EE247
Lecture 3

- Last lecture's summary
- Active Filters
  - Active biquads
    - Sallen- Key & Tow-Thomas
    - Integrator-based filters
      - Signal flowgraph concept
      - First order integrator-based filter
      - Second order integrator-based filter & biquads
  - High order & high Q filters
    - Cascaded biquads
      - Cascaded biquad sensitivity to component variations
    - Ladder type filters

Summary of Last Lecture

- Nomenclature
- Filter specifications
  - Quality factor
  - Frequency characteristics
  - Group delay
- Filter types
  - Butterworth
  - Chebyshev I
  - Chebyshev II
  - Elliptic
  - Bessel
- Group delay comparison example
- RLC filters

Integrated Filters

- Implementation of RLC filters in CMOS technologies requires on-chip inductors
  - Integrated L<10nH with Q<10
  - Combined with max. cap. 10pF
  - →*LC filters in the monolithic form feasible: freq>500MHz*
- Analog/Digital interface circuitry require fully integrated filters with critical frequencies << 500MHz
- Hence:

⇨ Need to build active filters built without inductors

Filters
2nd Order Transfer Functions (Biquads)

- Biquadratic (2nd order) transfer function:

$$H(s) = \frac{1}{1 + \frac{s}{\omega_P Q_P} + \frac{s^2}{\omega_P^2}}$$

$$|H(j\omega)|_{\omega=0} = 1$$
$$|H(j\omega)|_{\omega\to\infty} = 0$$
$$|H(j\omega)|_{\omega=\omega_P} = Q_P$$

Biquad poles @: $s = -\frac{\omega_P}{2Q_P}\left(1 \pm \sqrt{1-4Q_P^2}\right)$

for $Q_P \le \frac{1}{2}$ *poles are real, complex otherwise*

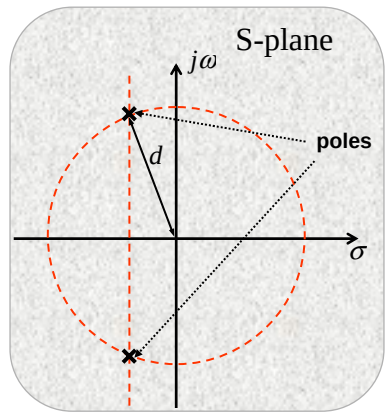
Biquad Complex Poles

$Q_P > \frac{1}{2} \quad \rightarrow$ Complex conjugate poles:

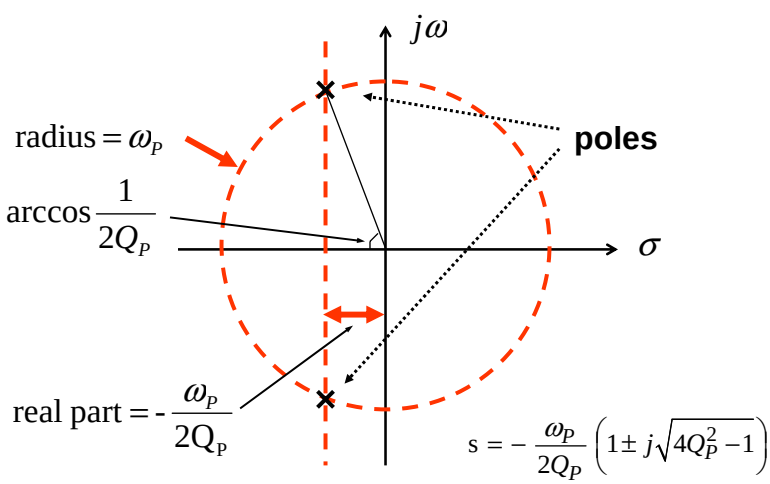
$$s = -\frac{\omega_P}{2Q_P}\left(1 \pm j\sqrt{4Q_P^2 - 1}\right)$$

Distance from origin in s-plane:

$$d^2 = \left(\frac{\omega_P}{2Q_P}\right)^2 \left(1 + 4Q_P^2 - 1\right)$$
$$= \omega_P^2$$



s-Plane

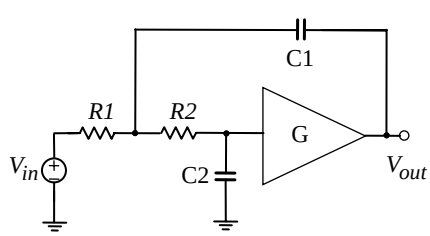


$$s = -\frac{\omega_P}{2Q_P}\left(1 \pm j\sqrt{4Q_P^2 - 1}\right)$$

Implementation of Biquads

- Passive RC: only *real poles* - can't implement *complex conjugate poles*
- Terminated LC
  - Low power, since it is passive
  - Only fundamental noise sources → load and source resistance
  - As previously analyzed, not feasible in the monolithic form for $f < 500MHz$
- Active Biquads
  - Many topologies can be found in filter textbooks!
  - Widely used topologies:
    - Single-opamp biquad: *Sallen-Key*
    - Multi-opamp biquad: *Tow-Thomas*
    - Integrator based biquads

Active Biquad
Sallen-Key Low-Pass Filter



$$H(s) = \frac{G}{1 + \frac{s}{\omega_P Q_P} + \frac{s^2}{\omega_P^2}}$$

$$\omega_P = \frac{1}{\sqrt{R_1 C_1 R_2 C_2}}$$

$$Q_P = \frac{\omega_P}{\frac{1}{R_1 C_1} + \frac{1}{R_2 C_1} + \frac{1-G}{R_2 C_2}}$$

- Single gain element
- Can be implemented both in discrete & monolithic form
- "Parasitic sensitive"
- Versions for LPF, HPF, BP, …
  - → Advantage: Only one opamp used
  - → Disadvantage: Sensitive to parasitic – all pole no zeros

Addition of Imaginary Axis Zeros

- Sharpen transition band
- Can "notch out" interference
- High-pass filter (HPF)
- Band-reject filter

$$H(s) = K \frac{1 + \left(\frac{s}{\omega_Z}\right)^2}{1 + \frac{s}{\omega_P Q_P} + \left(\frac{s}{\omega_P}\right)^2}$$

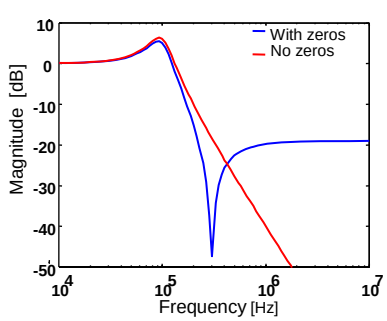
$$|H(j\omega)|_{\omega \to \infty} = K \left(\frac{\omega_P}{\omega_Z}\right)^2$$

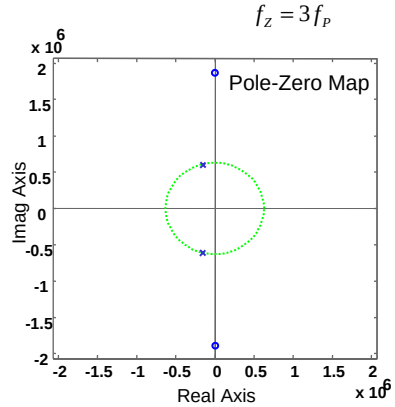
<u>Note:</u> Always represent transfer functions as a product of a gain term, poles, and zeros (pairs if complex). Then all coefficients have a physical meaning, and readily identifiable units.

Imaginary Zeros

- Zeros substantially sharpen transition band
- At the expense of reduced stop-band attenuation at high frequency

$f_P = 100kHz$
$Q_P = 2$
$f_Z = 3 f_P$



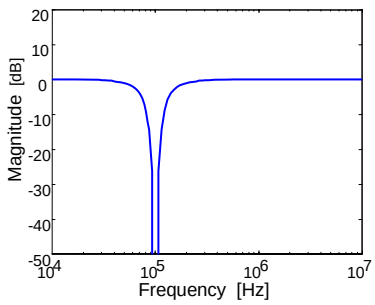


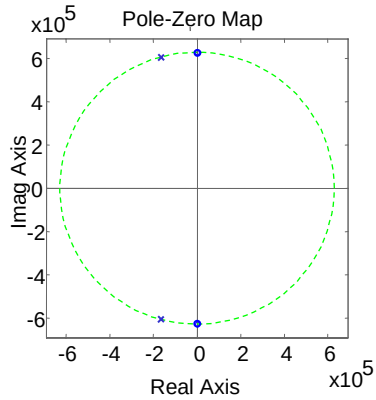
Moving the Zeros

$f_P = 100kHz$

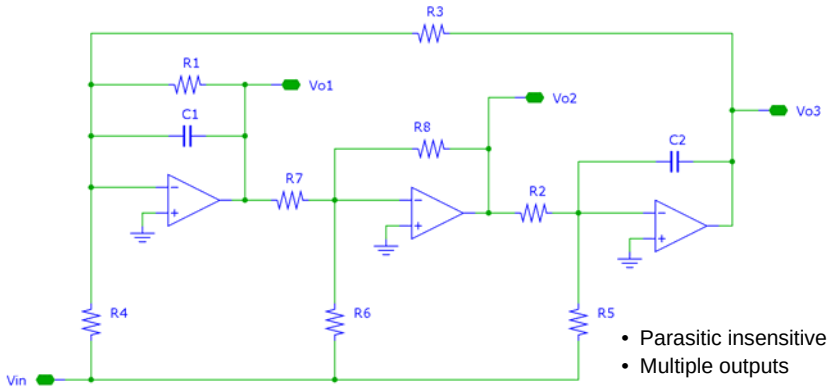
$Q_P = 2$

$\underline{f_Z = f_P}$





Tow-Thomas Active Biquad



- Parasitic insensitive
- Multiple outputs

Ref: P. E. Fleischer and J. Tow, "Design Formulas for biquad active filters using three operational amplifiers," Proc. IEEE, vol. 61, pp. 662-3, May 1973.

Frequency Response

$$\frac{V_{o1}}{V_{in}} = -k_2 \frac{(b_2 a_1 - b_1)s + (b_2 a_0 - b_0)}{s^2 + a_1 s + a_0}$$

$$\frac{V_{o2}}{V_{in}} = \frac{b_2 s^2 + b_1 s + b_0}{s^2 + a_1 s + a_0}$$

$$\frac{V_{o3}}{V_{in}} = -\frac{1}{k_1\sqrt{a_0}} \frac{(b_0 - b_2 a_0)s + (a_1 b_0 - a_0 b_1)}{s^2 + a_1 s + a_0}$$

- V_{o2} implements a general biquad section with arbitrary poles and zeros
- V_{o1} and V_{o3} realize the same poles but are limited to at most one finite zero

Component Values

$$b_0 = \frac{R_8}{R_3 R_5 R_7 C_1 C_2}$$

$$b_1 = \frac{1}{R_1 C_1}\left(\frac{R_8}{R_6} - \frac{R_1 R_8}{R_4 R_7}\right)$$

$$b_2 = \frac{R_8}{R_6}$$

$$a_0 = \frac{R_8}{R_2 R_3 R_7 C_1 C_2}$$

$$a_1 = \frac{1}{R_1 C_1}$$

$$k_1 = \sqrt{\frac{R_2 R_8 C_2}{R_3 R_7 C_1}}$$

$$k_2 = \frac{R_7}{R_8}$$

given a_i, b_i, k_i, C_1, C_2 and R_8

$$R_1 = \frac{1}{a_1 C_1}$$

$$R_2 = \frac{k_1}{\sqrt{a_0} C_2}$$

$$R_3 = \frac{1}{k_1 k_2} \frac{1}{\sqrt{a_0} C_1}$$

$$R_4 = \frac{1}{k_2} \frac{1}{a_1 b_2 - b_1} \frac{1}{C_1}$$

$$R_5 = \frac{k_1 \sqrt{a_0}}{b_0 C_2}$$

$$R_6 = \frac{R_8}{b_2}$$

$$R_7 = k_2 R_8$$

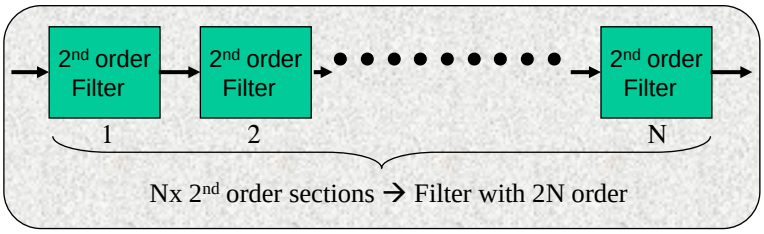
it follows that

$$\omega_P = \sqrt{\frac{R_8}{R_2 R_3 R_7 C_1 C_2}}$$

$$Q_P = \omega_P R_1 C_1$$

Higher-Order Filters

- Higher-order filters (N>2) can be built with cascade of 2nd order biquads, e.g. Sallen-Key,or Tow-Thomas



As will be shown later:
High-Q high-order filters built with cascade of 2nd order sections
→ Highly sensitive to component variations
→ Good alternative: Integrator-based ladder type filters

Integrator Based Filters

- Main building block for this category of filters → integrator
- By using signal flowgraph techniques → conventional filter topologies can be converted to integrator based type filters
- Next few pages:
  - Signal flowgraph techniques
  - 1st order integrator based filter
  - 2nd order integrator based filter
  - High order and high Q filters

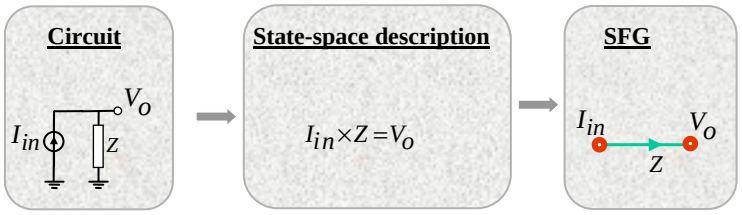
What is a Signal Flowgraph (SFG)?

- SFG → Topological network representation consisting of nodes & branches- used to convert one form of network to a more suitable form (e.g. passive RLC filters to integrator based filters)
- Any network described by a set of linear differential equations can be expressed in SFG form
- For a given network, many different SFGs exists
- Choice of a particular SFG is based on practical considerations such as type of available components

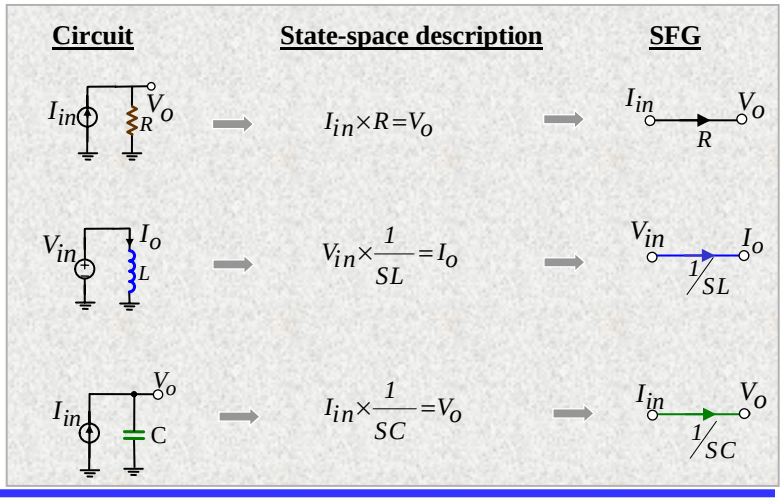
*Ref: W.Heinlein & W. Holmes, "Active Filters for Integrated Circuits", Prentice Hall, Chap. 8, 1974.

What is a Signal Flowgraph (SFG)?

- Signal flowgraph consist of nodes & branches:
  - Nodes represent variables (*V & I* in our case)
  - Branches represent transfer functions (we will call the transfer function *branch multiplication factor* or *BMF*)
- To convert a network to its SFG form, *KCL* & *KVL* is used to derive state space description
- Simple example:



Signal Flowgraph (SFG) Examples



Signal Flowgraph (SFG) Rules

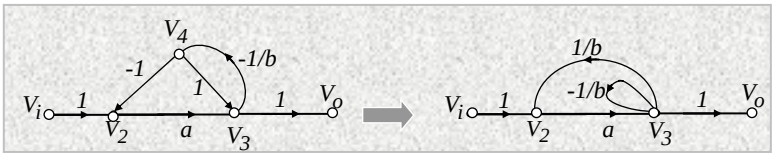
- Two parallel branches can be replaced by a single branch with overall *BMF* equal to sum of two *BMF*s

- A node with only one incoming branch & one outgoing branch can be replaced by a single branch with *BMF* equal to the product of the two *BMF*s

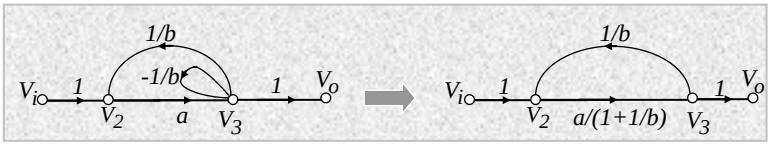
- An intermediate node can be multiplied by a factor (*k*). *BMF*s for incoming branches have to be multiplied by *k* and outgoing branches divided by *k*

Signal Flowgraph (SFG) Rules

- Simplifications can often be achieved by shifting or eliminating nodes

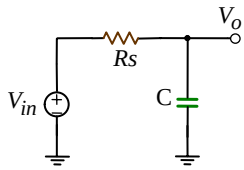


•A self-loop branch with *BMF* *y* can be eliminated by multiplying the *BMF* of <u>incoming</u> branches by *1/(1-y)*



Integrator Based Filters
1st Order LPF

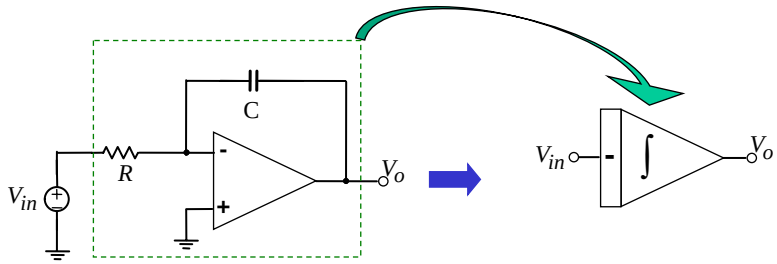
- Conversion of simple lowpass RC filter to integrator-based type by using signal flowgraph technique



$$\frac{V_o}{V_{in}} = \frac{1}{1+sRC}$$

What is an Integrator?

Example: Single-Ended Opamp-RC Integrator



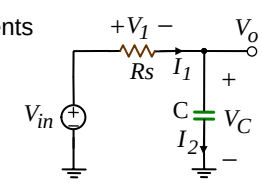
$$V_o = -V_{in} \times \frac{1}{sRC} \quad , \quad V_o = -\frac{1}{RC}\int V_{in}\,dt \qquad \tau = RC$$

Note: Practical integrator in CMOS technology has input & output both in the form of voltage and not current → Consideration for SFG derivation

Integrator Based Filters

1st Order LPF

1. Start from circuit prototype-
   - Name voltages & currents for all components



2. Use KCL & KVL to derive state space description- to have BMFs in the integrator form
   - Capacitor voltage expressed as function of its current $V_{Cap.}=f(I_{Cap.})$
   - Inductor current as a function of its voltage $I_{Ind.}=f(V_{Ind.})$

3. Use state space description to draw signal flowgraph (SFG) (see next page)

Integrator Based Filters

First Order LPF

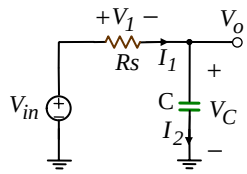
$$V_1 = V_{in} - V_C$$

$$V_C = I_2 \times \frac{1}{sC}$$ *Integrator form*

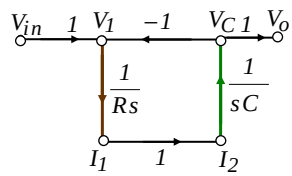
$$V_o = V_C$$

$$I_1 = V_1 \times \frac{1}{Rs}$$

$$I_2 = I_1$$



SFG



- All voltages & currents → nodes of SFG
- Voltage nodes on top, corresponding current nodes below each voltage node

Normalize

- Since integrators - the main building blocks- require in & out signals in the voltage form (not current)
  - → Convert all currents to voltages by multiplying current nodes by a scaling resistance R^*
  - → Corresponding *BMF*s should then be scaled accordingly

$$V_1 = V_{in} - V_o \qquad I_1 = \frac{V_1}{Rs} \qquad V_o = \frac{I_2}{sC} \qquad I_2 = I_1$$

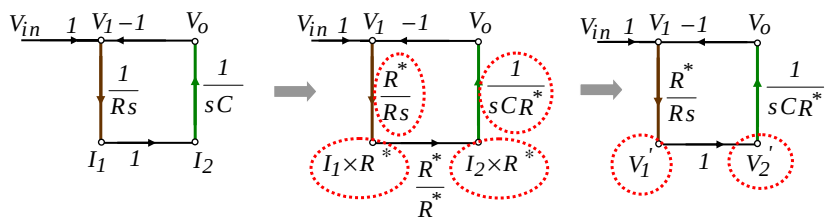
$$\Rightarrow$$

$$V_1 = V_{in} - V_o \qquad I_1 R^* = \frac{R^*}{Rs} V_1 \qquad V_o = \frac{I_2 R^*}{sC\ R^*} \qquad I_2 R^* = I_1 R^*$$

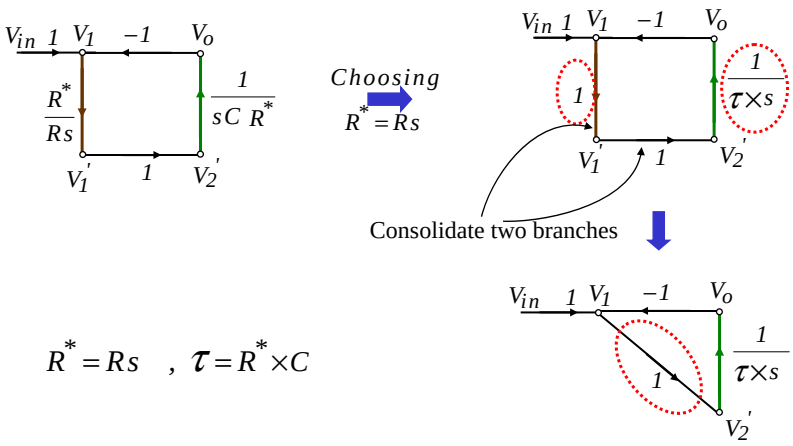
$$\Rightarrow \boxed{I_x R^* = V_x'} \Rightarrow$$

$$V_1 = V_{in} - V_o \qquad V_1' = \frac{R^*}{Rs} V_1 \qquad V_o = \frac{V_2'}{sC\ R^*} \qquad V_2' = V_1'$$

Normalize

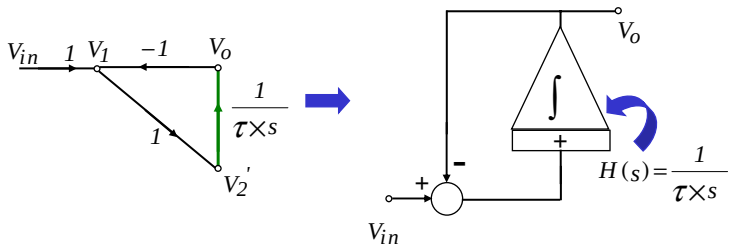


Synthesis



$$R^* = Rs \quad , \quad \tau = R^* \times C$$

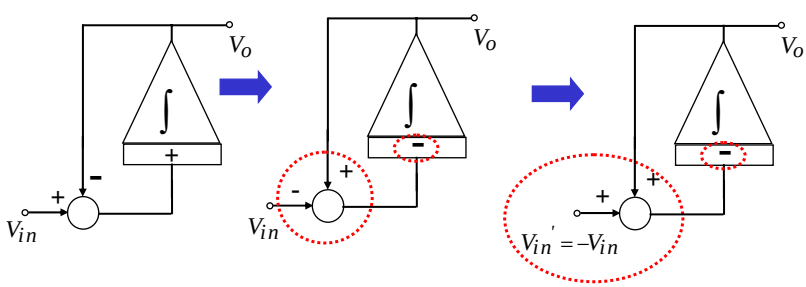
First Order Integrator Based Filter



1st Order Filter Built with Opamp-RC Integrator

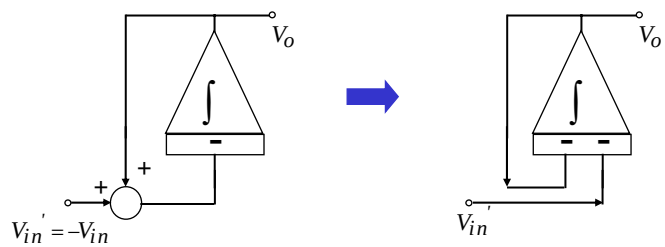
- Single-ended Opamp-RC integrator has a sign inversion from input to output

  → Convert SFG accordingly by modifying BMF

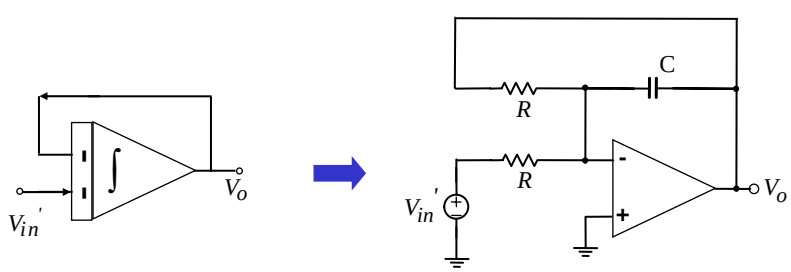


1st Order Filter
Built with Opamp-RC Integrator

- To avoid requiring an additional opamp to perform summation at the input node:



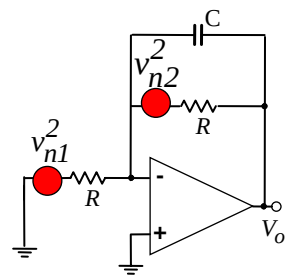
1st Order Filter
Built with Opamp-RC Integrator (continued)



$$\frac{V_o}{V_{in}'} = -\frac{1}{1+sRC}$$

Opamp-RC 1st Order Filter Noise

Identify noise sources (here it is resistors & opamp)
Find transfer function from each noise source to the output (opamp noise next page)



$$\overline{v_o^2} = \sum_{m=1}^{k} \int_0^{\infty} |H_m(f)|^2 S_m(f)\, df$$

$$S_i(f) \rightarrow Noise\ spectral\ density\ of\ m^{th}\ noise\ source$$

$$|H_1(f)|^2 = |H_2(f)|^2 = \frac{1}{1+(2\pi fRC)^2}$$

$$v_{n1}^2 = v_{n2}^2 = 4KTR\Delta f$$

$$\sqrt{\overline{v_o^2}} = \sqrt{2\frac{kT}{C}}$$

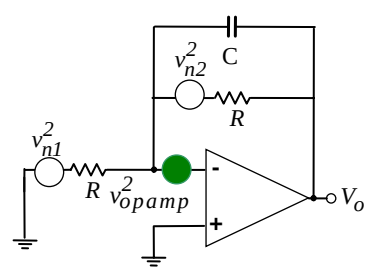
$$\alpha = 2$$

α

Typically, α increases as filter order increases

Opamp-RC Filter Noise Opamp Contribution

- So far only the fundamental noise sources are considered.
- In reality, noise associated with the opamp increases the overall noise.
- For a well-designed filter opamp is designed such that noise contribution of opamp << contribution of other noise sources
- The bandwidth of the opamp affects the opamp noise contribution to the total noise



Integrator Based Filter

2nd Order RLC Filter

•State space description:

$$V_R = V_L = V_C = V_o$$

$$V_C = \frac{I_C}{sC}$$

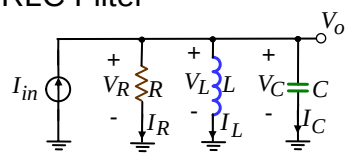
$$I_R = \frac{V_R}{R}$$

$$I_L = \frac{V_L}{sL}$$

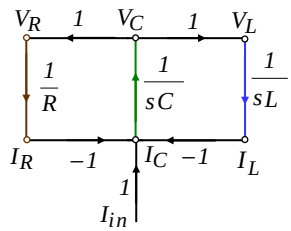
$$I_C = I_{in} - I_R - I_L$$

Integrator form

• Draw signal flowgraph (SFG)

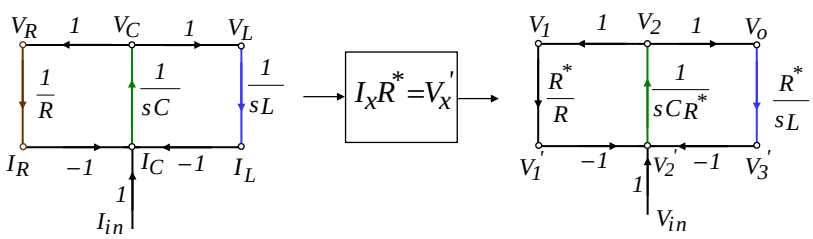


SFG

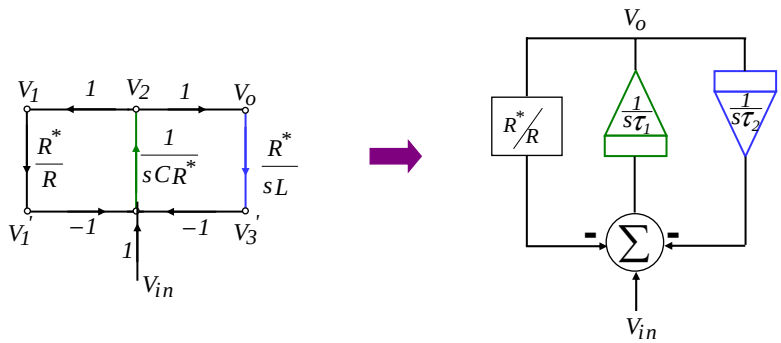


Normalize

- Convert currents to voltages by multiplying all current nodes by the scaling resistance R^*

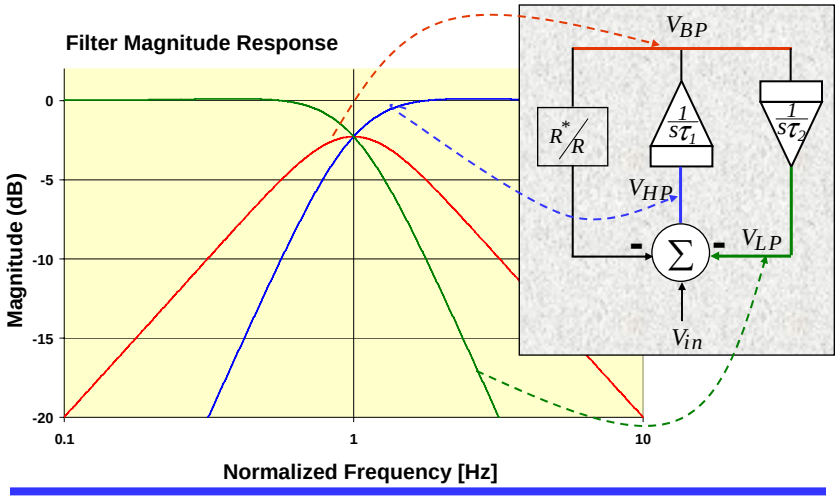


Synthesis



$\tau_1 = R^* \times C \qquad \tau_2 = L / R^*$

Second Order Integrator Based Filter



Second Order Integrator Based Filter

$$\frac{V_{BP}}{V_{in}} = \frac{\tau_2 s}{\tau_1\tau_2 s^2 + \beta\tau_2 s + 1}$$

$$\frac{V_{LP}}{V_{in}} = \frac{1}{\tau_1\tau_2 s^2 + \beta\tau_2 s + 1}$$

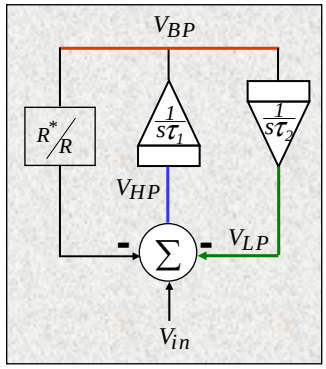
$$\frac{V_{HP}}{V_{in}} = \frac{\tau_1\tau_2 s^2}{\tau_1\tau_2 s^2 + \beta\tau_2 s + 1}$$

$$\tau_1 = R^* \times C \qquad \tau_2 = L / R^*$$

$$\beta = R^* / R$$

$$\omega_0 = 1 / \sqrt{\tau_1\tau_2} = 1 / \sqrt{L\,C}$$

$$Q = 1/\beta \times \sqrt{\tau_1 / \tau_2}$$



From matching point of view desirable:

$$\tau_1 = \tau_2 \rightarrow Q = R / R^*$$

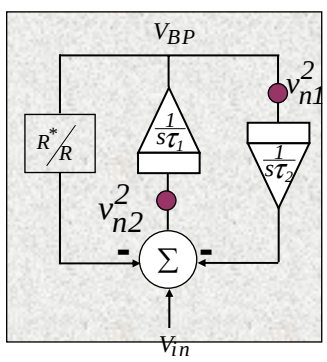
Second Order Bandpass Filter Noise

$$\overline{v_o^2} = \sum_{m=1}^{k} \int_0^\infty |H_m(f)|^2 S_m(f)\,df$$

- Find transfer function of each noise source to the output
- Integrate contribution of all noise sources
- Here it is assumed that opamps are noise free (not usually the case!)

$$v_{n1}^2 = v_{n2}^2 = 4KTRdf$$

$$\boxed{\sqrt{\overline{v_o^2}} = \sqrt{\underbrace{2Q}_{\alpha}\frac{kT}{C}}}$$

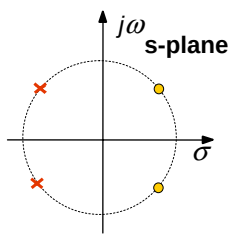


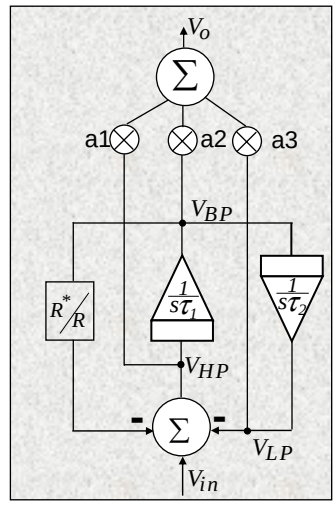
Typically, α increases as filter order increases
Note the noise power is directly proportion to *Q*

Second Order Integrator Based Filter Biquad

- By combining outputs can generate general biquad function:

$$\frac{V_0}{V_{in}} = \frac{a_1 \tau_1 \tau_2 s^2 + a_2 \tau_2 s + a_3}{\tau_1 \tau_2 s^2 + \beta \tau_2 s + 1}$$





Summary
Integrator Based Monolithic Filters

- Signal flowgraph techniques utilized to convert RLC networks to integrator based active filters
- Each reactive element (L& C) replaced by an integrator
- Fundamental noise limitation determined by integrating capacitor value:
  - For lowpass filter: $\sqrt{\overline{v_o^2}} = \sqrt{\alpha \frac{k\,T}{C}}$
  - Bandpass filter: $\sqrt{\overline{v_o^2}} = \sqrt{\alpha\, Q\, \frac{k\,T}{C}}$

where α is a function of filter order and topology

Higher Order Filters

- How do we build higher order filters?
  - Cascade of biquads and 1st order sections
    - Each complex conjugate pole built with a biquad and real pole with 1st order section
    - Easy to implement
    - In the case of high order high Q filters → highly sensitive to component variations
  - Direct conversion of high order ladder type RLC filters
    - SFG techniques used to perform exact conversion of ladder type filters to integrator based filters
    - More complicated conversion process
    - Much less sensitive to component variations compared to cascade of biquads

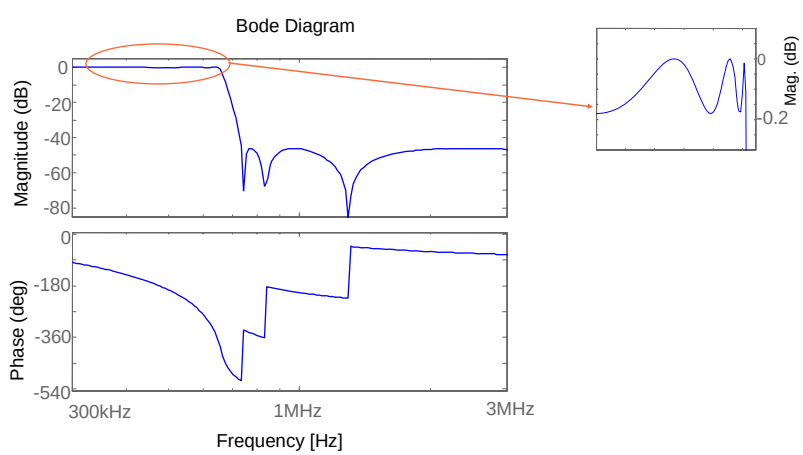
- Lecture 3 ended here

Higher Order Filters
Cascade of Biquads

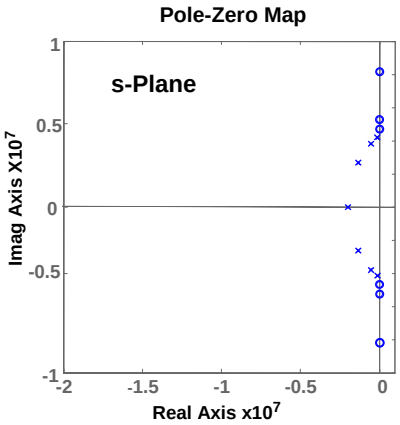
Example: LPF filter for CDMA baseband receiver

- LPF with
  - fpass = 650 kHz Rpass = 0.2 dB
  - fstop = 750 kHz Rstop = 45 dB
  - Assumption: Can compensate for phase distortion in the digital domain
- 7th order Elliptic Filter
- Implementation with cascaded Biquads
  Goal: Maximize dynamic range
  - Pair poles and zeros
  - Highest Q poles with closest zeros is a good starting point, but not necessarily optimum
  - Ordering:
    Lowest Q poles first is a good start

Filter Overall Frequency Response



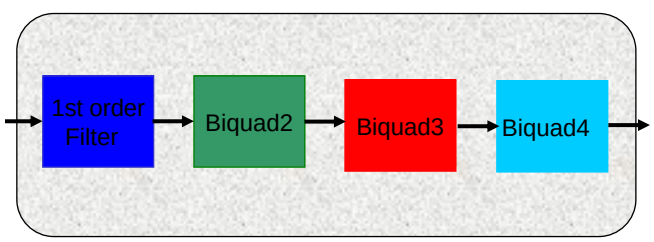
Pole-Zero Map



Q_{pole}	f_{pole} [kHz]
16.7902	659.496
3.6590	611.744
1.1026	473.643
	319.568

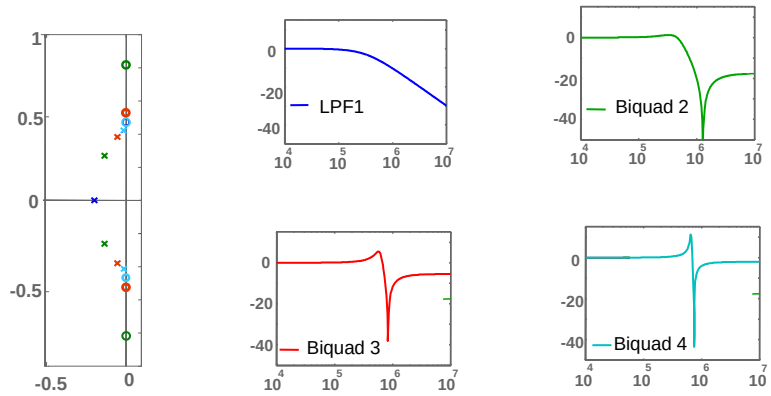
f_{zero} [kHz]
1297.5
836.6
744.0

CDMA Filter
Built with Cascade of 1st and 2nd Order Sections

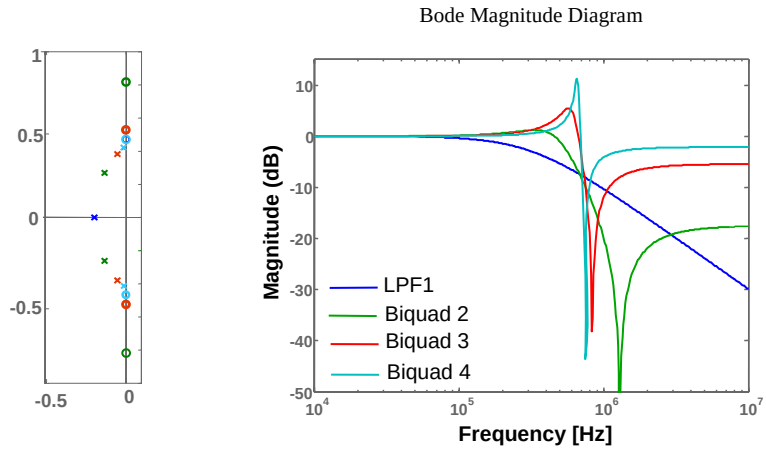


- 1st order filter implements the single real pole
- Each biquad implements a pair of complex conjugate pole and a pair of imaginary zero

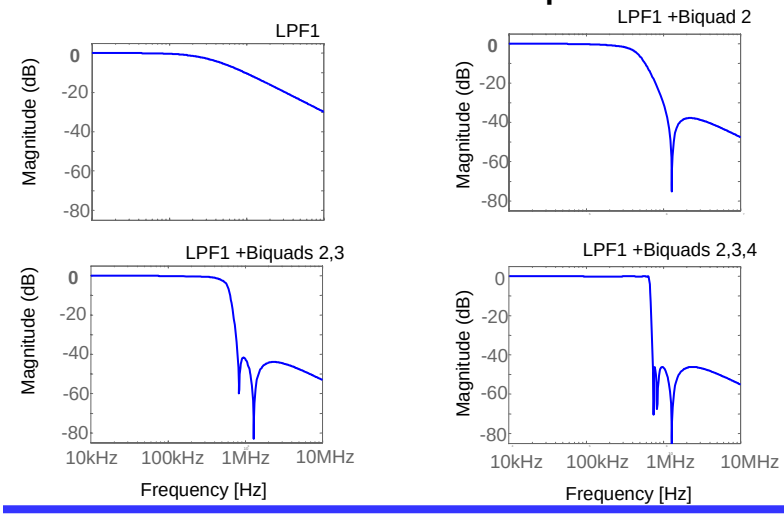
Biquad Response



Biquad Response



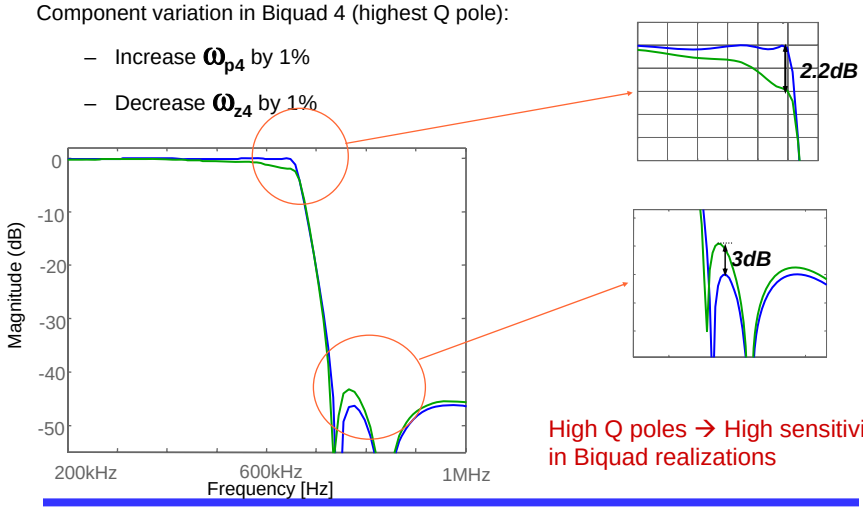
Intermediate Outputs



Sensitivity

Component variation in Biquad 4 (highest Q pole):

- Increase ω_{p4} by 1%
- Decrease ω_{z4} by 1%



High Q poles → High sensitivity in Biquad realizations