# EE247
# Lecture 11

# Data Converters



## Material Covered in EE247

- ✓ Filters
  - Continuous-time filters
    - Biquads & ladder type filters
    - Opamp-RC, Opamp-MOSFET-C, gm-C filters
    - Automatic frequency tuning
  - Switched capacitor (SC) filters
- Data Converters
  - D/A converter architectures
  - A/D converter
    - Nyquist rate ADC- Flash, Pipeline ADCs,….
    - Oversampled converters
    - Self-calibration techniques
- Systems utilizing analog/digital interfaces
  - Wireline communication systems- ISDN, XDSL…
  - Wireless communication systems- Wireless LAN, Cellular telephone,…
  - Disk drive electronics
  - Fiber-optics systems

# Data Converter Topics

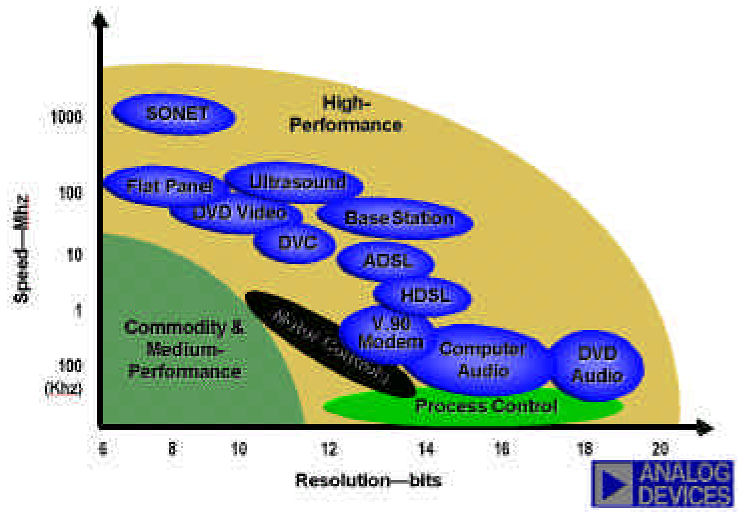
- Basic Operation of Data Converters
  - Uniform sampling and reconstruction
  - Uniform amplitude quantization
- Characterization and Testing
- Common ADC/DAC Architectures
- Selected Topics in Converter Design
  - Practical Implementations
  - Desensitization to Analog Circuit Non-Idealities
- Figures of Merit and Performance Trends



# Suggested Reference Texts


- R. v. d. Plassche, *CMOS Integrated Analog-to-Digital and Digital-to-Analog Converters*, 2nd ed., Kluwer, 2003.
- B. Razavi, *Data Conversion System Design,* IEEE Press, 1995.
- S. Norsworthy et al (eds), *Delta-Sigma Data Converters,* IEEE Press, 1997.
  Extensive treatment of oversampled converters including stability, tones, bandpass converters.
- J. G. Proakis, D. G. Manolakis, *Digital Signal Processing*, Prentice Hall, 1995.

# Converter Applications






# Example: Typical Cell Phone

Contains in integrated form:

- 4 Rx filters
- 4 Tx filters
- 4 Rx ADCs
- 4 Tx DACs

} Dual Standard, I/Q

- 3 Auxiliary ADCs
- 8 Auxiliary DACs

} Audio, Tx/Rx power control, Battery charge control, display, ...

**Total: Filters → 8**

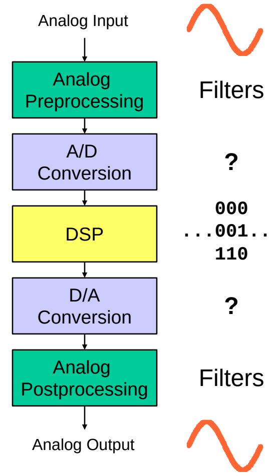
**ADCs → 7**

**DACs → 12**

# Data Converter Basics

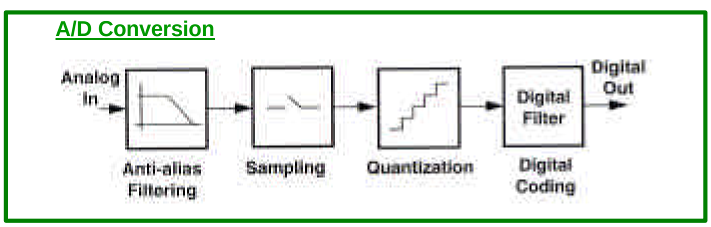
- DSP is wonderful, but...
- Real world signals are analog:
  - Continuous time
  - Continuous amplitude
- DSP can only process:
  - Discrete time
  - Discrete amplitude

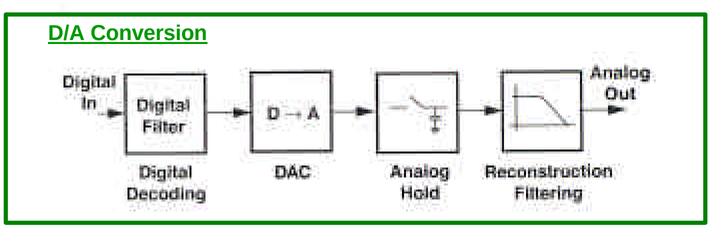
    → Need for data conversion from analog to digital and digital to analog






# A/D & D/A Conversion



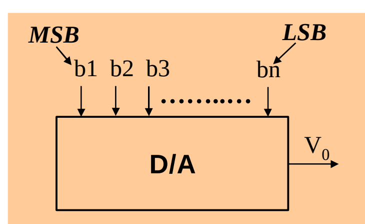
# Data Converters

- Stand alone data converters
  - Used in variety of systems
  - Example: Analog Devices AD9235 12bit/ 65Ms/s ADC- Applications:
    - Ultrasound equipment
    - IF sampling in wireless receivers
    - Hand-held scopemeters
    - Low cost digital oscilloscopes
- Embedded data converters
  - Cost, reliability, and performance → integration of data conversion interfaces along with DSPs
  - Main issues
    - Feasibility of integrating sensitive analog functions in a technology optimized for digital performance
    - Down scaling of supply voltage
    - Interference & spurious signal pick-up from on-chip digital circuitry
    - Portable applications dictate low power consumption



# D/A Converter Transfer Characteristic

- For an ideal digital-to-analog converter with uniform, binary digital encoding & a unipolar output range from 0 to $V_{FS}$




$$V_0 = V_{FS}\sum_{i=1}^{N}\frac{bi}{2^i} = \Delta\sum_{i=1}^{N} bi \times 2^{N-i} \quad , bi = 0 \ \ or \ \ 1$$

where $N = \#$ of bits
$V_{FS} =$ full scale output
$\Delta =$ step size

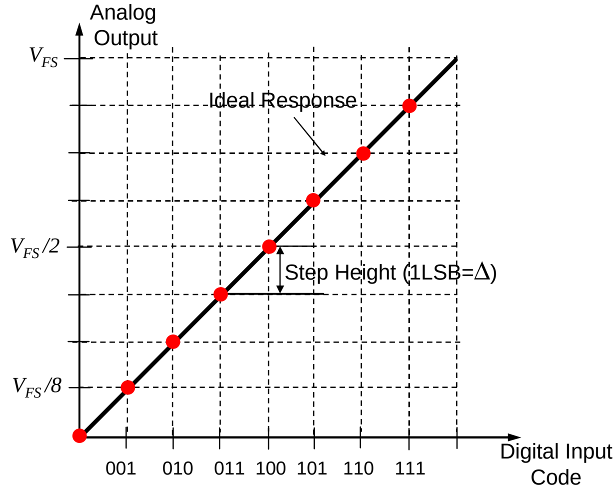
$$Note: V_0(bi = 1, all\ i) = V_{FS} - \Delta$$
$$= V_{FS}\left(1 - \frac{1}{2^N}\right)$$

Example: $N = 3$

$$V_0 = \Delta\left(b1.2^2 + b2.2^1 + b3.2^0\right)$$

# Ideal D/A Transfer Characteristic

- Ideal DAC introduces no error!
- One-to-one mapping from input to output



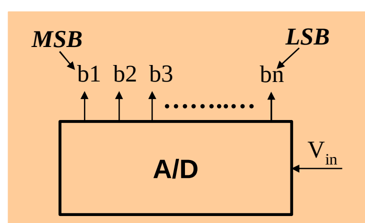



# A/D Converter Transfer Characteristic

- For an ideal analog-to-digital converter with uniform, binary digital encoding & a unipolar input range for 0 to $V_{FS}$

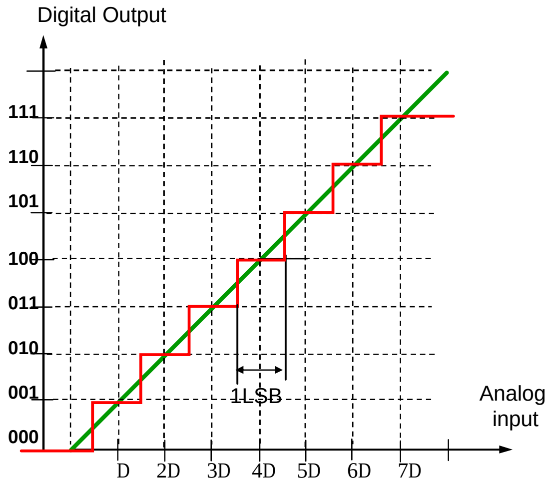
$$\begin{aligned} where\ m &= \#\ of\ bits \\ V_{FS} &= full\ scale\ output \\ \Delta &= step\ size \end{aligned}$$

$$Note: D(bi = 1, all\ i) \rightarrow V_{FS} - \Delta \rightarrow V_{FS}\left(1 - \frac{1}{2^m}\right)$$



# Ideal A/D Transfer Characteristic

- Ideal ADC introduces error
  →(+-1/2Δ)
  $\Delta = V_{FS}/2^m$
  $m$ = # of bits
- This error is called ``quantization error``



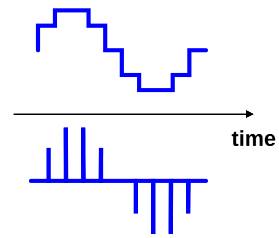



# Data Converter Performance Metrics

- Data Converters are typically characterized by static, time-domain, & frequency domain performance metrics :
  - Static
    - Monotonicity
    - Offset
    - Gain error
    - Differential nonlinearity (DNL)
    - Integral nonlinearity (INL)
  - Dynamic
    - Delay, settling time
    - Aperture uncertainty
    - Distortion- harmonic content
    - Signal-to-noise ratio (SNR), Signal-to-(noise+distortion) ratio (SNDR)
    - Idle channel noise
    - Dynamic range & spurious-free dynamic range (SFDR)

# What is a discrete time signal?

- ❑ A signal that changes only at discrete time instances?
- ❑ A continous time signal multiplied with a train of infinitely narrow unit pulses?
- ❑ A vector whose element indices correspond to discrete instances in time?
- ❑ All of the above?



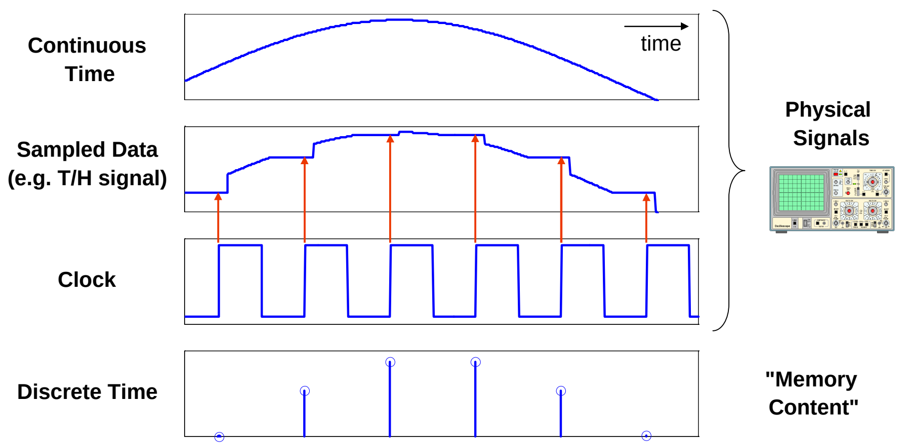

[1.2  2.0  2.5  0.1 ...]



# Discrete Time Signals

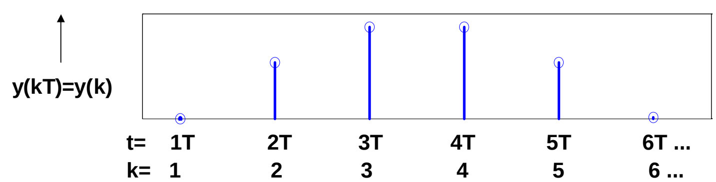
- A sequence of numbers (or vector) with discrete index time instants
- Intermediate signal values <u>not defined</u> (not the same as equal to zero!)
- Mathematically convenient, non-physical
- We will use the term "*sampled data*" for related signals that occur in real, physical interface circuits

# Typical Sampling Process
# CT ⇒ SD ⇒ DT






# Uniform Sampling




- Samples spaced T seconds in time
- Sampling Period T ⇔ Sampling Frequency $f_s=1/T$
- Problem: Multiple continuous time signals can yield exactly the same discrete time signal (aliasing)

# Summary
# Data Converters

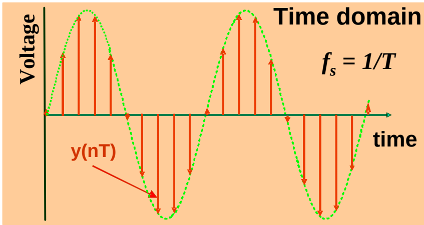
- ADC/DACs need to *sample/reconstruct* to convert from continuous time to discrete time signals and back
- We distinguish between purely mathematical discrete time signals and "sampled data signals" that carry information in actual circuits
- Question: How do we ensure that sampling/reconstruction preserves information

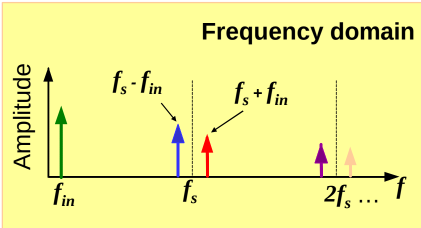


# Aliasing

- The frequencies $f_x$ and $Nf_s \pm f_x$, N integer, are indistinguishable in the discrete time domain
- Undesired frequency interaction and translation due to sampling is called aliasing
- If aliasing occurs, no signal processing operation downstream of the sampling process can recover the original continuous time signal!
- Let's look at this in the frequency domain...

## Sampling Sine Waves

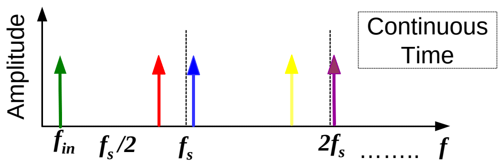




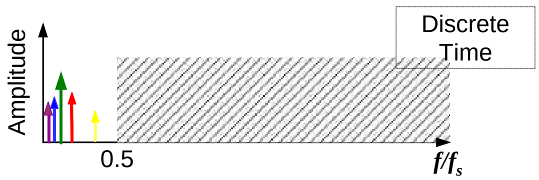

# Frequency Domain Interpretation

Signal scenario before sampling




Signal scenario after sampling → DT

→Signals @ $nf_S \pm f_{max\_signal}$ fold back into band of interest
→Aliasing

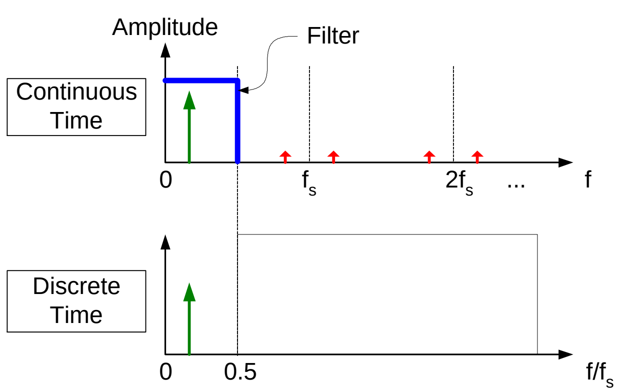


# Aliasing

- Multiple continuous time signals can produce identical series of sampled voltages
- The folding back of signals from $nf_S \pm f_{sig}$ down to $f_{fin}$ is called <u>aliasing</u>
  - Sampling theorem: $f_s > 2f_{max\_Signal}$
- If aliasing occurs, no signal processing operation downstream of the sampling process can recover the original continuous time signal



# Brick Wall Anti-Aliasing Filter



# How to Avoid Aliasing

- Must obey sampling theorem:

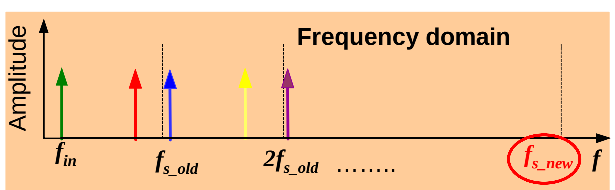
$$f_{max\_Signal} < f_s/2$$

- Two possibilities:
  1. Sample fast enough to cover all spectral components, including "parasitic" ones outside band of interest
  2. Limit $f_{max\_Signal}$ through filtering

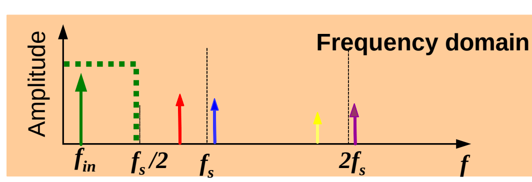


# How to Avoid Aliasing

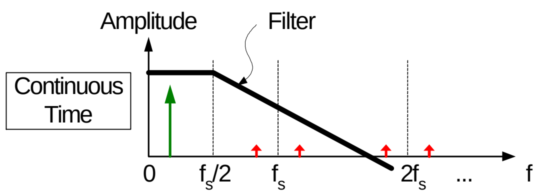
**1- Push sampling frequency to x2 of the highest freq.**
**→ Oversampled converters almost!**




**2- Pre-filter signal to eliminate signals above 1/2 sampling frequency- then sample**



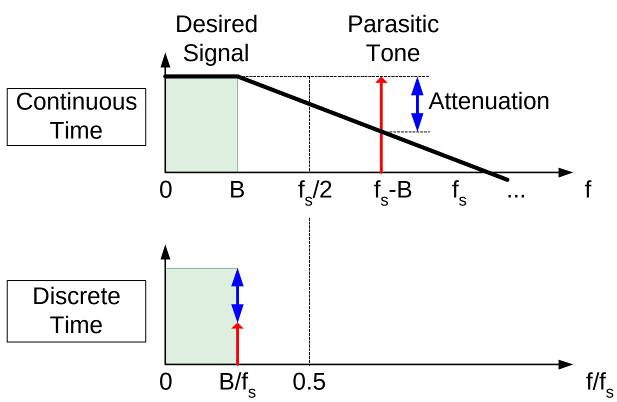
# Practical Anti-Aliasing Filter



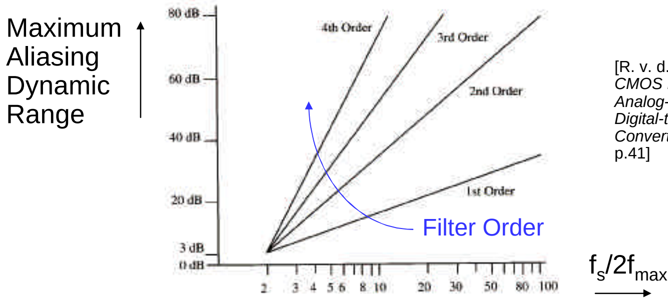

- Practical filter: Nonzero "transition band"
- In order to make this work, we need to sample faster than 2x the signal bandwidth
- "Oversampling"



# Practical Anti-Aliasing Filter



# How much Oversampling?




[R. v. d. Plassche, *CMOS Integrated Analog-to-Digital and Digital-to-Analog Converters*, 2nd ed., p.41]

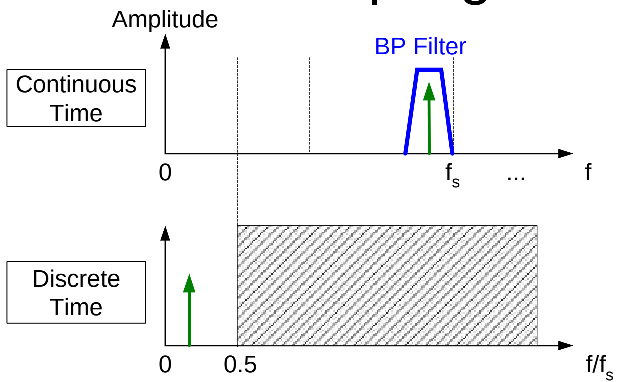
- Tradeoff: Sampling speed vs. filter order



# Data Converter Classification

- $f_s > 2f_{max}$ Nyquist Sampling
  - "Nyquist Converters"
  - Actually always slightly oversampled
- $f_s >> 2f_{max}$ Oversampling
  - "Oversampled Converters"
  - Anti alias filtering is often trivial
  - Oversampling is also used to reduce quantization noise, see later in the course...
- $f_s < 2f_{max}$ Undersampling (sub-sampling)

# Sub-Sampling

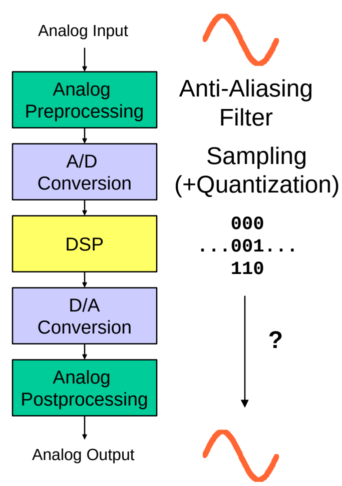



- Sampling at a rate less than Nyquist rate → aliasing
- For signals centered @ an intermediate frequency → Not destructive!
- Sub-sampling can be exploited to mix a narrowband RF or IF signal down to lower frequencies

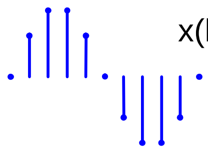


# Where Are We Now?

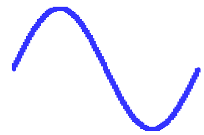
- We now know how to preserve signal information in CT→DT transition
- How do we go back from DT→ CT?



# Ideal Reconstruction




$\Rightarrow$



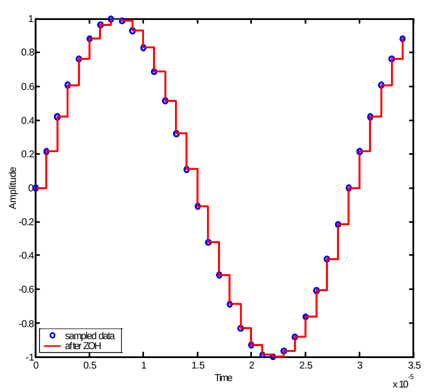

- The DSP books tell us:

$$x(t) = \sum_{k=-\infty}^{\infty} x(k) \cdot g(t - kT) \qquad g(t) = \frac{\sin(2pBt)}{2pBt}$$

- Unfortunately not all that practical...

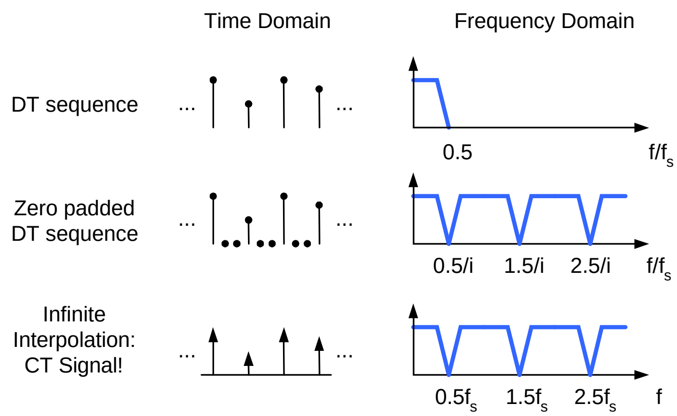


# Zero-Order Hold Reconstruction




- How about just creating a staircase, i.e. hold each discrete time value until new information becomes available
- What does this do the frequency content of the signal?
- Let's analyze this in two steps...

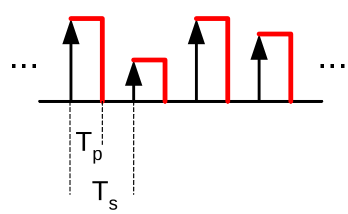
# 1) DT→ CT: Infinite Zero Padding





# 2) Effect of Hold Pulse

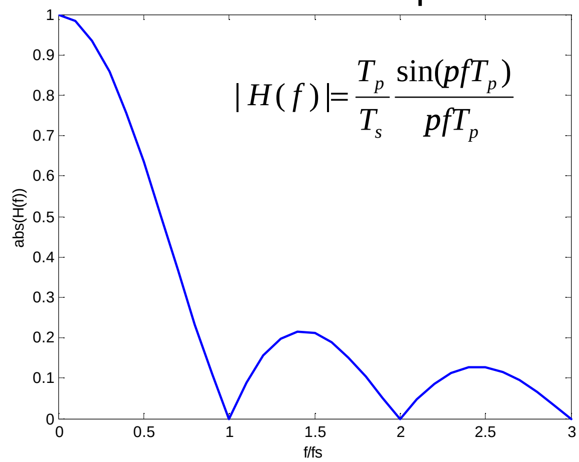



- Usubg the Fourier transform of a rectangular impulse we get:

$$|H(f)| = \frac{T_p}{T_s}\frac{\sin(pfT_p)}{pfT_p}$$

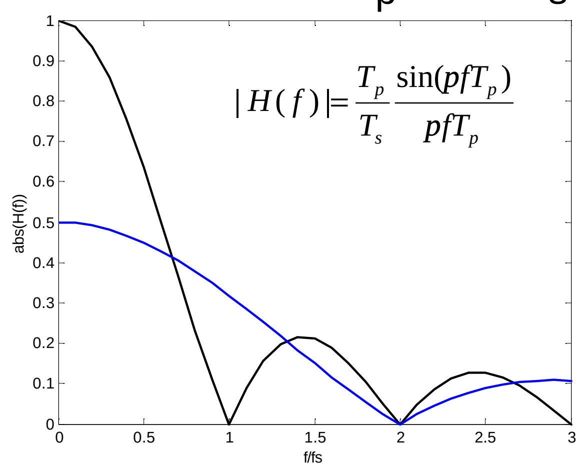
# Hold Pulse $T_p=T_s$



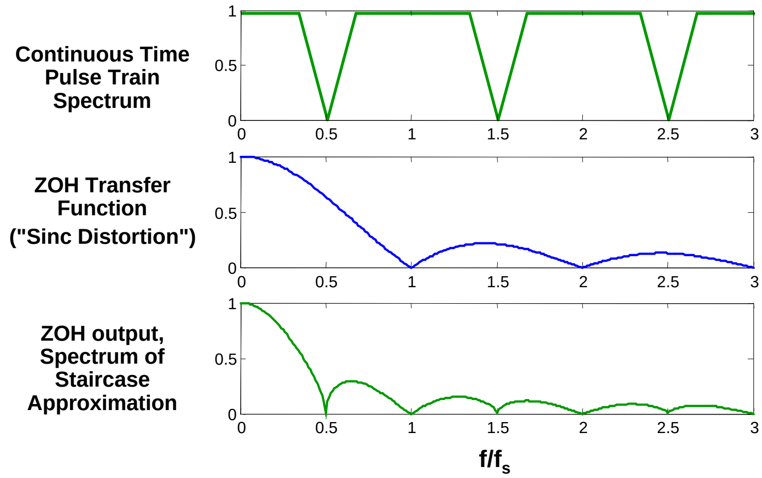





# Hold Pulse $T_p=0.5T_s$

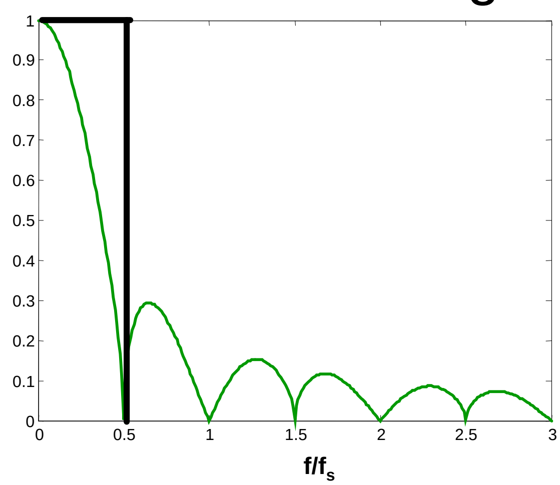


# ZOH Spectral Distortion






# Smoothing Filter




Again:

- A brick wall filter would be nice
- Oversampling helps to reduce filter order

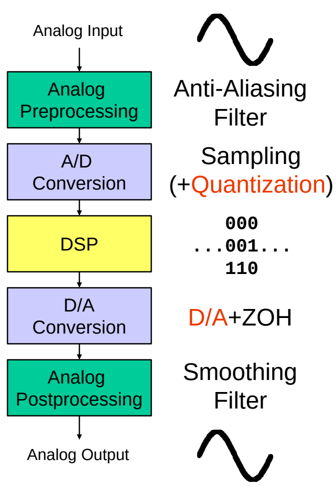
# Summary

- Sampling theorem $f_s > 2f_{max}$, usually dictates anti-aliasing filter
- If theorem is met, CT signal can be recovered from DT without loss of information
- ZOH and smoothing filter reconstruct CT from DT signal
- Oversampling helps reduce order & complexity of anti-aliasing & smoothing filters



# Next Topic

- Done with "Quantization in time"
- Next: Quantization in amplitude



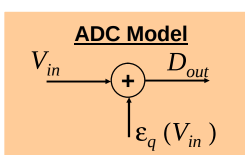
# Amplitude Quantization

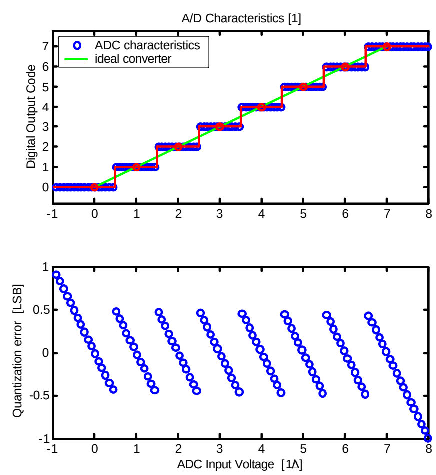
- Amplitude quantization
  → quantization “noise”
- Static ADC/DAC performance measures
  – Offset
  – Gain
  – INL
  – DNL



# Ideal ADC ("Quantizer")

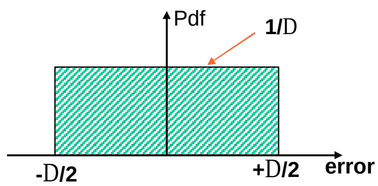
- Quantization step Δ (= 1 LSB)
- E.g. N = 3 Bits
- Full-scale input range:
  -0.5Δ … ($2^N$-0.5)Δ
- Quantization error:
  bounded by –Δ/2 … +Δ/2
  for inputs within full-scale range



# Quantization Error PDF

- Uniformly distributed from –Δ/2 … +Δ/2 provided that
  - Busy input
  - Amplitude is many LSBs
  - No overload
- Not Gaussian!




- Zero mean
- Variance

$$\overline{e^2} = \int_{-\Delta/2}^{+\Delta/2} \frac{e^2}{\Delta} de = \frac{\Delta^2}{12}$$

- Spectral density white if the joint pdf of the input at different sample times is smooth

Ref: W. R. Bennett, “Spectra of quantized signals,” Bell Syst. Tech. J., vol. 27, pp. 446-72, July 1988.

B. Widrow, “A study of rough amplitude quantization by means of Nyquist sampling theory,” IRE Trans. Circuit Theory, vol. CT-3, pp. 266-76, 1956.



# Signal-to-Quantization Noise Ratio

- If certain conditions are met (!) the quantization error can be viewed as being "random", and is often referred to as “noise”
- In this case, we can define a peak “signal-to-quantization noise ratio”, SQNR, for sinusoidal inputs:

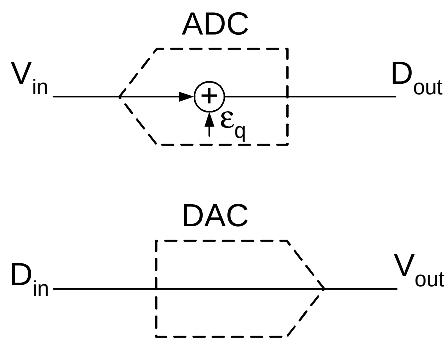
$$SQNR = \frac{\frac{1}{2}\left(\frac{2^N \Delta}{2}\right)^2}{\frac{\Delta^2}{12}} = 1.5\times 2^{2N}$$

$$= 6.02N + 1.76 \text{ dB}$$

| e.g. | N | SQNR |
|---|---|---|
| | 8 | 50 dB |
| | 12 | 74 dB |
| | 16 | 98 dB |
| | 20 | 122 dB |

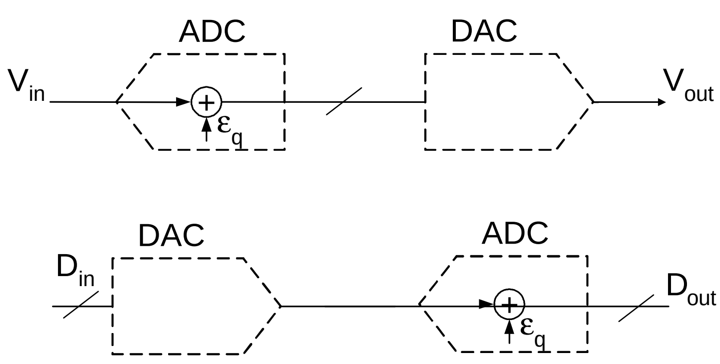
- Actual converters do not quite achieve this performance due to other errors, including
  - Electronic noise
  - Deviations from the ideal quantization levels

# Static, Ideal Macro Models






# Cascade of Data Converters



# Static Converter Errors

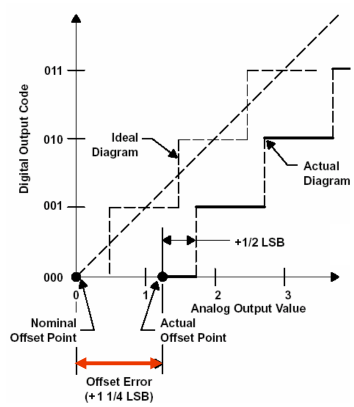
Deviations of characteristic from ideality

- Offset
- Gain error
- Differential Nonlinearity, DNL
- Integral Nonlinearity, INL

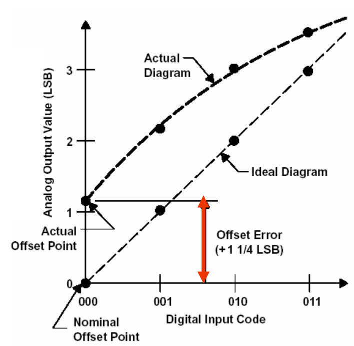


# Offset Errors

**ADC**

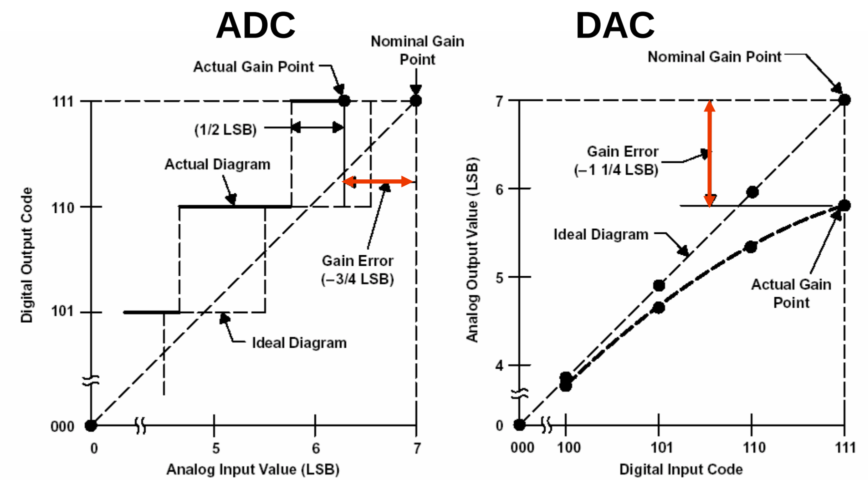



**DAC**




Ref: "Understanding Data Converters," Texas Instruments Application Report SLAA013, Mixed-Signal Products, 1995.

# Gain Errors

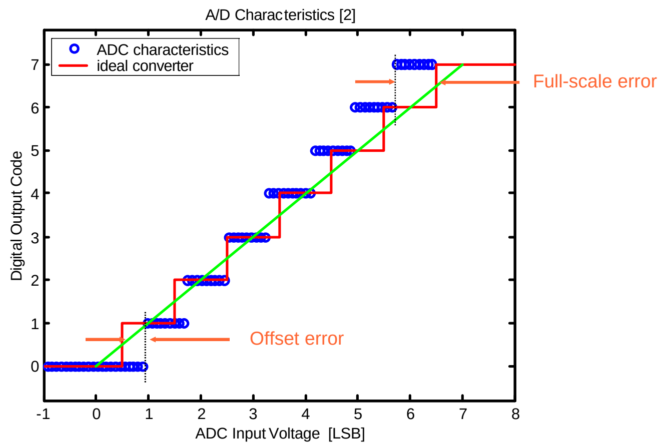





# Offset and Gain Errors

- Alternative Specification in % Full Scale = 100% * (LSB value)/ $2^N$
- Non-trivial to build a converter with extremely good gain/offset specs
- Typically gain/offset is most easily compensated by the digital pre/post-processor
- More interesting: Linearity →DNL, INL

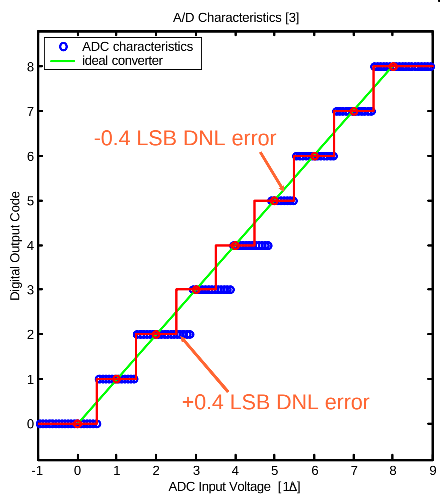
# Offset and Gain Error



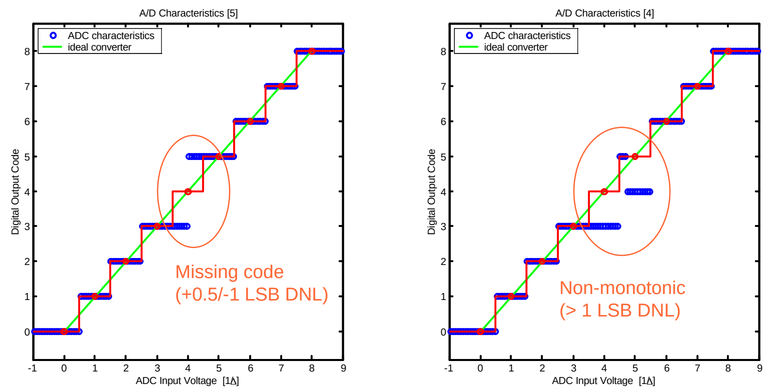



# ADC Differential Nonlinearity

DNL = deviation of code width from Δ (1LSB)

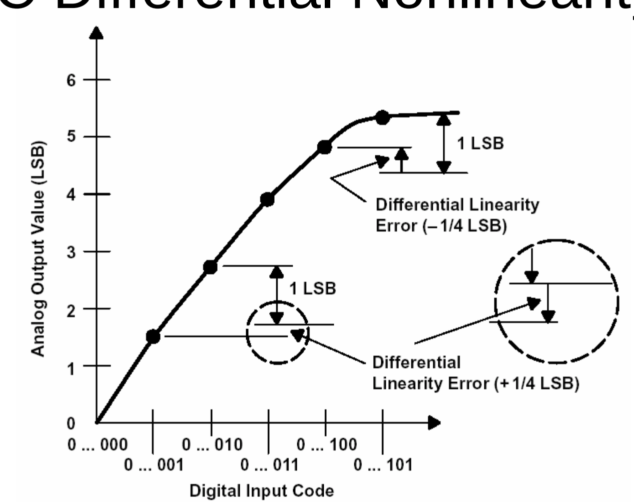


# ADC Differential Nonlinearity






# DAC Differential Nonlinearity



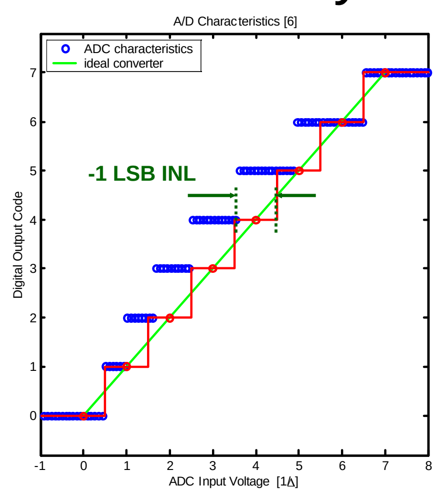
# Impact of DNL on Performance

- Same as a somewhat larger quantization error, consequently degrades SQNR
- How much – later in the course...
- People sometimes speak of "DNL noise", i.e. "additional quantization noise due to DNL"



# ADC Integral Nonlinearity

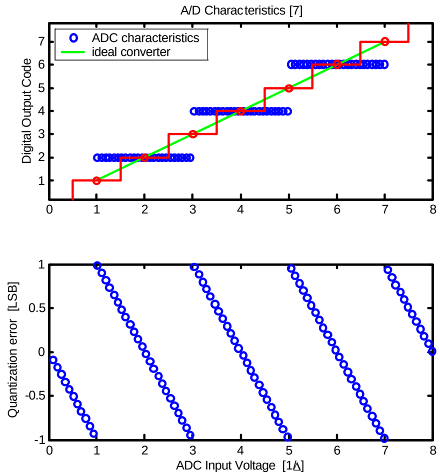
- INL = deviation of code transition from its ideal location
- A straight line through the endpoints is usually used as reference,
  i.e. offset and gain errors are ignored in INL calculation
- Note that INL errors can be much larger than DNL errors and vice-versa



# Large DNL Errors

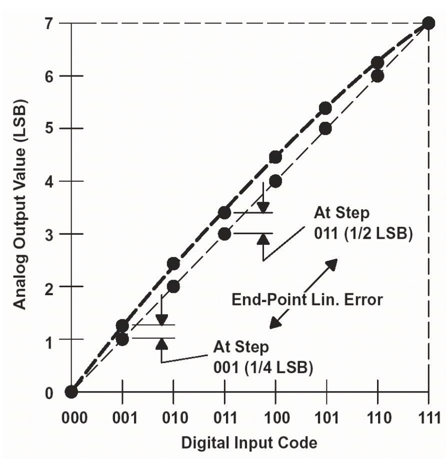
A converter with DNL larger than 1LSB could be equivalent an ideal ADC with 1 bit less resolution

At right:
alternating DNL –1/+1 LSB

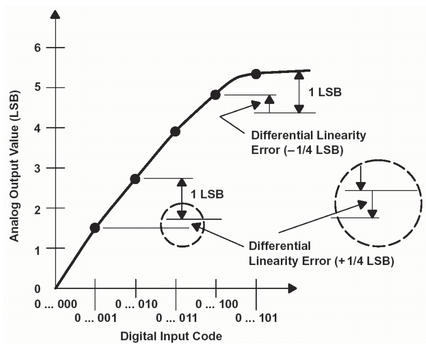


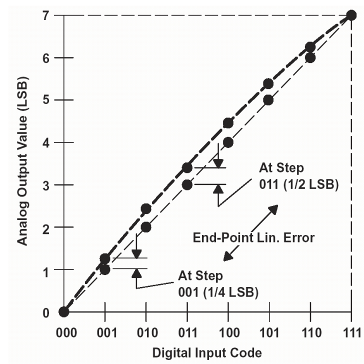



# DAC Integral Nonlinearity



# DAC DNL and INL

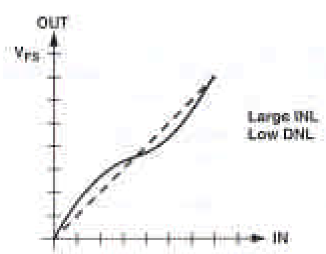


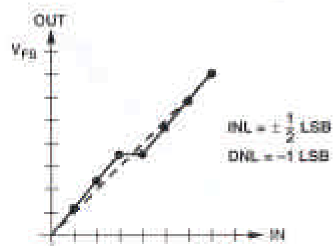

* Ref: "Understanding Data Converters," Texas Instruments Application Report SLAA013, Mixed-Signal Products, 1995.



# Example: INL & DNL




Large INL & Small DNL




Large DNL & Small INL

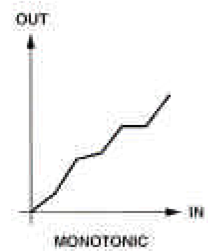
# Monotonicity

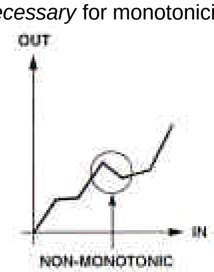
- Monotonicity guaranteed if

  $| INL | = 0.5$ LSB

  The best fit straight line is taken as the reference for determining the INL.
- This implies

  $| DNL | = 1$ LSB
- Note: these conditions are *sufficient* but not *necessary* for monotonicity




* Ref: R. J. van de Plassche, *Integrated Analog-to-Digital and Digital-to-Analog Converters*, Kluwer Academic Publishers, 2nd ed., 2003.