EE247
Lecture 8

- Finishing Continuous-time filters
  - Various Gm-C Filter implementations
  - Comparison of continuous-time filter topologies
- Switched-Capacitor Filters
  - “Analog” sampled-data filters:
    - Continuous amplitude
    - Quantized time
  - Applications:
    - First commercial product: Intel 2912 voice-band CODEC chip, 1979
    - Oversampled A/D and D/A converters
    - Stand-alone filters
      E.g. National Semiconductor LMF100

BiCMOS Gm-Cell

- MOSFET in triode mode:

$$I_d = \frac{mC_{ox}}{2}\frac{W}{L}\left[2\left(V_{gs} - V_{th}\right)V_{ds} - V_{ds}^2\right]$$

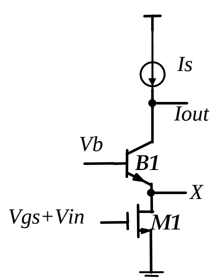
- Note that if Vds is kept constant:

$$g_m = \frac{\partial I_d}{\partial V_{gs}} = mC_{ox}\frac{W}{L}V_{ds}$$

- Linearity performance function of how constant *Vds* can be held
  - Gain @ Node X must be minimized

$$A_x = g_m^{M1} / g_m^{B1}$$

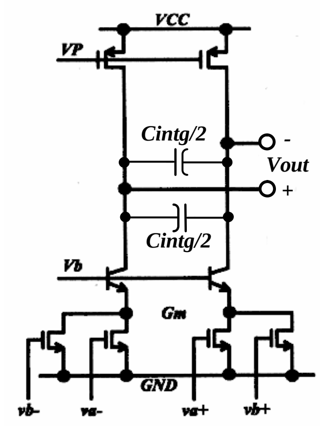
- Since for a given current, *gm* of BJT is larger compared to MOS preferable to have BJT
- Extra pole at node X



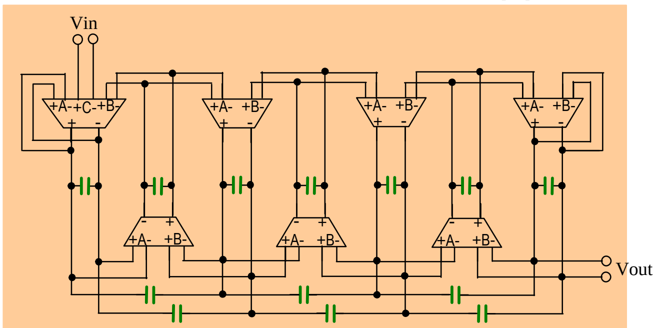
gm can be varied by changing *Vb* and thus *Vds*

BiCMOS Gm-C Integrator

- Differential- needs common-mode feedback ckt
- Freq. tuned by varying Vb
- Design tradeoffs:
  - Extra poles at the input device drain junctions
  - Input devices have to be small to minimize parasitic poles
  - Results in high input-referred offset voltage → could drive ckt into non-linear region
  - Small devices → high 1/f noise



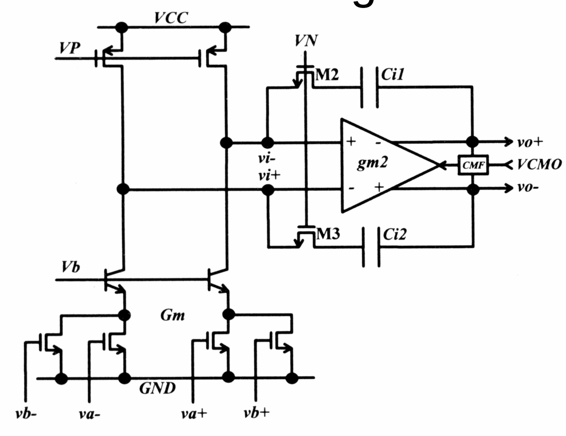
7th Order Elliptic Gm-C LPF For CDMA RX Baseband Application



- Gm-Cell in previous page used to build a 7th order elliptic filter for CDMA baseband applications (650kHz corner frequency)
- In-band dynamic range of <50dB achieved

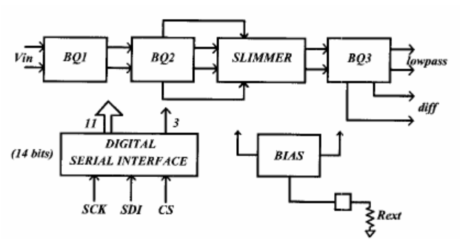
BiCMOS Gm-OTA-C Integrator

- Used to build filter for disk-drive applications
- Since high frequency of operation, time-constant sensitivity to parasitic caps significant.
  → Opamp used
- M2 & M3 added to compensate for phase lag (provides phase lead)



Ref: C. Laber and P.Gray, "A 20MHz 6th Order BiCMOS Parasitic Insensitive Continuous-time Filter & Second Order Equalizer Optimized for Disk Drive Read Channels," *IEEE Journal of Solid State Circuits*, Vol. 28, pp. 462-470, April 1993.

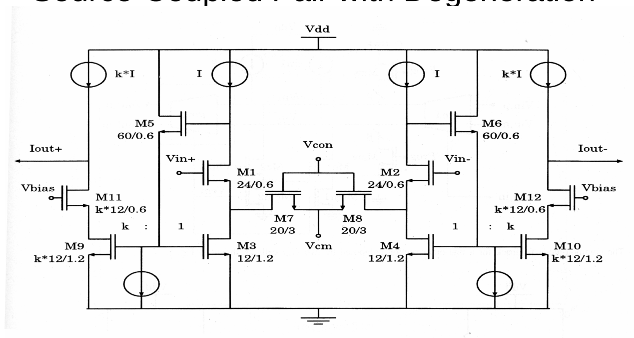
6th Order BiCMOS Continuous-time Filter & Second Order Equalizer for Disk Drive Read Channels



- Gm-C-opamp of the previous page used to build a 6th order filter for Disk Drive
- Filter consists of 3 biquads with max. Q of 2 each
- Performance in the order of 40dB SNDR achieved for variable corner frequency in discrete steps up to 20MHz

Ref: C. Laber and P.Gray, "A 20MHz 6th Order BiCMOS Parasitic Insensitive Continuous-time Filter & Second Order Equalizer Optimized for Disk Drive Read Channels," *IEEE Journal of Solid State Circuits*, Vol. 28, pp. 462-470, April 1993.

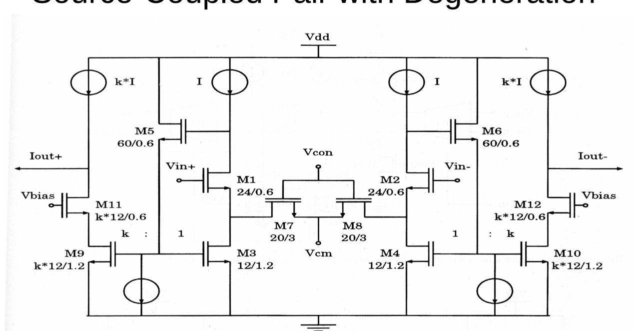
Gm-Cell
Source-Coupled Pair with Degeneration



– Gm-cell intended for low Q disk drive filter

Ref: I.Mehr and D.R.Welland, "A CMOS Continuous-Time Gm-C Filter for PRML Read Channel Applications at 150 Mb/s and Beyond", IEEE Journal of Solid-State Circuits, April 1997, Vol.32, No.4, pp. 499-513.

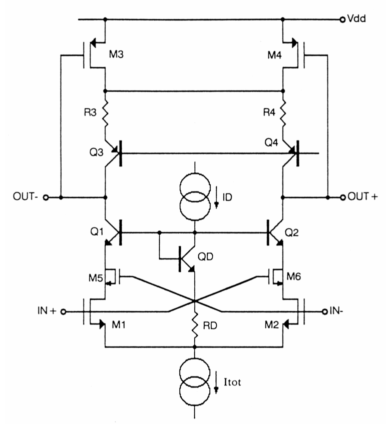
Gm-Cell
Source-Coupled Pair with Degeneration



– M7,8 operating in triode mode determine the gm of the cell
– Feedback provided by M5,6 maintains the gate-source voltage of M1,2 constant by forcing their current to be constant→ helps linearize *rds* of M7,8
– Current mirrored to the output via M9,10 with a factor of *k*
– Performance level of about 50dB SNDR at fcorner of 25MHz achieved

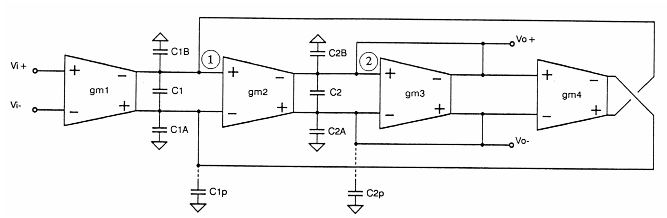
BiCMOS Gm-C Integrator

- Needs higher supply voltage compared to the previous design
- M5 & M6 configured as capacitors added to compensate for RHP zero due to Cgd of M1 & M2 (moves it to LHP) size of M5-6 is 1/3 of M1-2
- Current ID used to tune filter critical frequency
- M3, M4 operate in triode mode and added to provide CMFB



Ref: R. Alini, A. Baschirotto, and R. Castello, "Tunable BiCMOS Continuous-Time Filter for High-Frequency Applications," *IEEE Journal of Solid State Circuits,* Vol. 27, No. 12, pp. 1905-1915, Dec. 1992.

BiCMOS Gm-C Filter For Disk-Drive Application



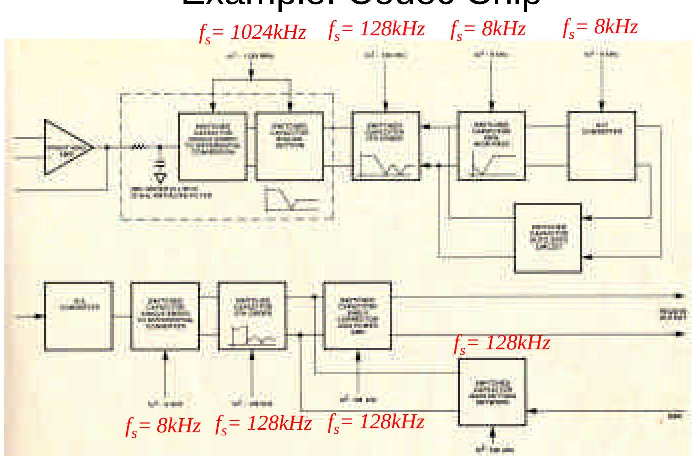
- Using the integrators shown in the previous page
- Biquad filter for disk drives
- $gm1=gm2=gm4=2gm3$
- $Q=2$
- Tunable from 8MHz to 32MHz

Ref: R. Alini, A. Baschirotto, and R. Castello, "Tunable BiCMOS Continuous-Time Filter for High-Frequency Applications," *IEEE Journal of Solid State Circuits,* Vol. 27, No. 12, pp. 1905-1915, Dec. 1992.

Summary
Continuous-Time Filters

- Opamp RC filters
  - Good linearity → High dynamic range (60-90dB)
  - Only discrete tuning possible
  - Medium usable signal bandwidth (<10MHz)
- Opamp MOSFET-C
  - Linearity compromised (typical dynamic range 40-60dB)
  - Continuous tuning possible
  - Low usable signal bandwidth (<5MHz)
- Opamp MOSFET-RC
  - Improved linearity compared to Opamp MOSFET-C (D.R. 50-90dB)
  - Continuous tuning possible
  - Low usable signal bandwidth (<5MHz)
- Gm-C
  - Highest frequency performance (at least an order of magnitude higher compared to the rest <100MHz)
  - Dynamic range not as high as Opamp RC but better than Opamp MOSFET-C (40-70dB)

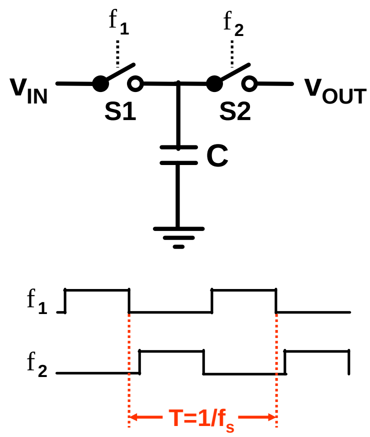
Switched-Capacitor Filters
Example: Codec Chip



Ref: D. Senderowicz et. al, “A Family of Differential NMOS Analog Circuits for PCM Codec Filter Chip,” *IEEE Journal of Solid-State Circuits*, Vol.-SC-17, No. 6, pp.1014-1023, Dec. 1982.

Switched-Capacitor Resistor

- Capacitor C is the "switched capacitor"
- Non-overlapping clocks ϕ_1 and ϕ_2 control switches S1 and S2, respectively
- v_{IN} is sampled at the falling edge of ϕ_1
  - Sampling frequency f_S
- Next, ϕ_2 rises and the voltage across C is transferred to v_{OUT}
- Why is this a resistor?



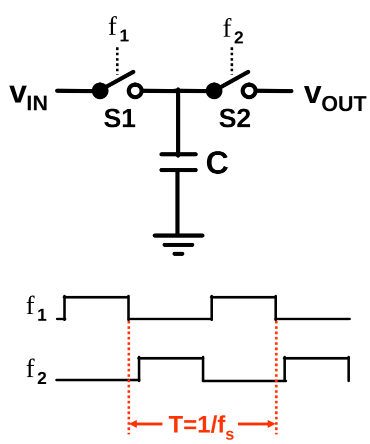
Switched-Capacitor Resistors

- Charge transferred from v_{IN} to v_{OUT} during each clock cycle is:

  $$Q = C(v_{IN} - v_{OUT})$$

- Average current flowing from v_{IN} to v_{OUT} is:

  $$i = Q/t = Q x f_S$$
  $$i = f_S C(v_{IN} - v_{OUT})$$



Switched-Capacitor Resistors

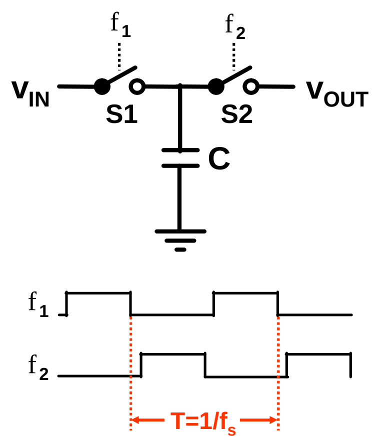
$$i = f_S C(v_{IN} - v_{OUT})$$

With the current through the switched capacitor resistor proportional to the voltage across it, the equivalent “switched capacitor resistance” is:

$$R_{eq} = \frac{1}{f_s C}$$

Example

$$f = 1MHz, C = 1pF$$
$$\rightarrow R_{eq} = 1Mega\Omega$$

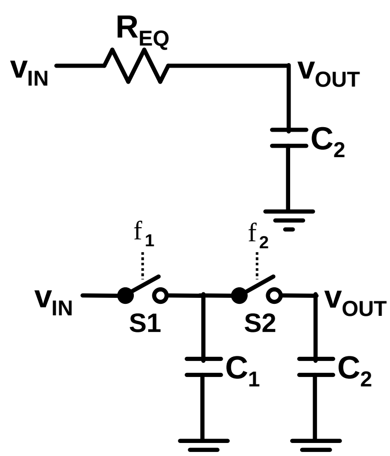


Switched-Capacitor Filter

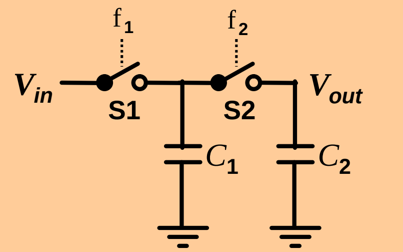
- Let’s build a “SC” filter …
- We’ll start with a simple RC LPF
- Replace the physical resistor by an equivalent SC resistor
- 3-dB bandwidth:

$$w_{-3dB} = \frac{1}{R_{eq}C_2} = f_s \times \frac{C_1}{C_2}$$

$$f_{-3dB} = \frac{1}{2p} f_s \times \frac{C_1}{C_2}$$

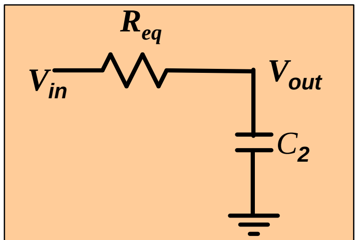


Switched-Capacitor Filter Advantage versus Continuous-Time Filters



$$f_{-3dB} = \frac{1}{2p} f_s \times \frac{C_1}{C_2}$$

- Corner freq. proportional to:
  System clock (accurate to few ppm)
  C ratio accurate → < 0.1%



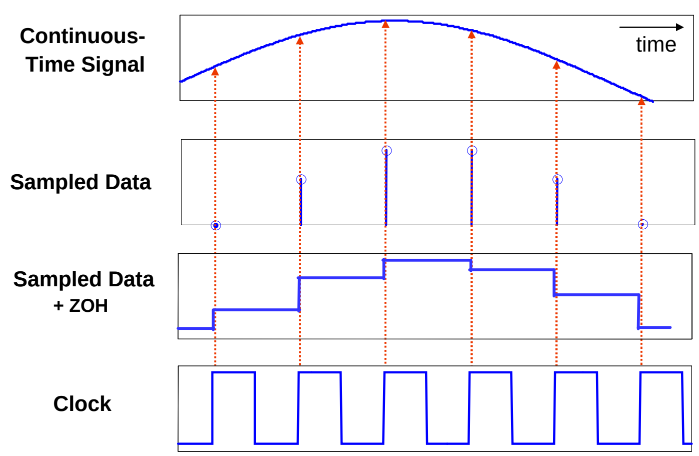
$$f_{-3dB} = \frac{1}{2p} \times \frac{1}{R_{eq} C_2}$$

- Corner freq. proportional to:
  Absolute value of Rs & Cs
  Poor accuracy → 20 to 50%

Main advantage of SC filter inherent corner frequency accuracy

Typical Sampling Process

Continuous-Time(CT) ⇒ Sampled Data (SD)

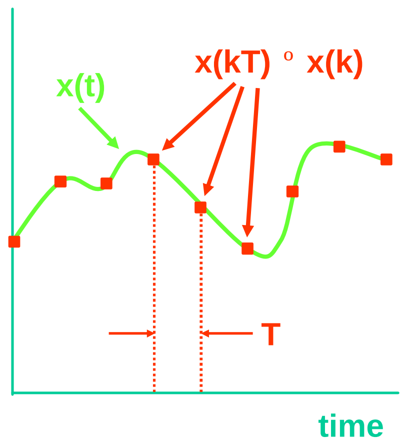


Uniform Sampling

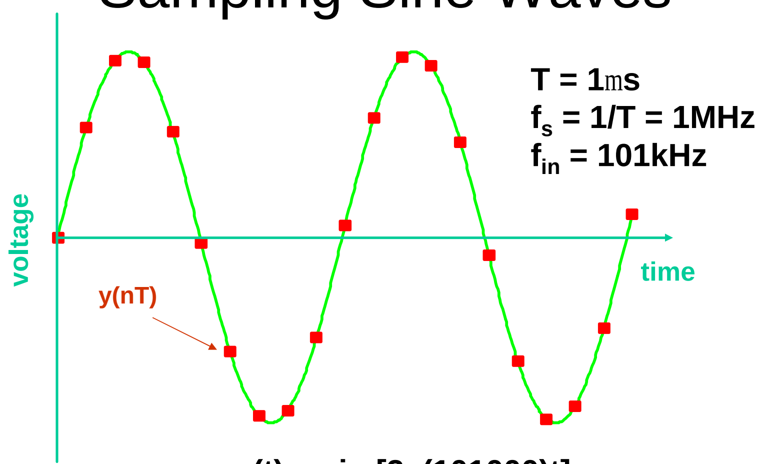
Nomenclature:

Continuous time signal $x(t)$
Sampling interval T
Sampling frequency $f_s = 1/T$
Sampled signal $x(kT) = x(k)$

- Problem: Multiple continuous time signals can yield exactly the same discrete time signal
- Let's look at samples taken at 1μs intervals of several sinusoidal waveforms …

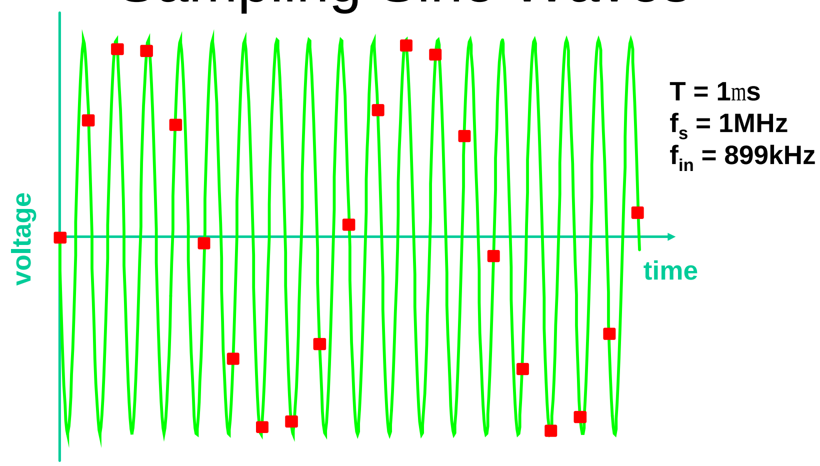


Sampling Sine Waves



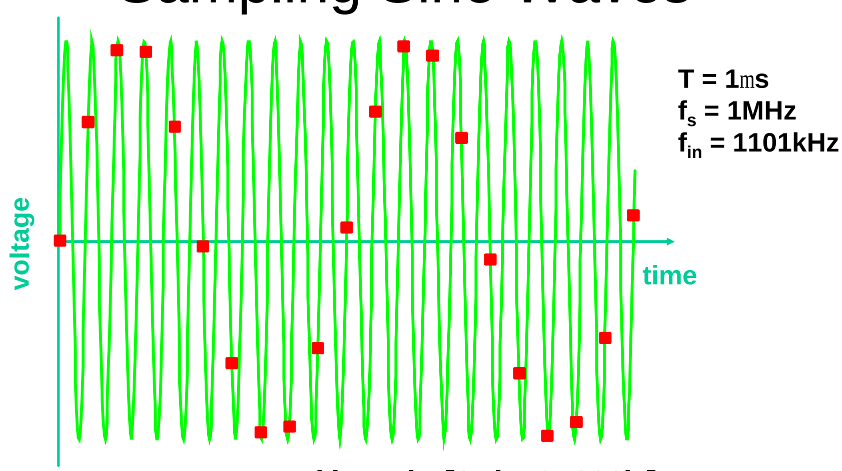
v(t) = sin [2p(101000)t]

Sampling Sine Waves



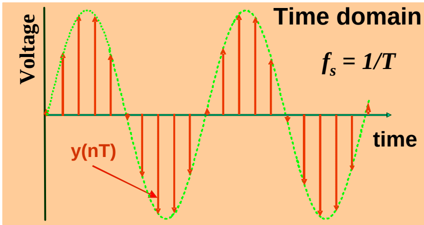
$$v(t) = -\sin[2\pi(899000)t]$$

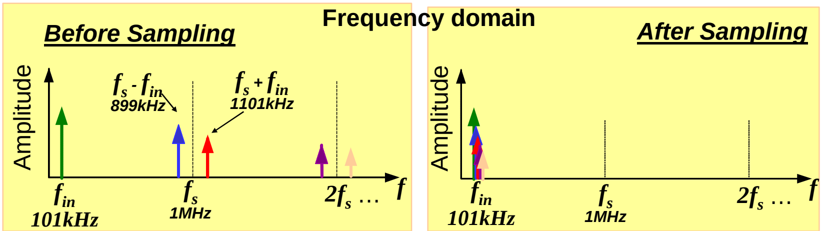
Sampling Sine Waves



$$v(t) = \sin[2\pi(1101000)t]$$

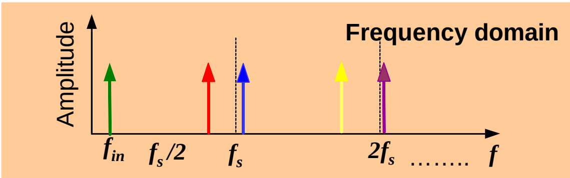
Sampling Sine Waves



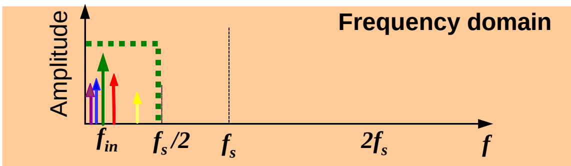


Frequency Domain Interpretation

Signal scenario before sampling



Signal scenario after sampling & filtering
→Signals @ $nf_S \pm f_{max\_signal}$ fold back into band of interest
→Aliasing



Aliasing

- Multiple continuous time signals can produce identical series of sampled voltages
- The folding back of signals from $nf_S \pm f_{sig}$ down to f_{fin} is called aliasing
  - Sampling theorem: $f_s > 2f_{max\_Signal}$
- If aliasing occurs, no signal processing operation downstream of the sampling process can recover the original continuous time signal

How to Avoid Aliasing

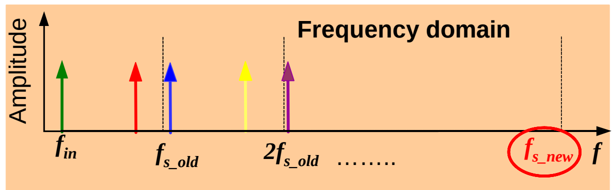
- Must obey sampling theorem:

$$f_{max\_Signal} < f_s/2$$

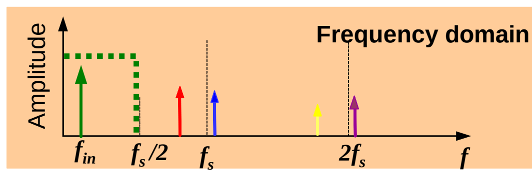
- Two possibilities:
  1. Sample fast enough to cover all spectral components, including "parasitic" ones outside band of interest
  2. Limit f_{max_Signal} through filtering

How to Avoid Aliasing

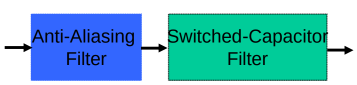
1- Push sampling frequency to x2 of the highest freq.
→ In most cases not practical

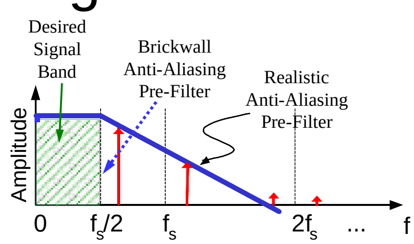


2- Pre-filter signal to eliminate signals above 1/2 sampling frequency- then sample



Anti-Aliasing Filter

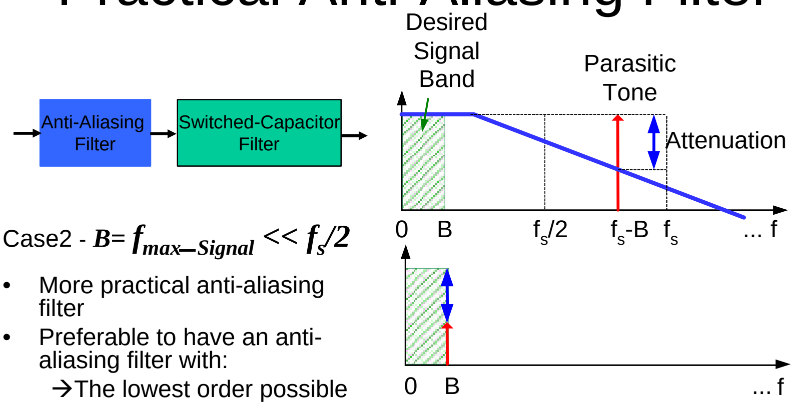




Case1- $B = f_{max-Signal} = f_s/2$

- Non-practical since an extremely high order anti-aliasing filter (close to an ideal brickwall filter) is required
- Practical anti-aliasing filter →Nonzero filter "transition band"
- In order to make this work, we need to sample much faster than 2x the signal bandwidth
  →"Oversampling"

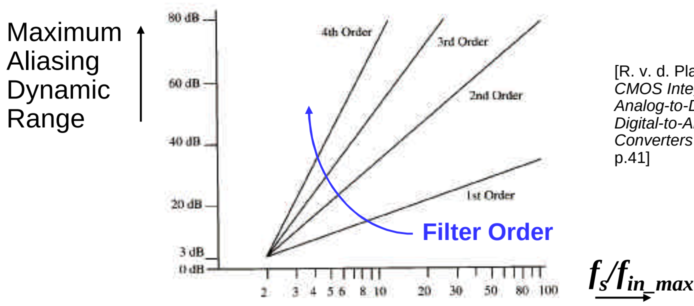
Practical Anti-Aliasing Filter



Case2 - $B = f_{max\text{—}Signal} << f_s/2$

- More practical anti-aliasing filter
- Preferable to have an anti-aliasing filter with:
  - →The lowest order possible
  - →No frequency tuning required (if frequency tuning is required then why use SC filter, just use the prefilter!?)

Tradeoff
Oversampling Ratio versus Anti-Aliasing Filter Order

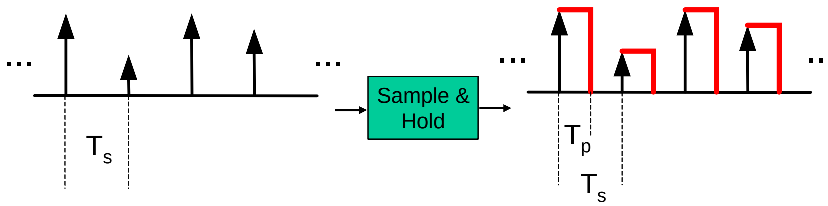


[R. v. d. Plassche, *CMOS Integrated Analog-to-Digital and Digital-to-Analog Converters*, 2nd ed., p.41]

* *Assumption→ anti-aliasing filter is Butterworth type*

→Tradeoff: Sampling speed vs. filter order

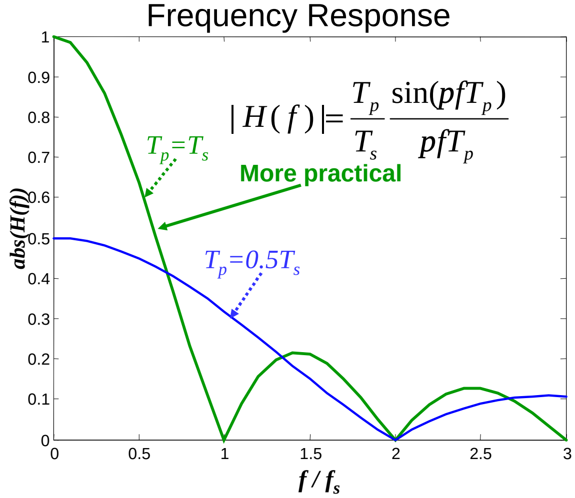
Effect of Sample & Hold



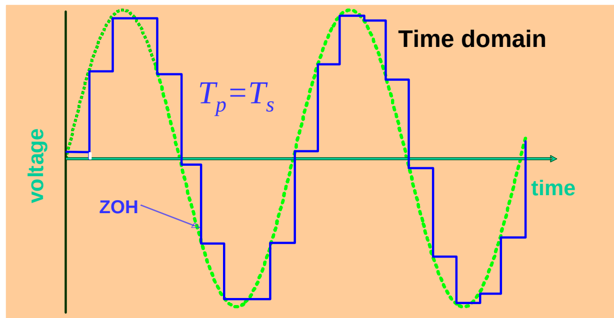
•Using the Fourier transform of a rectangular impulse:

$$|H(f)| = \frac{T_p}{T_s} \frac{\sin(pfT_p)}{pfT_p}$$

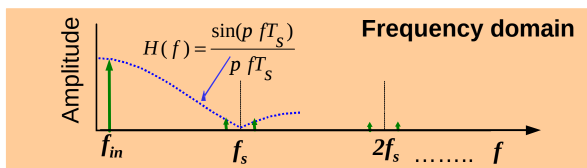
Effect of Sample & Hold on Frequency Response



Sample & Hold Effect
(Reconstruction of Analog Signals)



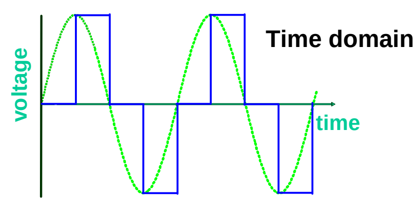
Magnitude droop due to *sinx/x* effect



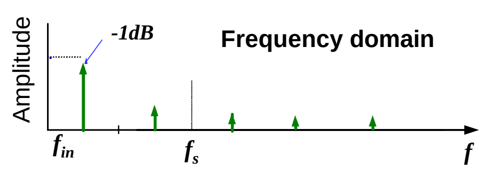
Sample & Hold Effect
(Reconstruction of Analog Signals)

Magnitude droop due to *sinx/x* effect:

Case 1) $f_{sig}=f_s/4$



Droop= *-1dB*

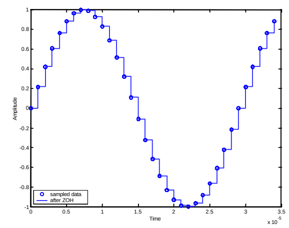


Sample & Hold Effect
(Reconstruction of Analog Signals)

Magnitude droop due to ***sinx/x*** effect:

Case 2)
$f_{sig}=f_s/32$

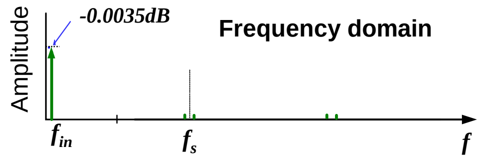
Time domain



Droop= *-0.0035dB*

→ ***High oversampling ratio desirable***

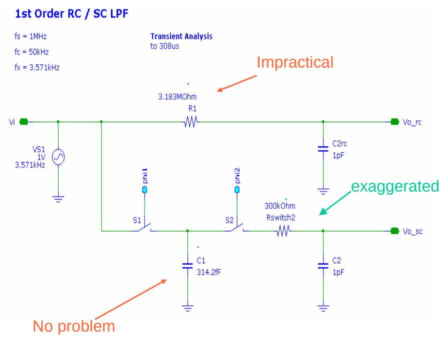
Frequency domain

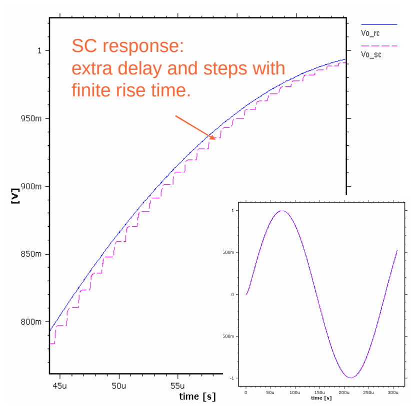


Summary

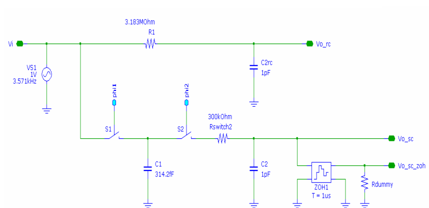
- Sampling theorem → $f_s > 2f_{max\_Signal}$
- Signals at frequencies $nf_S \pm f_{sig}$ fold back down to desired signal band, f_{sig}
  - → This is called aliasing & usually dictates use of anti-aliasing pre-filters
- Oversampling helps reduce order of anti-aliasing filter
- S/H function shapes the frequency response with *sinx/x*
  - → Need to pay attention to droop in passband due to *sinx/x*
- If the above requirements is not met, CT signal can NOT be recovered from SD or DT without loss of information

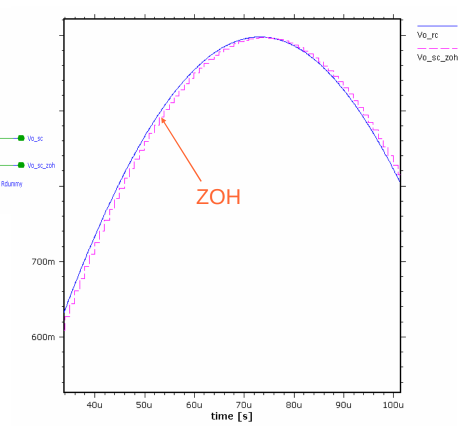
1^{st} Order Filter Transient Analysis





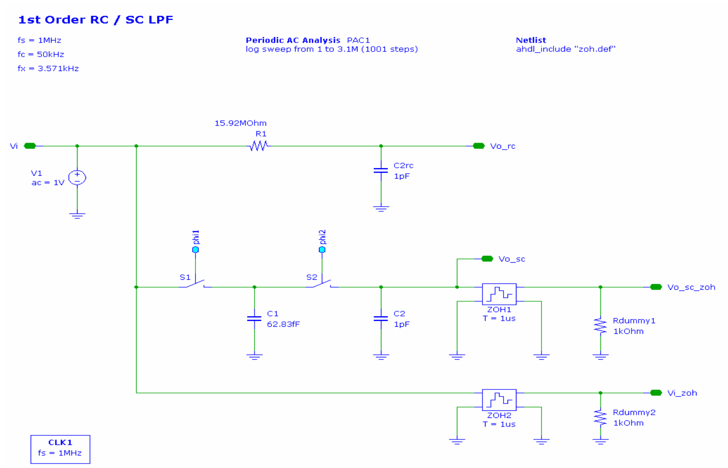
1^{st} Order Filter Transient Analysis



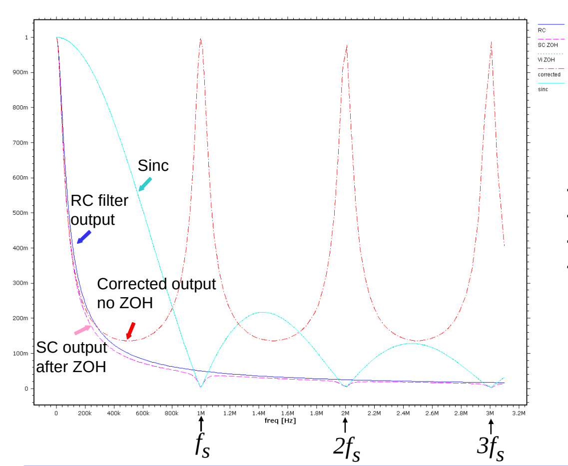


- ZOH: pick signal after settling (usually at end of clock phase)
- Adds delay and sin(x)/x distortion
- When in doubt, use a ZOH in periodic ac simulations

Periodic AC Analysis



Magnitude Response



. RC filter output
. SC output after ZOH
. Input after ZOH
. Corrected output
- (2) over (3)
- Repeats filter shape around nf_s
- Identical to RC for $f << f_s/2$

Periodic AC Analysis

- SPICE frequency analysis
  - ac — linear, time-invariant circuits
  - pac — linear, time-variant circuits
- SpectreRF statements

```
V1 ( Vi 0 ) vsource type=dc dc=0 mag=1 pacmag=1
PSS1 pss period=1u errpreset=conservative
PAC1 pac start=1 stop=1M lin=1001
```

- Output
  - Divide results by sinc(f/f_s) to correct for ZOH distortion

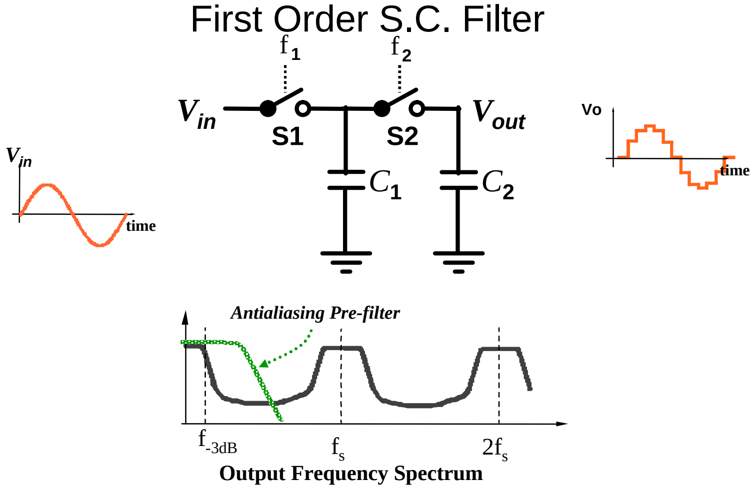
Spectre Circuit File

```
rc_pac
simulator lang=spectre
ahdl_include "zoh.def"

S1 ( Vi c1 phi1 0 ) relay ropen=100G rclosed=1 vt1=-500m vt2=500m
S2 ( c1 Vo_sc phi2 0 ) relay ropen=100G rclosed=1 vt1=-500m vt2=500m
C1 ( c1 0 ) capacitor c=314.159f
C2 ( Vo_sc 0 ) capacitor c=1p
R1 ( Vi Vo_rc ) resistor r=3.1831M
C2rc ( Vo_rc 0 ) capacitor c=1p
CLK1_Vphi1 ( phi1 0 ) vsource type=pulse val0=-1 val1=1 period=1u
                          width=450n delay=50n rise=10n fall=10n
CLK1_Vphi2 ( phi2 0 ) vsource type=pulse val0=-1 val1=1 period=1u
                          width=450n delay=550n rise=10n fall=10n
V1 ( Vi 0 ) vsource type=dc dc=0 mag=1 pacmag=1
PSS1 pss period=1u errpreset=conservative
PAC1 pac start=1 stop=3.1M log=1001
ZOH1 ( Vo_sc_zoh 0 Vo_sc 0 ) zoh period=1u delay=500n aperture=1n tc=10p
ZOH2 ( Vi_zoh 0 Vi 0 ) zoh period=1u delay=0 aperture=1n tc=10p
```

ZOH Circuit File

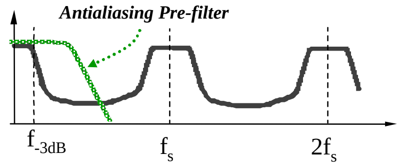
```
// Copy from the SpectreRF Primer

module zoh (Pout, Nout, Pin, Nin) (period, delay,
     aperture, tc)

node [V,I] Pin, Nin, Pout, Nout;
parameter real period=1 from (0:inf);
parameter real delay=0 from [0:inf);
parameter real aperture=1/100 from (0:inf);
parameter real tc=1/500 from (0:inf);
{
integer n; real start, stop;
node [V,I] hold;
  analog {
    // determine the point when aperture begins
    n = ($time() - delay + aperture) / period +
     0.5;
    start = n*period + delay - aperture;
    $break_point(start);

    // determine the time when aperture ends
    n = ($time() - delay) / period + 0.5;
    stop = n*period + delay;
    $break_point(stop);

    // Implement switch with effective series
    // resistence of 1 Ohm
    if ( ($time() > start) && ($time() <= stop))
      I(hold) <- V(hold) - V(Pin, Nin);
    else
      I(hold) <- 1.0e-12 * (V(hold) - V(Pin, Nin));

    // Implement capacitor with an effective
    // capacitance of tc
    I(hold) <- tc * dot(V(hold));

    // Buffer output
    V(Pout, Nout) <- V(hold);

    // Control time step tightly during
    // aperture and loosely otherwise
    if (($time() >= start) && ($time() <= stop))
      $bound_step(tc);
    else
      $bound_step(period/5);
  }
}
```


First Order S.C. Filter



Switched-Capacitor Filters → problem with aliasing

Example : Anti-Aliasing Filter



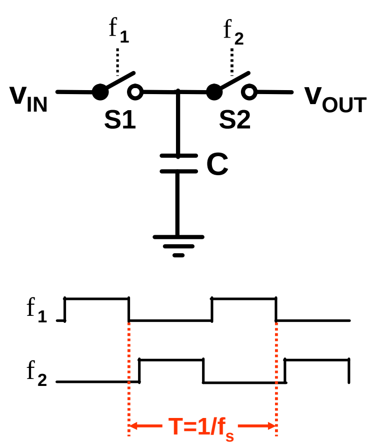
- Voice-band SC filter f_{-3dB} =4kHz & f_s =256kHz
- Anti-aliasing filter requirements:
  - Need 40dB attenuation at clock freq.
  - Incur no phase-error from 0 to 4kHz
  - Gain error 0 to 4kHz < 0.05dB
  - Allow +-30% variation for anti-aliasing corner frequency (no tuning)

    → 2-pole Butterworth LPF with nominal corner freq. of 17kHz & no tuning (12kHz to 22kHz corner frequency)

Ease of anti-aliasing → high ratio for $f_{sampling} / f_{-3dB}$

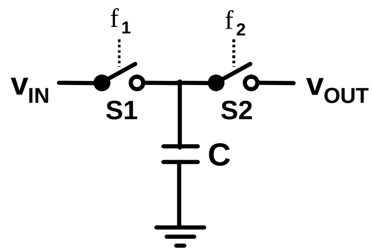
Switched-Capacitor Noise

- Resistance of switch S1 produces a noise voltage on C with variance kT/C
- The corresponding noise charge is $Q^2=C^2V^2=kTC$
- This charge is sampled when S_1 opens

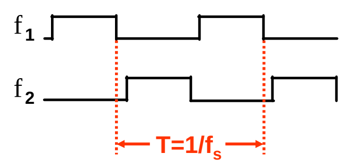


Switched-Capacitor Noise

- Resistance of switch S2 contributes to an uncorrelated noise charge on C at the end of ϕ_2



- Mean-squared noise charge transferred from v_{IN} to v_{OUT} each sample period is $Q^2=2kTC$



Switched-Capacitor Noise

- The mean-squared noise current due to S1 and S2's kT/C noise is :

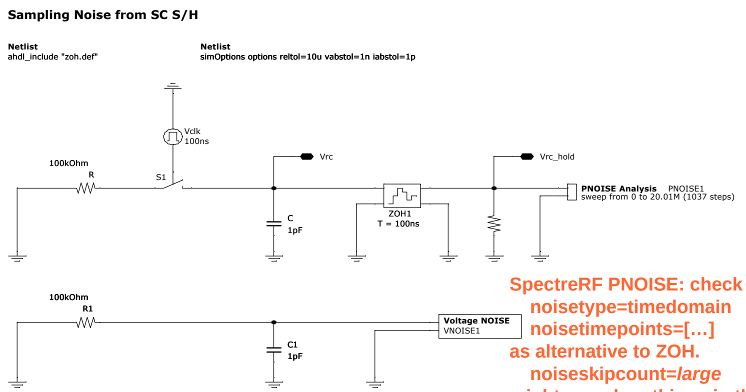
$$\overline{i^2} = \left(Qf_s\right)^2 = 2k_B T C f_s^2$$

- This noise is approximately white and distributed between 0 and $f_s/2$ (noise spectra → single sided by convention)
The spectral density of the noise is:

$$\frac{\overline{i^2}}{\Delta f} = \frac{2k_B T C f_s^2}{f_s/2} = 4k_B T C f_s = \frac{4k_B T}{R_{EQ}} \qquad using \qquad R_{EQ} = \frac{1}{f_s C}$$

→ **S.C. resistor noise equals a physical resistor noise with same value!**

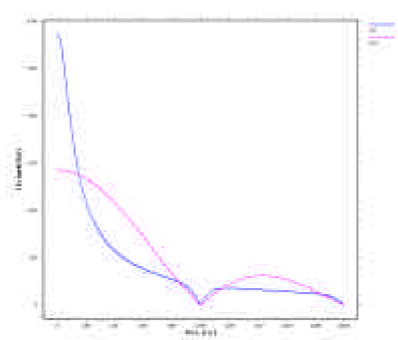
Periodic Noise Analysis



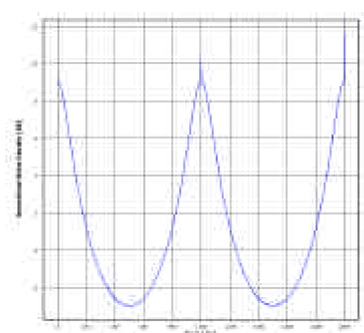
SpectreRF PNOISE: check
noisetype=timedomain
noisetimepoints=[…]
as alternative to ZOH.
noiseskipcount=*large*
might speed up things in this case.

```
PSS pss period=100n maxacfreq=1.5G errpreset=conservative
PNOISE ( Vrc_hold 0 ) pnoise start=0 stop=20M lin=500 maxsideband=10
```


Sampled Noise Spectrum

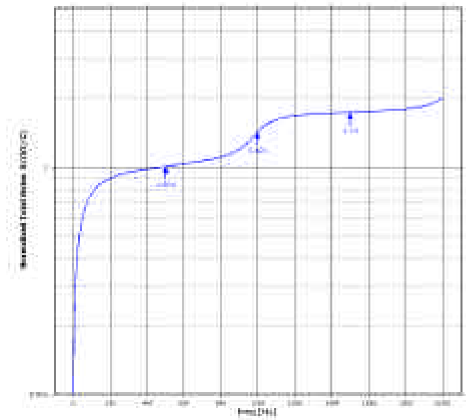


Density of sampled noise with sinc distortion.



Normalized density of sampled noise, corrected for sinc distortion.

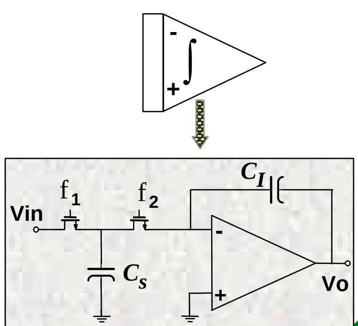
Total Noise

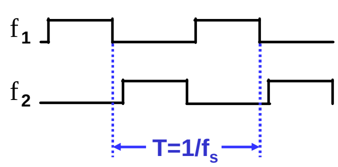


Sampled noise in
0 … $f_s/2$: 62.2μV rms

(expect 64μV for 1pF)

Switched-Capacitor Integrator



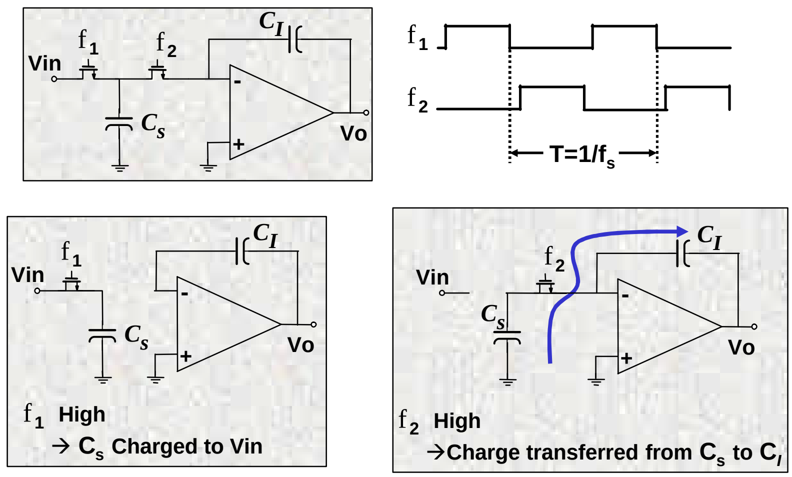


for $f_{signal} \ll f_{sampling}$ $V_0 = \frac{f_s \times C_s}{C_I} \int V_{in}\, dt$

$w_0 = f_s \times \frac{C_s}{C_I}$

Main advantage: No tuning needed
→ critical frequency function of ratio of caps & clock freq.

SC Integrator



Continuous-Time versus Discrete Time Design Flow

<u>Continuous-Time</u>

- Write differential equation
- Laplace transform (F(s))
- Let s=jω → F(jω)
- Plot |F(jω)|, phase(F(jω))

<u>Discrete-Time</u>

- Write difference equation, → relates output sequence to input sequence

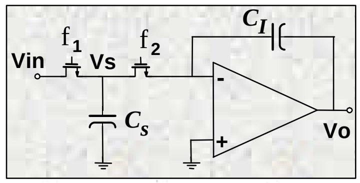
$$V_o(nT_s) = V_i\ [(n-1)T_s] - \ldots\ldots\ldots.$$

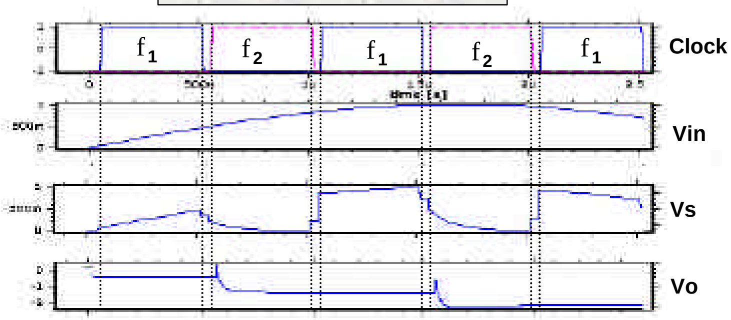
- Use delay operator Z^{-1} to transform the recursive realization to algebraic equation in Z domain

$$V_o\ (Z) = Z^{-1} V_i\ (Z) \ldots\ldots.$$

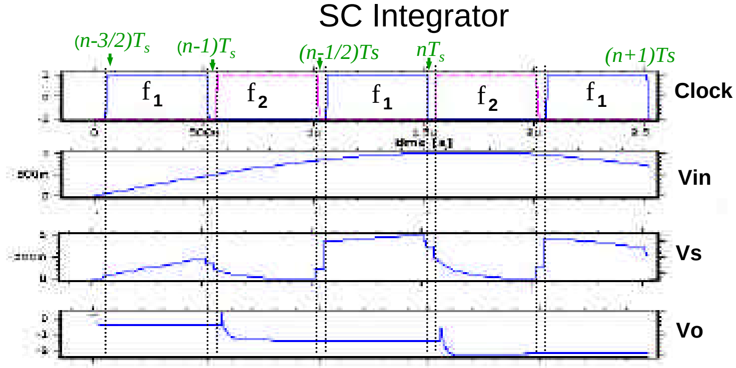
- Set $Z = e^{jwT}$
- Plot mag./phase versus frequency

SC Integrator





SC Integrator



$\Phi_1 \rightarrow$ $Q_s[(n-1)T_s] = C_s V_i[(n-1)T_s]$, $Q_I[(n-1)T_s] = Q_I[(n-3/2)T_s]$

$\Phi_2 \rightarrow$ $Q_s[(n-1/2)T_s] = 0$, $Q_I[(n-1/2)T_s] = Q_I[(n-3/2)T_s] + Q_s[(n-1)T_s]$

$\Phi_1 \rightarrow$ $Q_s[nT_s] = C_s V_i[nT_s]$, $Q_I[nT_s] = Q_I[(n-1)T_s] + Q_s[(n-1)T_s]$

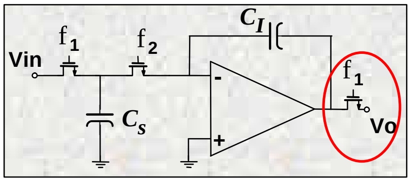
Since $V_o = -Q_I/C_I$ & $V_i = Q_s/C_s \rightarrow C_I V_o(nT_s) = C_I V_o[(n-1)T_s] - C_s V_i[(n-1)T_s]$

Discrete Time Design Flow

- Transforming the recursive realization to algebraic equation in Z domain:
  - Use Delay operator Z :

$$nT_S \ldots\ldots\ldots\ldots\ldots\ldots \rightarrow 1$$
$$\left[(n-1)T_S\right] \ldots\ldots\ldots\ldots \rightarrow Z^{-1}$$
$$\left[(n-1/2)T_S\right] \ldots\ldots\ldots \rightarrow Z^{-1/2}$$
$$\left[(n+1)T_S\right] \ldots\ldots\ldots\ldots \rightarrow Z^{+1}$$
$$\left[(n+1/2)T_S\right] \ldots\ldots\ldots \rightarrow Z^{+1/2}$$

SC Integrator



$$-C_I V_o(nT_s) = -C_I V_o\left[(n-1)T_s\right] + C_s V_{in}\left[(n-1)T_s\right]$$
$$V_o(nT_s) = V_o\left[(n-1)T_s\right] - \frac{C_s}{C_I} V_{in}\left[(n-1)T_s\right]$$
$$V_o(Z) = Z^{-1} V_o(Z) - Z^{-1}\frac{C_s}{C_I} V_{in}(Z)$$
$$\frac{V_o}{V_{in}}(Z) = -\frac{C_s}{C_I} \times \boxed{\frac{Z^{-1}}{1-Z^{-1}}}$$

DDI (Direct-Transform Discrete Integrator)

z-Plane

- Consider variable $Z=e^{sT}$ for any s in left-half-plane (LHP):

$S= - a+jb$

$Z= e^{-aT} . e^{jbT} = e^{-aT} (cosbT + jsin\ bT)$

$|Z|= e^{-aT}$, $angle(Z)= bT$

→ For values of S in LHP $|Z|<1$

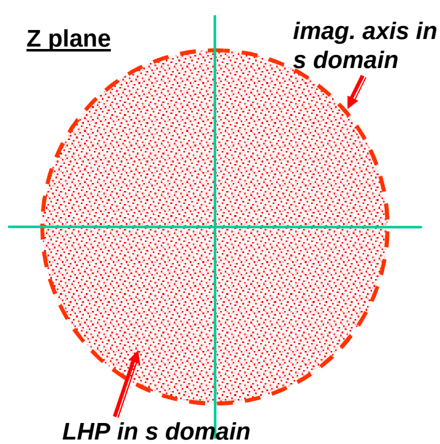
→ For $a=0$ (imag. axis in s-plane) $|Z|=1$ (unit circle)

if $angle(Z)=\pi=bT$ then $b=\pi/T=w$

Then $w=w_s/2$

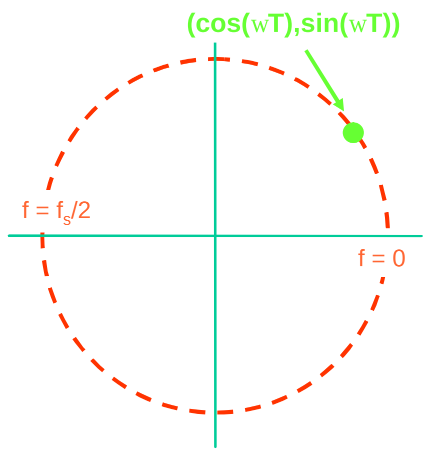
z-Domain Frequency Response

- LHP singularities in s-plane map into inside of unit-circle in Z domain
- RHP singularities in s-plane map into outside of unit-circle in Z domain
- The jω axis maps onto the unit circle



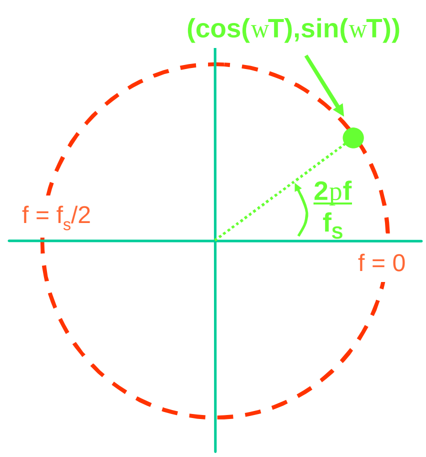
z-Domain Frequency Response

- Particular values:
  - f = 0 → z = 1
  - f = $f_s/2$ → z = -1
- The frequency response is obtained by evaluating H(z) on the unit circle at $z = e^{j\omega T} = \cos(\omega T)+j\sin(\omega T)$
- Once z=1 ($f_s/2$) is reached, the frequency response repeats, as expected

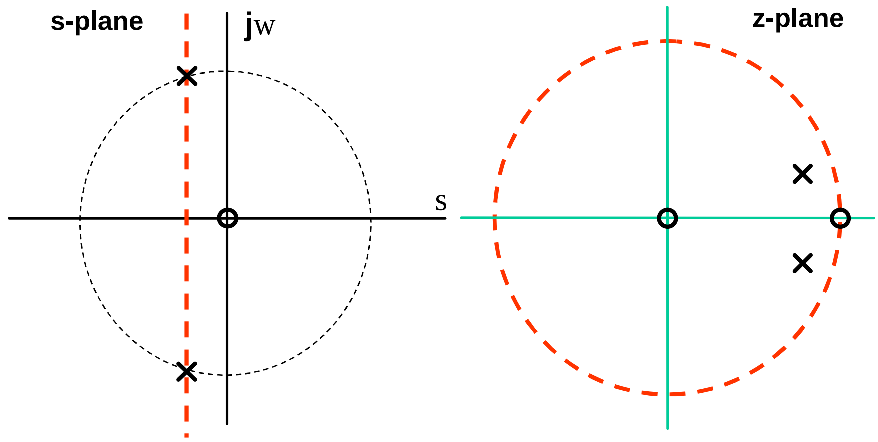


z-Domain Frequency Response

- The angle to the pole is equal to 360° (or 2π radians) times the ratio of the pole frequency to the sampling frequency



s-Plane versus z-Plane
Example: 2nd Order LDI Bandpass Filter



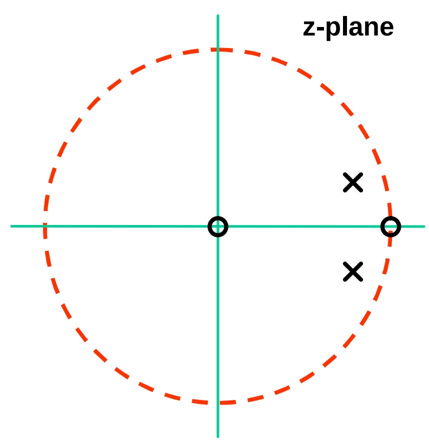
Pole-Zero Map in z-Plane

Zero from f→infinity in s-plane mapped to z=0, a non-physical frequency.

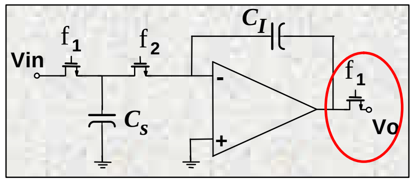
Zero from f→0 in s-plane mapped to z=+1

Distance from the pole to the unit circle is inversely proportional to pole Q

Pole on unit-circle → Q of infinity



DDI SC Integrator



$$\frac{V_o}{V_{in}}(Z) = -\frac{C_s}{C_I} \times \frac{Z^{-1}}{1-Z^{-1}} \quad , \quad Z = e^{jwT}$$

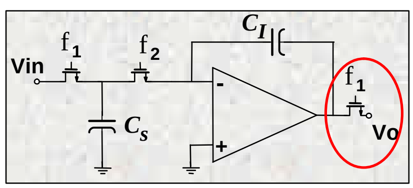
$$= \frac{C_s}{C_I} \times \frac{1}{1-e^{jwT}} = \frac{C_s}{C_I} \times \frac{e^{-jwT/2}}{e^{-jwT/2} - e^{jwT/2}}$$

$$= -j\frac{C_s}{C_I} \times e^{-jwT/2} \times \frac{1}{2\sin(wT/2)}$$

$$= -\frac{C_s}{C_I}\frac{1}{jwT} \times \frac{wT/2}{\sin(wT/2)} \times e^{-jwT/2}$$

Ideal Integrator) Magnitude Error Phase Error

DDI SC Integrator



$$\frac{V_o}{V_{in}}(Z) = -\frac{C_s}{C_I}\frac{1}{jwT} \times \frac{wT/2}{\sin(wT/2)} \times e^{-jwT/2}$$

Ideal Integrator) Magnitude Error Phase Error

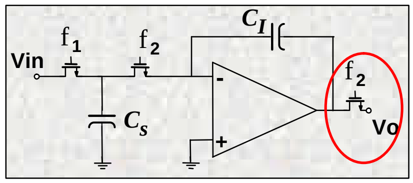
Example: Mag. & phase error for:

1- f / fs=1/12 → Mag. Error = 1% or 0.1dB
Phase error=15 degree
Q_{intg} = -3.8

2- f / fs=1/32 → Mag. Error=0.16% or 0.014dB
Phase error=5.6 degree
Q_{intg} = -10.2

DDI Integrator
→ magnitude error no problem
phase error major problem

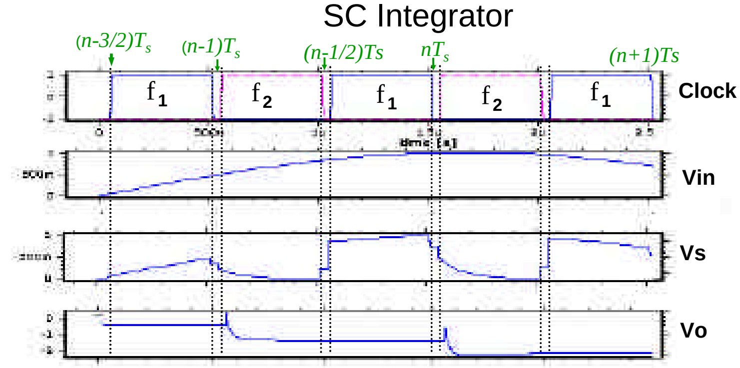
SC Integrator



Sample output ½ clock cycle earlier

→ Sample output on f_2

SC Integrator



$\Phi_1 \rightarrow$ $Q_s[(n-1)T_s] = C_s V_i[(n-1)T_s]$, $Q_I[(n-1)T_s] = Q_I[(n-3/2)T_s]$

$\Phi_2 \rightarrow$ $Q_s[(n-1/2)T_s] = 0$, $Q_I[(n-1/2)T_s] = Q_I[(n-3/2)T_s] + Q_s[(n-1)T_s]$

$\Phi_1 \rightarrow$ $Q_s[nT_s] = C_s V_i[nT_s]$, $Q_I[nT_s] = Q_I[(n-1)T_s] + Q_s[(n-1)T_s]$

$\Phi_2 \rightarrow$ $Q_s[(n+1/2)T_s] = 0$, $Q_I[(n+1/2)T_s] = Q_I[(n-1/2)T_s] + Q_s[nT_s]$