparator input (VR-S/Hout), current sent to preamp → comparator in amplify mode



Ref: S. Lewis, et al., "A Pipelined 5-Msample/s 9-bit Analog-to-Digital Converter" IEEE JSSC, NO. 6, Dec. 1987

## Bipolar Comparator Example

- Used in 8bit 400Ms/s & 6bit 2Gb/s flash ADC
- Signal amplification during  $\phi_1$  high, latch operates when  $\phi_1$  low
- Input buffers suppress kick-back & input current
- Separate ground and supply buses for front-end preamp → kick-back noise reduction



Ref: Y. Akazawa, et al., "A 400MSPS 8b flash AD conversion LSI," IEEE International Solid-State Circuits Conference, vol. XXX, pp. 98 - 99, February 1987

Ref: T. Wakimoto, et al., "Si bipolar 2GS/s 6b flash A/D conversion LSI," IEEE International Solid-State Circuits Conference, vol. XXXI, pp. 232 - 233, February 1988