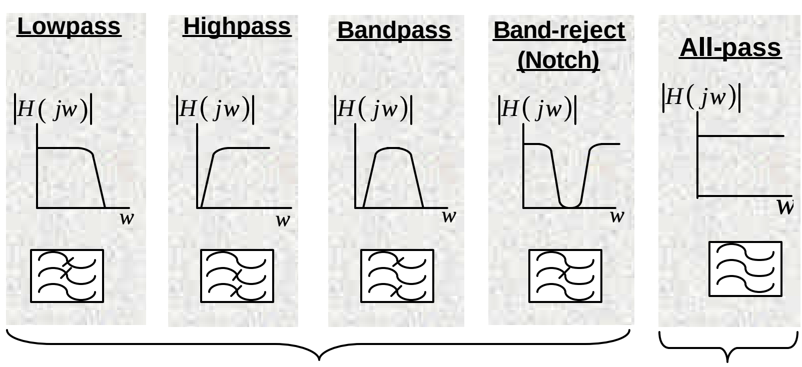
EE247
Lecture 2

- Material covered today:
  - Nomenclature
  - Filter specifications
    - Quality factor
    - Frequency characteristics
    - Group delay
  - Filter types
    - Butterworth
    - Chebyshev I
    - Chebyshev II
    - Elliptic
    - Bessel
  - Group delay comparison example

Nomenclature
Filter Types



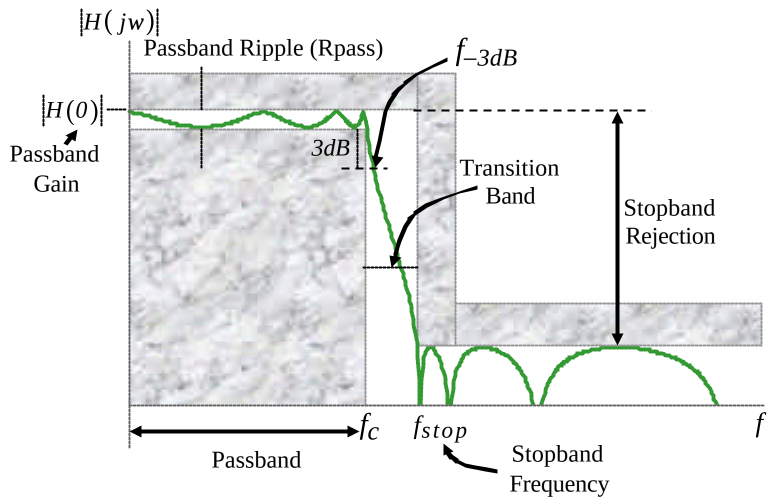
Provide frequency selectivity

Phase shaping or equalization

Filter Specifications

- Frequency characteristics (lowpass filter):
  - Passband ripple (Rpass)
  - Cutoff frequency or *-3dB* frequency
  - Stopband rejection
  - Passband gain
- Phase characteristics:
  - Group delay
- SNR (Dynamic range)
- SNDR (Signal to Noise+Distortion ratio)
- Linearity measures: IM3 (intermodulation distortion), HD3 (harmonic distortion), IIP3 or OIP3 (Input-referred or output-referred third order intercept point)
- Power/pole & Area/pole

Lowpass Filter Frequency Characteristics



Quality Factor (Q)

- The term Quality Factor (Q) has different definitions:
  - Component quality factor (inductor & capacitor Q)
  - Pole quality factor
  - Bandpass filter quality factor

- Next 3 slides clarifies each

Component Quality Factor (Q)

- For any component with a transfer function:

$$H(jw) = \frac{1}{R(w) + jX(w)}$$

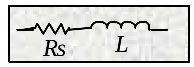
- Quality factor is defined as:

$$Q = \frac{X(w)}{R(w)} \rightarrow \frac{Energy\ Stored}{Average\ Power\ Dissipation}\quad per unit time$$

Inductor & Capacitor Quality Factor

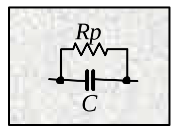
- Inductor Q :

$$Y_L = \frac{1}{Rs + jwL} \qquad Q_L = \frac{wL}{Rs}$$

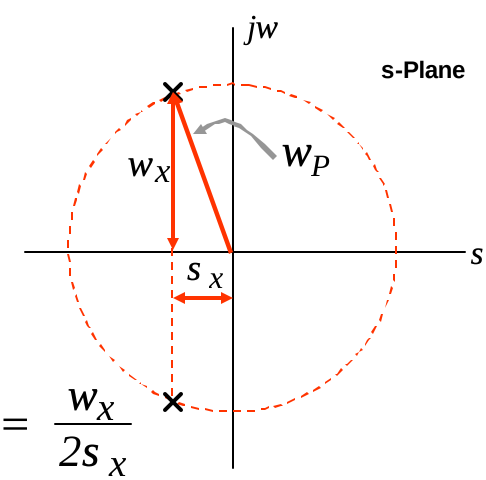


- Capacitor Q :

$$Z_C = \frac{1}{\frac{1}{Rp} + jwC} \qquad Q_C = wCRp$$

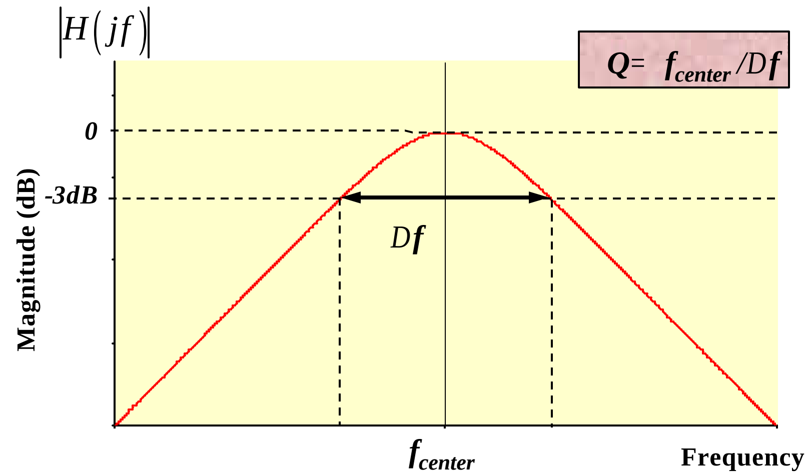


Pole Quality Factor



$$Q_{Pole} = \frac{w_x}{2s_x}$$

Bandpass Filter Quality Factor (Q)



What is Group Delay?

- Consider a continuous time filter with s-domain transfer function G(s):

$$G(j\omega) \equiv |G(j\omega)| e^{j\theta(\omega)}$$

- Let us apply a signal to the filter input composed of sum of two sinewaves at slightly different frequencies ($\Delta\omega << \omega$):

$$v_{IN}(t) = A_1 \sin(\omega t) + A_2 \sin[(\omega + \Delta\omega)\, t]$$

- The filter output is:

$$v_{OUT}(t) = A_1 |G(j\omega)| \sin[\omega t + \theta(\omega)] +$$

$$A_2 |G[j(\omega + \Delta\omega)]| \sin[(\omega + \Delta\omega) t + \theta(\omega + \Delta\omega)]$$

What is Group Delay?

$$v_{OUT}(t) = A_1 ½G(jw)½ \sin \left\{ w \left[t + \frac{q(w)}{w} \right] \right\} +$$

$$+ A_2 ½G[j(w+Dw)]½ \sin \left\{ (w+Dw) \left[t + \frac{q(w+Dw)}{w+Dw} \right] \right\}$$

Since $\frac{Dw}{w} << 1$ then $\left[\frac{Dw}{w}\right]^2 \rightarrow 0$

$$\frac{q(w+Dw)}{w+Dw} @ \left[q(w) + \frac{dq(w)}{dw} Dw \right] \left[\frac{1}{w} \left(1 - \frac{Dw}{w} \right) \right]$$

$$@ \frac{q(w)}{w} + \left(\frac{dq(w)}{dw} - \frac{q(w)}{w} \right) \frac{Dw}{w}$$

What is Group Delay?
Signal Magnitude and Phase Impairment

$$v_{OUT}(t) = A_1 ½G(jw)½ \sin \left\{ w \left[t + \frac{q(w)}{w} \right] \right\} +$$

$$+ A_2 ½G[j(w+Dw)]½ \sin \left\{ (w+Dw) \left[t + \frac{q(w)}{w} + \left(\frac{dq(w)}{dw} - \frac{q(w)}{w} \right) \frac{Dw}{w} \right] \right\}$$

- If the second term in the phase of the 2nd sinwave is non-zero, then the filter's output at frequency $\omega+\Delta\omega$ is time-shifted differently than the filter's output at frequency ω
  → "Phase distortion"
- If the second term is zero, then the filter's output at frequency $\omega+\Delta\omega$ and the output at frequency ω are each delayed in time by $-\theta(\omega)/\omega$
- $\tau_{PD} \equiv -\theta(\omega)/\omega$ is called the "phase delay" and has units of time

What is Group Delay?
Signal Magnitude and Phase Impairment

- Phase distortion is avoided only if:

$$\frac{dq(w)}{dw} - \frac{q(w)}{w} = 0$$

- Clearly, if $\theta(\omega)=k\omega$, k a constant, → no phase distortion
- This type of filter phase response is called "linear phase"

  →Phase shift varies linearly with frequency
- $\tau_{GR} \equiv -d\theta(\omega)/d\omega$ is called the "group delay" and also has units of time. For a linear phase filter $\tau_{GR} \equiv \tau_{PD} = k$

  → $\tau_{GR} = \tau_{PD}$ implies linear phase
- Note: Filters with $\theta(\omega)=k\omega+c$ are also called linear phase filters, but they're not free of phase distortion

What is Group Delay?
Signal Magnitude and Phase Impairment

- If $\tau_{GR} = \tau_{PD}$ → No phase distortion

$$v_{OUT}(t) = A_1 \, ½G(jw)½ \, \sin\left[w\left(t - t_{GR}\right)\right] +$$

$$+ A_2 \, ½G[\, j(w+Dw)]½ \, \sin\left[(w+Dw)\left(t - t_{GR}\right)\right]$$

- If also $|G(j\omega)| = |G[j(\omega+\Delta\omega)]|$ for all input frequencies within the signal-band, V_{OUT} is a scaled, time-shifted replica of the input, with no "signal magnitude distortion" :
- In most cases neither of these conditions are realizable exactly

Summary
Group Delay

- Phase delay is defined as:

  $\tau_{PD} \equiv -\theta(\omega)/\omega$ [time]
- Group delay is defined as :

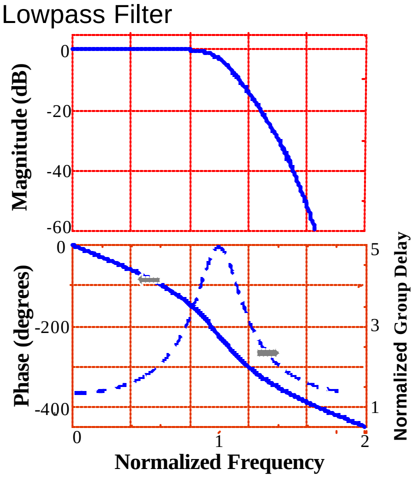
  $\tau_{GR} \equiv -d\theta(\omega)/d\omega$ [time]
- If $\theta(\omega)=k\omega$, k a constant, → no phase distortion
- For a linear phase filter $\tau_{GR} \equiv \tau_{PD} = k$

Filter Types
Butterworth Lowpass Filter

- Maximally flat amplitude within the filter passband

$$\left.\frac{d^N |H(jw)|}{dw}\right|_{w=0} = 0$$

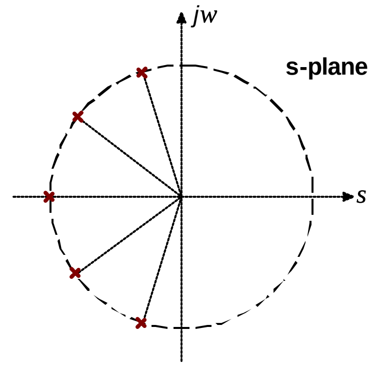
- Moderate phase distortion



Example: 5th Order Butterworth filter

Butterworth Lowpass Filter

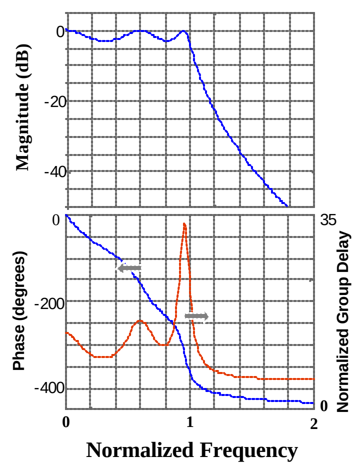
- All poles
- Poles located on the unit circle with equal angles



Example: 5th Order Butterworth filter

Filter Types
Chebyshev I Lowpass Filter

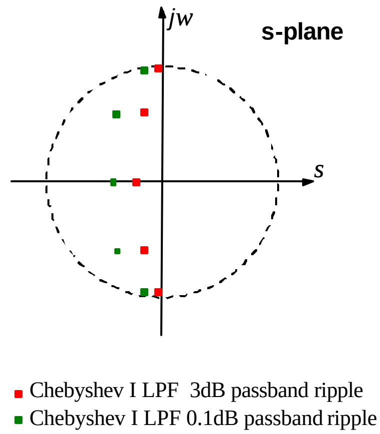
- Chebyshev I filter
  - Equal-ripple passband
  - Sharper transition band compared to Butterworth
  - Poorer group delay



Example: 5th Order Chebyshev filter

Chebyshev I Lowpass Filter Characteristics

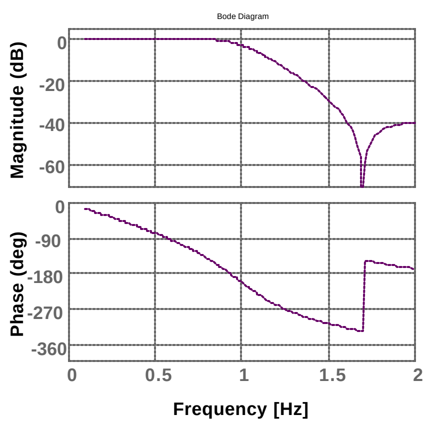
- All poles
- Poles located on an ellipse inside the unit circle
- Allowing more ripple in the passband:
  - ⇨Narrower transition band
  - ⇨Sharper cut-off
  - ⇨Higher pole Q



Example: 5th Order Chebyshev I Filter

Filter Types
Cheybshev II Lowpass

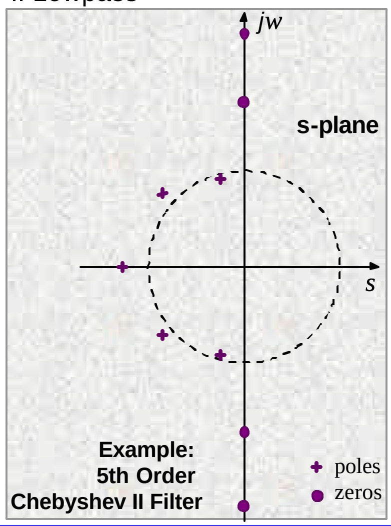
- Chebyshev II filter
  - Ripple in stopband
  - Sharper transition band compared to Butterworth
  - Passband group delay superior to Chebyshev I



Example: 5th Order Chebyshev II filter

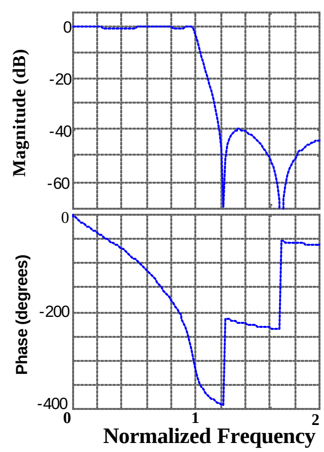
Filter Types
Cheybshev II Lowpass

- Both poles & zeros
  - No. of poles *n*
  - No. of zeros *n-1*
- Poles located both inside & outside of the unit circle
- Zeros located on *jw* axis
- Ripple in the stopband only



Filter Types
Elliptic Lowpass Filter

- Elliptic filter
  - Ripple in passband
  - Ripple in the stopband
  - Sharper transition band compared to Butterworth & both Chebyshevs
  - Poorer group delay



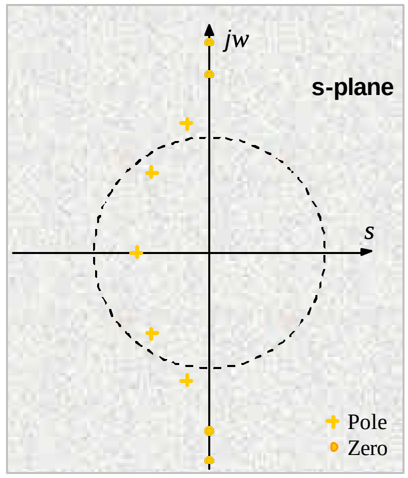
Example: 5th Order Elliptic filter

Filter Types
Elliptic Lowpass Filter

- Both poles & zeros
  – No. of poles *n*
  – No. of zeros *n-1*
- Zeros located on *jw* axis
- Sharp cut-off

  ⇨Narrower transition band

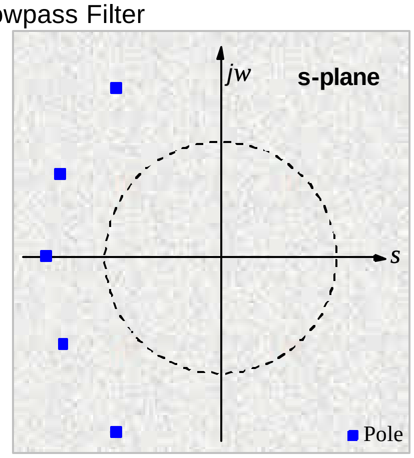
  ⇨Pole Q higher compared to the previous filters



Example: 5th Order Elliptic Filter

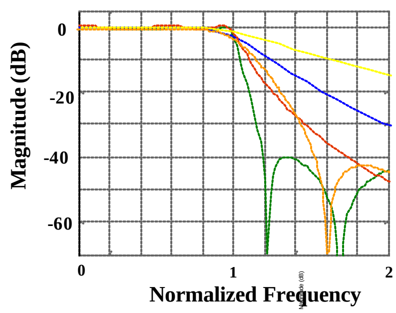
Filter Types
Bessel Lowpass Filter

- Bessel
  – All poles
  – Maximally flat group delay
  – Poor amplitude attenuation
  – Poles outside unit circle (s-plane)
  – Relatively low Q poles



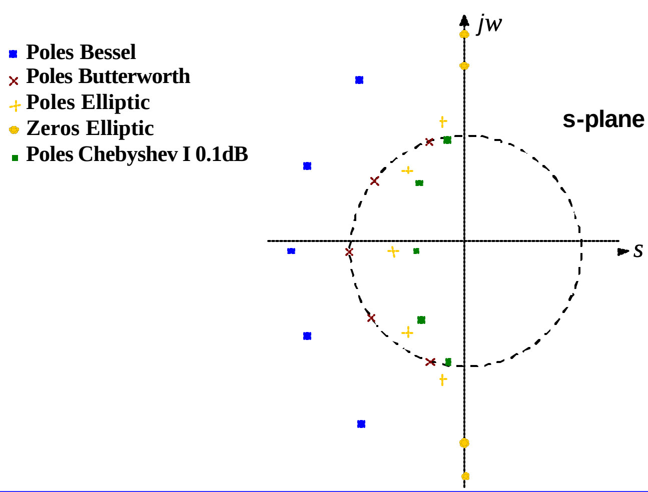
Example: 5th Order Bessel filter

Filter Types
Comparison of Various LPF Magnitude Response

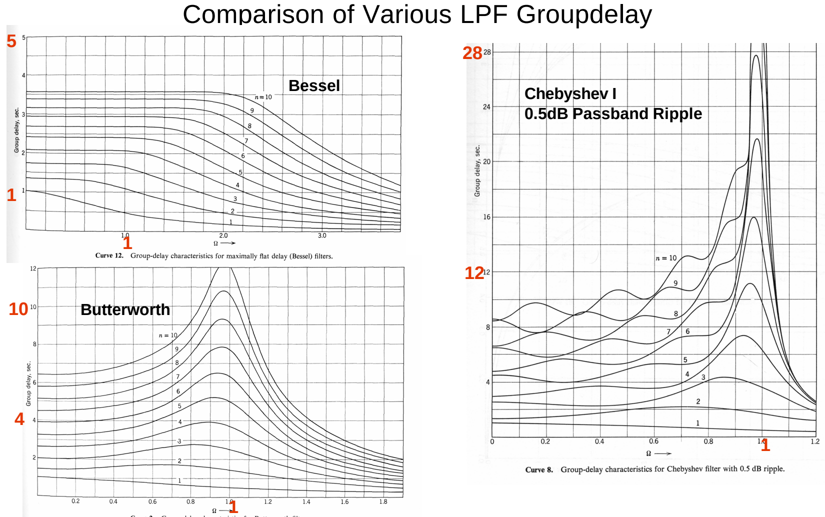


All 5th order filters with same corner freq.

Filter Types
Comparison of Various LPF Singularities



Comparison of Various LPF Groupdelay

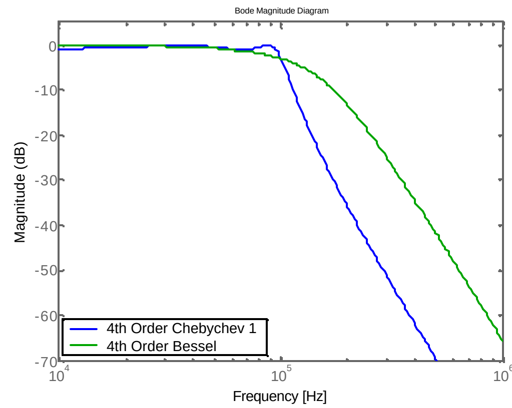


Ref: A. Zverev, *Handbook of filter synthesis*, Wiley, 1967.

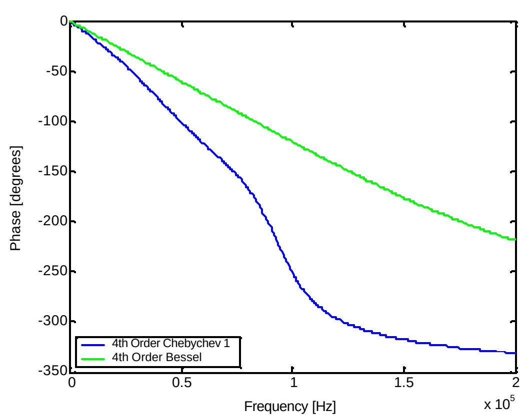
Group Delay Comparison Example

- Lowpass filter with 100kHz corner frequency
- Chebyshev I versus Bessel
  - Both filters 4th order- same ***-3dB*** point
  - Passband ripple of ***1dB*** allowed for Chebyshev I

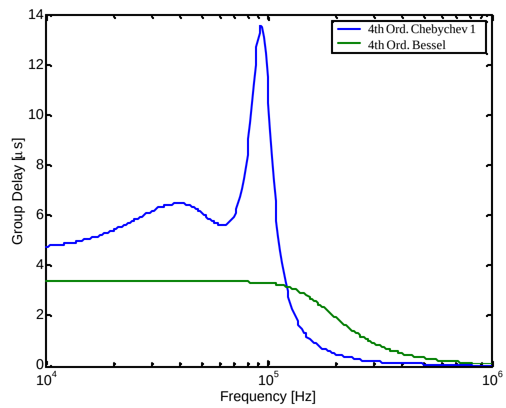
Magnitude Response



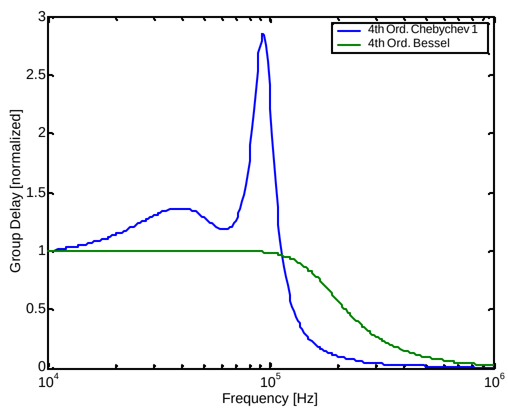
Phase Response



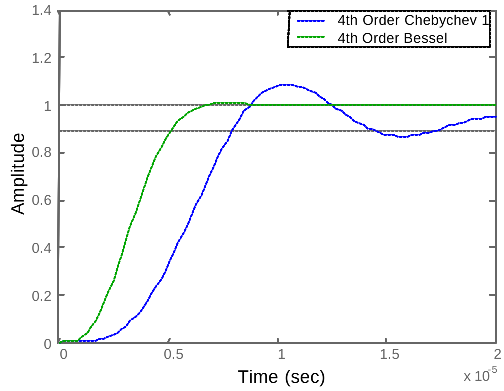
Group Delay



Normalized Group Delay



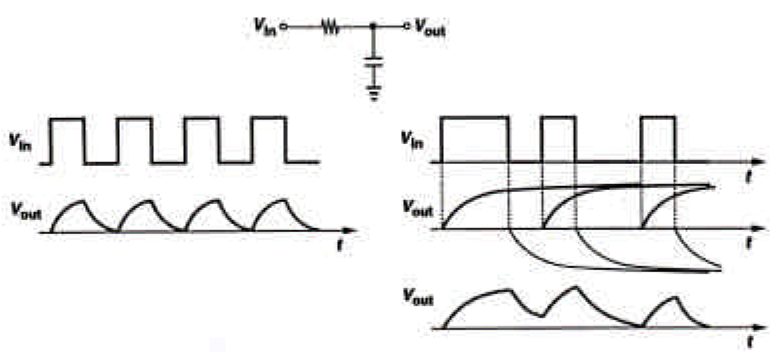
Step Response



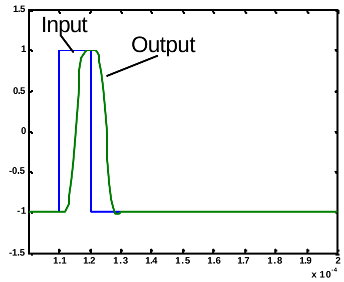
Intersymbol Interference (ISI)

ISI→ Broadening of pulses resulting in interference between successive transmitted pulses

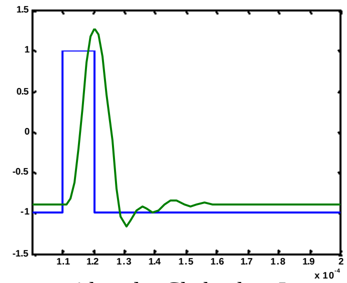
Example: Simple RC filter



Pulse Broadening
Bessel versus Chebyshev



8th order Bessel

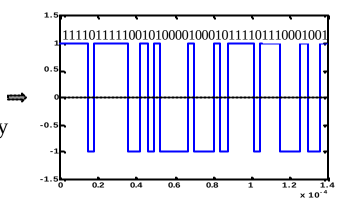


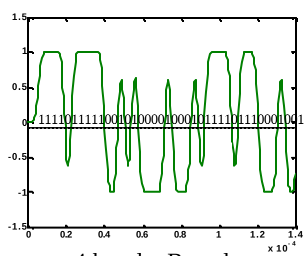
4th order Chebyshev I

Chebyshev has more pulse broadening compared to Bessel → More ISI

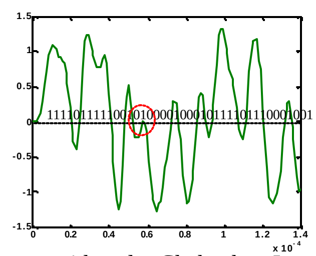
Response to Random Data
Chebyshev versus Bessel

Input Signal:
130kHz max.
signal spectral density



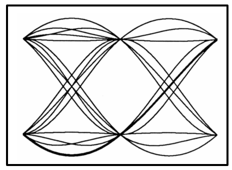


4th order Bessel



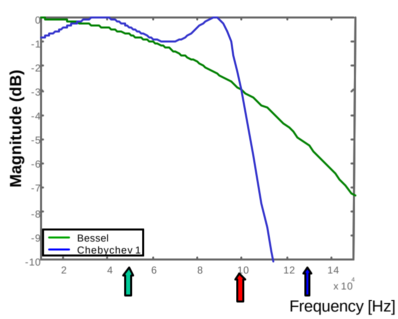
4th order Chebyshev I

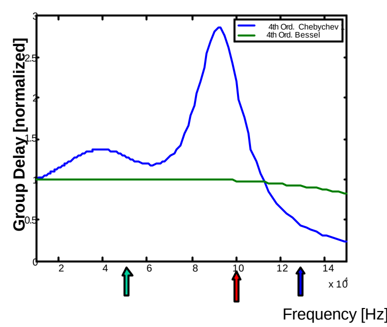
Measure of Signal Degradation
Eye Diagram



- Eye diagram is a useful graphical illustration for signal degradation
- Consists of many overlaid traces of a signal using an oscilloscope where the symbol timing serves as the scope trigger
- It is a visual summary of all possible intersymbol interference waveforms
  - The vertical opening → immunity to noise
  - Horizontal opening → timing jitter

Measure of Signal Degradation
Eye Diagram



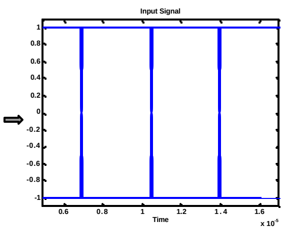


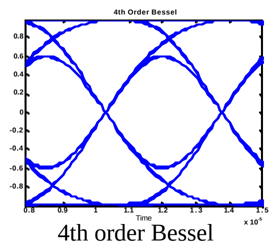
- Random data with max. power spectral density of:
  - 50kHz
  - 100kHz
  - 130kHz

Eye Diagram
Chebyshev versus Bessel

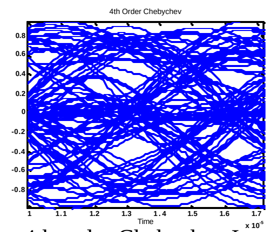
Input Signal
Random data
maximum power
spectral density → 130kHz

⇨



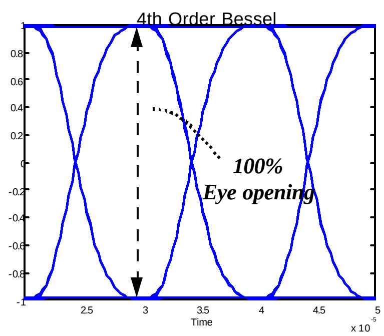


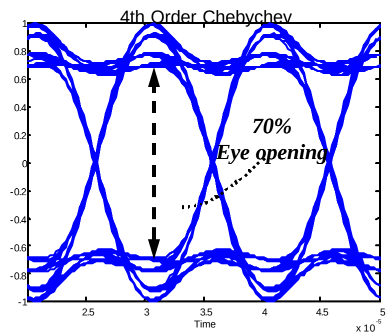
4th order Bessel



4th order Chebyshev I

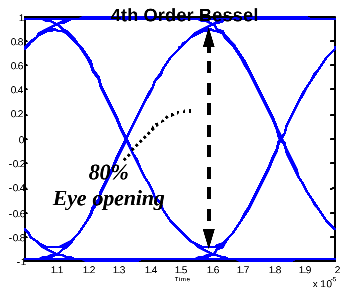
Eye Diagrams

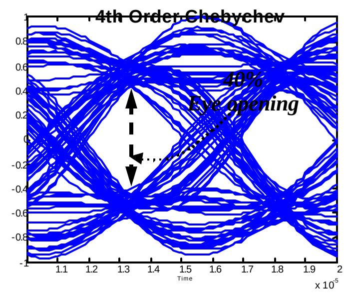




Random data maximum power spectral density → 50kHz

Eye Diagrams





Random data maximum power spectral density → 100kHz

Filter with constant group delay → More open eye → Lower BER (bit-error-rate)

Summary
Filter Types

– Filters with high signal attenuation per pole ⇨ poor phase response

– For a given signal attenuation requirement of preserving constant groupdelay →Higher order filter
  - In the case of passive filters ⇨ higher component count
  - Case of integrated active filters ⇨ higher chip area & power dissipation

– In cases where filter is followed by ADC and DSP
  - Possible to digitally correct for phase non-linearities incurred by the analog circuitry by using phase equalizers

Summary
Filter Types

- Filters with high signal attenuation per pole → poor phase response
- For a given signal attenuation requirement of preserving constant groupdelay →Higher order filter
  - In the case of passive filters → higher component count
  - Case of integrated active filters → higher chip area & power dissipation
- In cases where filter is followed by ADC and DSP →possible to digitally correct for phase non-linearities incurred by the analog circuitry by using digital phase equalizers

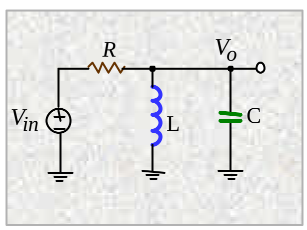
RLC Filters

•Bandpass filter:

$$\frac{V_o}{V_{in}} = \frac{\frac{s}{RC}}{s^2 + \frac{w_o}{Q}s + w_o^2}$$

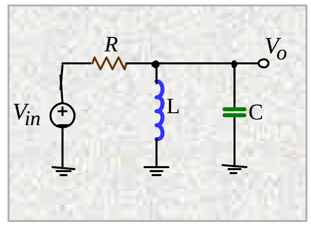
$$w_o = 1 \Big/ \sqrt{LC}$$

$$Q = w_o RC = \frac{R}{L w_o}$$



RLC Filters

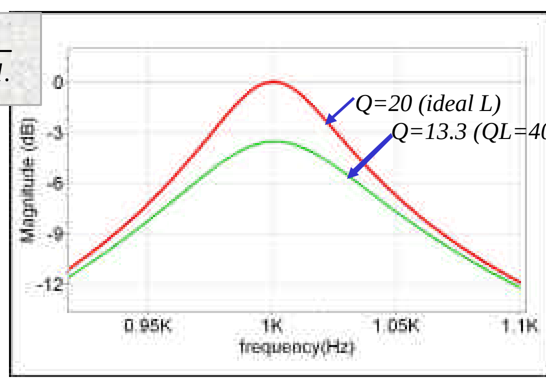
- Design a bandpass filter with:
  - Center frequency of 1kHz
  - Q of 20



- Assume that the inductor has series R resulting in an inductor Q of 40
- What is the effect of finite inductor Q on the overall Q?

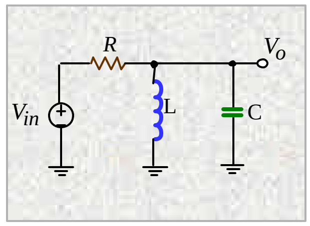
RLC Filters
Effect of Component Finite Q

$$\frac{1}{Q_{filt}} = \frac{1}{Q_{filt}^{ideal}} + \frac{1}{Q_{ind.}}$$



⇨**Component Q must be much higher compared to desired filter Q**

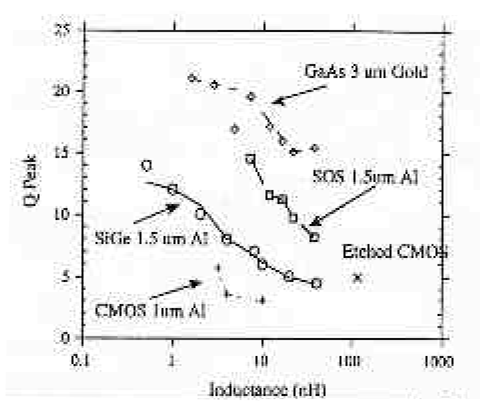
RLC Filters



Question:

Can RLC filters be integrated on-chip?

Monolithic Inductors
Feasible Quality Factor & Value



⇨ Feasible monolithic inductor in CMOS tech. <10nH with Q <7

❖Ref: "Radio Frequency Filters", Lawrence Larson; Mead workshop presentation 1999

Monolithic LC Filters

- Monolithic inductor in CMOS tech.
  – L<10nH with Q<7
- Max. capacitor
  – C< 10pF

⇨*LC filters in the monolithic form feasible:*
- freq >500MHz
- Only low quality factor filters

Learn more in EE242

Monolithic Filters

- Desirable to integrate filters with critical frequencies << 500MHz
- Per previous slide LC filters not a practical option in the integrated form
- Good alternative:

⇨Integrator based filters