

EE247

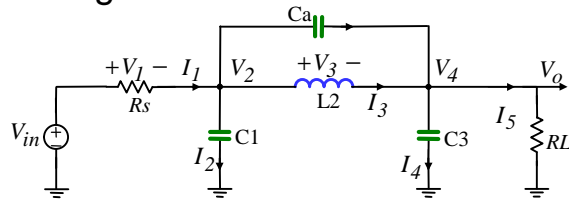
Lecture 5

- Summary last lecture
- Continuous-time filters
 - Facts about monolithic Rs & Cs and its effect on integrated filter characteristics
 - Opamp MOSFET-C filters
 - Opamp MOSFET-RC filters
 - Gm-C filters
- Frequency tuning for continuous-time filters
 - Trimming
 - Automatic frequency tuning
 - Continuous tuning
 - Periodic tuning

Summary Last Lecture

- High Q high order filters
 - Transmission zero implementation
 - Example
- Various integrator topologies utilized in monolithic filters
 - Resistor + C based filters
 - Transconductance (gm) + C based filters
 - Switched-capacitor filters
- Effect of integrator non-idealities on filter behavior

Summary Last Lecture Transmission zero Implementation for Integrator Based Ladder Filters



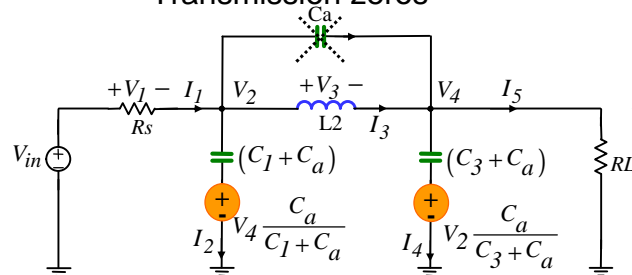
- Use KCL & KVL to derive :

$$V_2 = \frac{I_1 - I_3}{s(C_1 + C_a)} + V_4 \frac{C_a}{C_1 + C_a}$$

$$V_4 = \frac{I_3 - I_5}{s(C_3 + C_a)} + V_2 \frac{C_a}{C_3 + C_a}$$

Voltage Controlled Voltage Source!

Summary Last Lecture Integrator Based Ladder Filters Transmission zeros

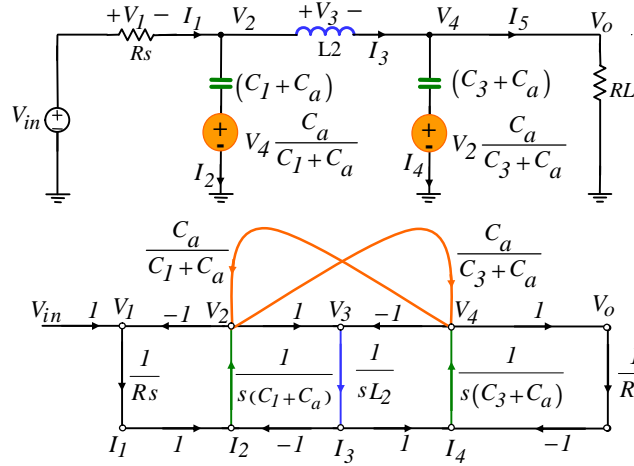


- Replace *shunt capacitor* with *voltage controlled voltage sources*:

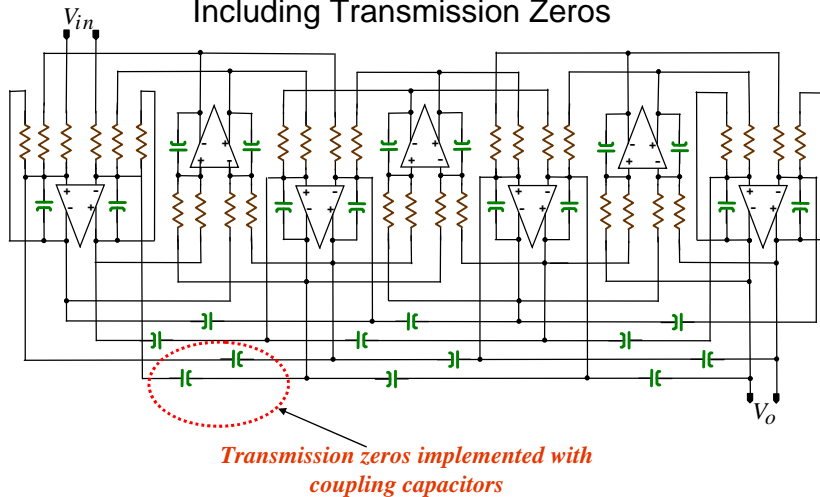
$$V_2 = \frac{I_1 - I_3}{s(C_1 + C_a)} + V_4 \frac{C_a}{C_1 + C_a}$$

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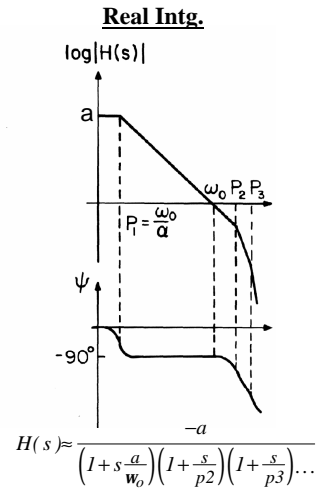
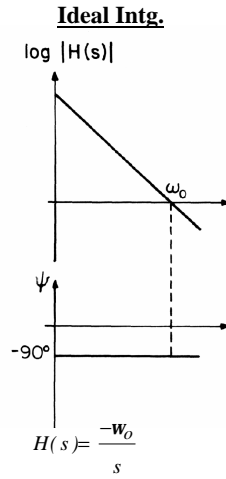
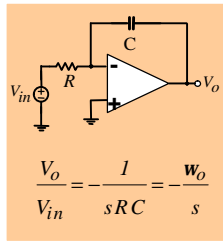
Summary Last Lecture
 Integrator Based Ladder Filters
 Transmission zeros



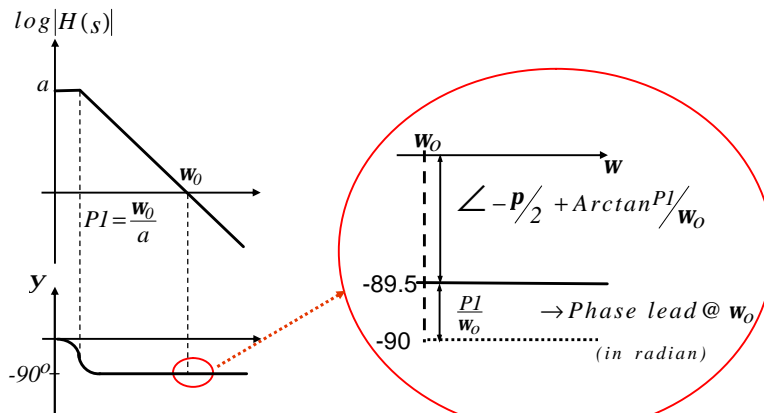
Summary Last Lecture
 Seventh Order Differential Lowpass Filter
 Including Transmission Zeros



Summary Last Lecture Effect of Integrator Non-Idealities on Filter Performance

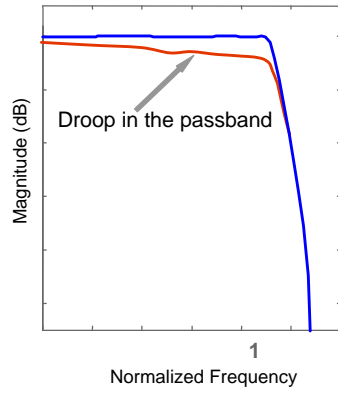


Effect of Integrator Finite DC Gain on Q

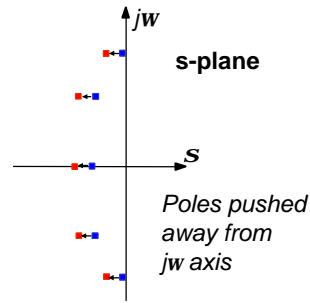


Example: $P/\omega_0 = 1/a = 1/100$

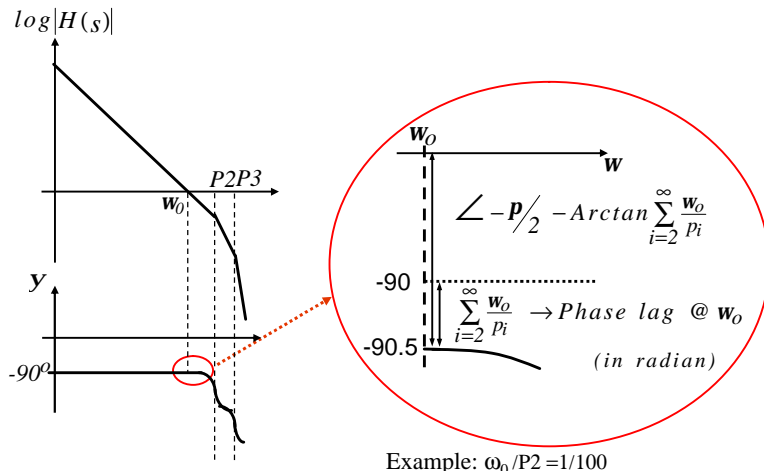
Effect of Integrator Finite DC Gain on Overall Filter Frequency Response



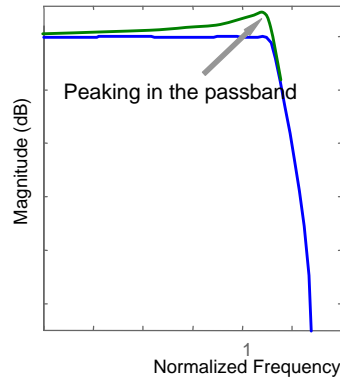
- Phase lead @ ω_0
→ Droop in the passband



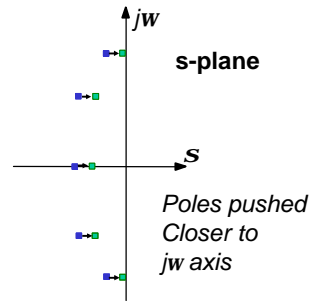
Effect of Integrator Non-Dominant Poles



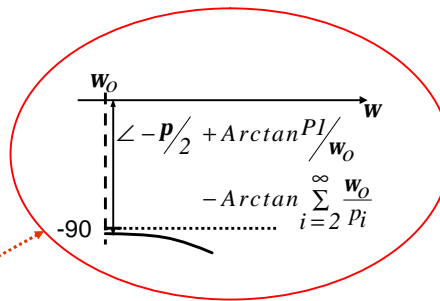
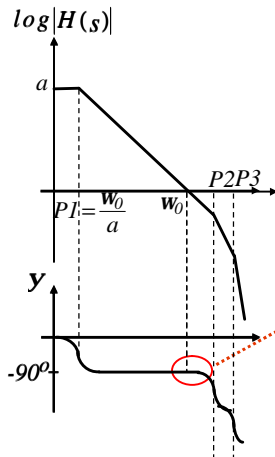
Effect of Integrator Non-Dominant Poles on Overall Filter Frequency Response



- Phase lag @ ω_0
 → Peaking in the passband
 In extreme cases could result in oscillation!



Effect of Integrator Non-Dominant Poles & Finite DC Gain on Q



Note that the two terms can cancel each other's effect

Summary

Effect of Integrator Non-Idealities on Q

$$Q_{ideal}^{intg.} = \infty$$

$$Q_{real}^{intg.} \approx \frac{1}{\frac{1}{a} - \omega_0 \sum_{i=2}^{\infty} \frac{1}{p_i}}$$

Phase lead @ ω_0
Phase lag @ ω_0

- Amplifier DC gain reduces the overall Q in the same manner as series/parallel resistance associated with passive elements
- Amplifier poles located above integrator unity-gain frequency enhance the Q!
 - If non-dominant poles close to unity-gain freq. → Oscillation
- Depending on the location of unity-gain-frequency, the two terms can cancel each other out!

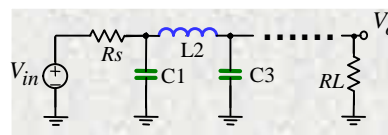
Few Facts About Monolithic R_s & C_s & G_m s

- Monolithic continuous-time filter critical frequency set by RxC or $GmxC$
 - Absolute value of integrated R_s & C_s & G_m s are quite variable
 - R_s vary due to doping and etching non-uniformities
 - Could vary by as much as $\sim \pm 30$ to 40% due to process & temperature variations
 - C_s vary because of oxide thickness variations and etching inaccuracies
 - Could vary $\sim \pm 10$ to 15%
 - G_m s typically function of mobility, oxide thickness, current, device geometry ...
 - Could vary $> \sim \pm 40\%$ or more with process & temp. & supply voltage
- Continuous-time filter critical frequency could vary by over $\pm 50\%$

Few Facts About Monolithic Rs & Cs

- While absolute value of monolithic R_s & C_s and g_m s are quite variable, with special attention paid to layout, C & R & g_m s quite well-matched
 - **Ratios** very accurate and stable over time and temperature
- With special attention to layout (e.g. interleaving, use of dummy devices, common-centroid geometries...):
 - Capacitor matching $\ll 0.1\%$
 - Resistor matching $< 0.1\%$
 - Gm matching $< 0.5\%$

Impact of Process Variations on Filter Characteristics



RLC Filters

Facts about RLC filters

- ω_{-3dB} determined by absolute value L_s & C_s
- Shape of filter depends on **ratios** of normalized L_s & C_s

$$C_1^{RLC} = C_r \times C_1^{Norm} = \frac{C_1^{Norm}}{R^* \times \omega_{-3dB}}$$

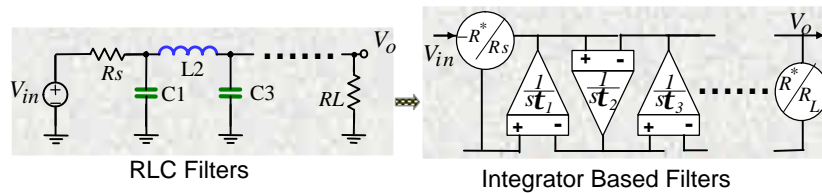
$$L_2^{RLC} = L_r \times L_2^{Norm} = \frac{L_2^{Norm} \times R^*}{\omega_{-3dB}}$$

Effect of Monolithic R & C Variations on Filter Characteristics

- Filter shape (whether Elliptic with 0.1dB Rpass or Butterworth..etc) is a function of *ratio* of *normalized Ls & Cs* in RLC filters
- Critical frequency (e.g. ω_{-3dB}) function of *absolute value* of *Ls & Cs*
- Absolute value of integrated *Rs & Cs & Gms* are quite variable
- *Ratios* very accurate and stable over time and temperature

→ What is the effect of on-chip component variations on monolithic filter frequency characteristics?

Impact of Process Variations on Filter Characteristics

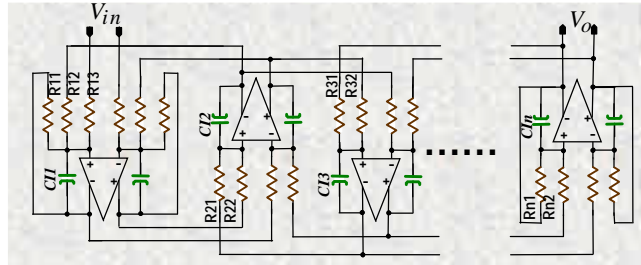


$$t_1 = C_1^{RLC} \cdot R^* = \frac{C_1^{Norm}}{\omega_{-3dB}}$$

$$t_2 = \frac{L_2^{RLC}}{R^*} = \frac{L_2^{Norm}}{\omega_{-3dB}}$$

$$\frac{t_1}{t_2} = \frac{C_1^{Norm}}{L_2^{Norm}}$$

Impact of Process Variations on Filter Characteristics



$$t_1^{int g} = C_{I1} \cdot R_{I1} = \frac{C_1^{Norm}}{\omega_{-3dB}}$$

$$t_2^{int g} = C_{I2} \cdot R_{I2} = \frac{L_2^{Norm}}{\omega_{-3dB}}$$

$$\frac{t_1^{int g}}{t_2^{int g}} = \frac{C_{I1} \cdot R_{I1}}{C_{I2} \cdot R_{I2}} = \frac{C_1^{Norm}}{L_2^{Norm}}$$

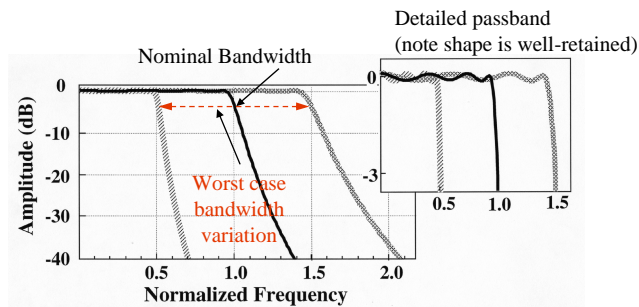
Variation in absolute value of integrated

Rs & Cs causes change in critical freq. (ω_{-3dB})

Since Ratios of Rs & Cs very accurate

→ Continuous time monolithic filters fully retain their shape

Example: LPF Worst Case Corner Frequency Variations



•While absolute value of on-chip RC (gm-C) time-constants vary by as much as 100% (process & temp.)

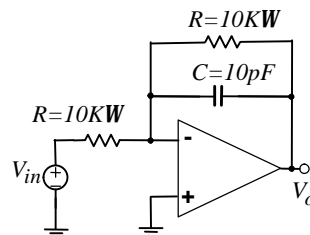
•With proper precautions, excellent matching can be achieved:

→ Well-preserved relative amplitude & phase vs freq. characteristics

→ **Need to adjust (tune) continuous-time filter critical frequencies only**

Tunable Opamp-RC Filters

- Example: A 1st order Opamp-RC filter is designed to have a corner frequency of 1.6MHz
- Assuming process variations of:
 - C varies by $\pm 10\%$
 - R varies by $\pm 25\%$
- Build the filter in such a way that the corner frequency can be adjusted post-manufacturing.



*Nominal R & C values
for 1.6MHz corner frequency*

Tunable Resistor

- Make provisions for either R or C to be adjustable (example adjustable R)
- Monolithic Rs can only be made adjustable in discrete steps (not continuous)
- Assuming expected process variations of:
 - Maximum C variations by $\pm 10\% \rightarrow C_{min}=9pF, C_{max}=11pF$
 - Maximum R variations by $\pm 25\% \rightarrow R_{min}=7.5K, R_{max}=12.5K$
 - \rightarrow Corner frequency varies by $\pm 35\%$

- Assuming $n=3$ bit (0 or 1) control signal for adjustment

$$R_{max} = R_{nom}(1+35\%) = 13.5K\Omega$$

$$R_{min} = R_{nom}(1-35\%) = 6.5K\Omega$$

$$R_1 = R_{min}$$

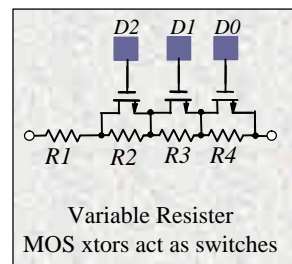
$$R_2 = (R_{max} - R_{min})/7 = 4K \rightarrow (2^{n-1} / (2^n - 1))$$

$$R_3 = (R_{max} - R_{min})/7 = 2K \rightarrow (2^{n-2} / (2^n - 1))$$

$$R_4 = (R_{max} - R_{min})/7 = 1K \rightarrow (2^{n-3} / (2^n - 1))$$

$$\text{Tuning resolution } 10\% \rightarrow (1K/10K)$$

If finer resolution needed add more bits & Rs

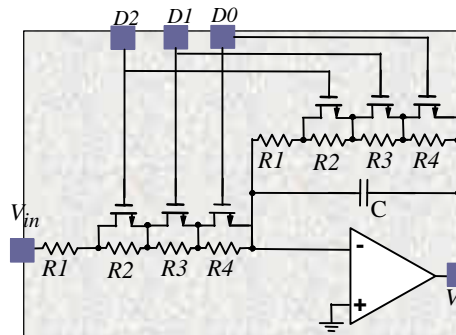


Tunable Opamp-RC Filter

D2	D1	D0	Rtotal
1	1	1	6.5K
1	1	0	7.5K
1	0	1	8.5K
.....			
.....			
0	0	0	13.5K

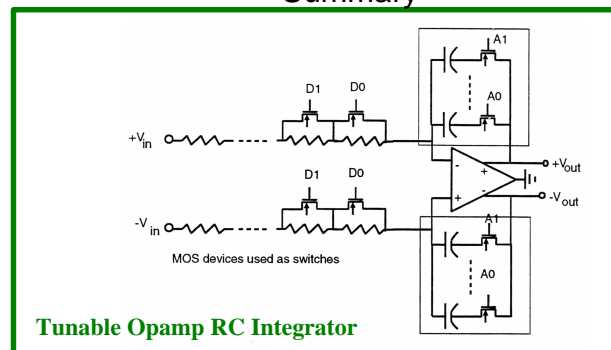
Post manufacturing:

- Set all Dx
- Measure -3dB frequency
 - If frequency too high
decrement D to D-1
 - If frequency too low
increment D to D+1
 - If frequency within 10% of
the desired corner freq. stop



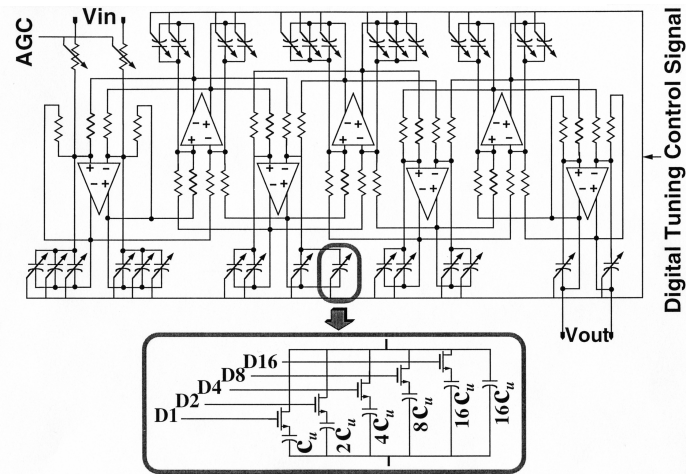
For higher order filters, all filter integrators tuned simultaneously

Tunable Opamp-RC Filters Summary



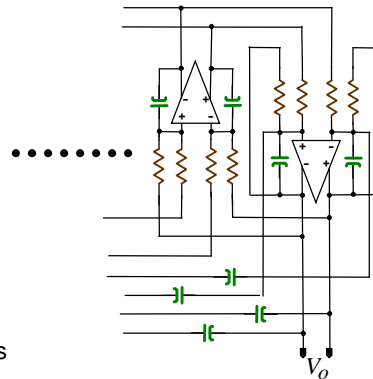
- Program Cs and/or Rs to freq. tune the filter
- All filter integrators tuned simultaneously
- Tuning in discrete steps & not continuous
- Tuning resolution limited
- Switch parasitic C & series R can affect the freq. response of the filter

Example: Tunable Low-Pass Opamp-RC Filter Adjustable Capacitors



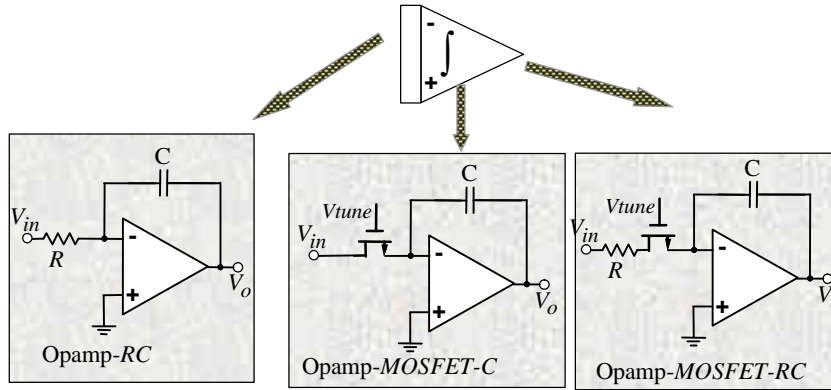
Opamp RC Filters

- Advantages
 - Since resistors are quite linear, linearity only a function of opamp linearity
 - good linearity
- Disadvantages
 - Opamps have to drive resistive load, low output impedance is required
 - High power consumption
 - Continuous tuning not possible
 - Tuning requires programmable R_s and/or C_s



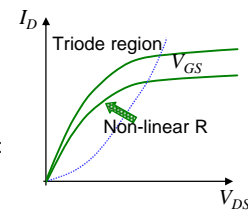
Integrator Implementation

Opamp-RC & Opamp-MOSFET-C & Opamp-MOSFET-RC



$$\frac{V_o}{V_{in}} = \frac{-w_o}{s} \quad \text{where} \quad w_o = \frac{I}{R_{eq}C}$$

Use of MOSFETs as Resistors



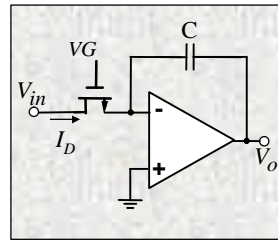
MOSFET IV characteristic:

Use of MOSFETs as Resistors Single-Ended Integrator

$$I_D = \mathbf{m}C_{ox} \frac{W}{L} \left[(V_{gs} - V_{th})V_{ds} - \frac{V_{ds}^2}{2} \right]$$

$$I_D = \mathbf{m}C_{ox} \frac{W}{L} \left[(V_{gs} - V_{th})V_i - \frac{V_i^2}{2} \right]$$

$$G = \frac{\partial I_D}{\partial V_i} = \mathbf{m}C_{ox} \frac{W}{L} (V_{gs} - V_{th} - V_i)$$



→ Tunable by varying VG:

Problem: Single-ended MOSFET-C Integrator → Effective R non-linear
Note that the non-linearity is mainly 2nd order type

Use of MOSFETs as Resistors Differential Integrator

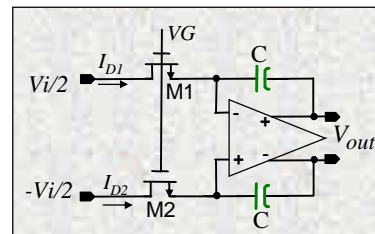
$$I_D = \mathbf{m}C_{ox} \frac{W}{L} \left(V_{gs} - V_{th} - \frac{V_{ds}}{2} \right) V_{ds}$$

$$I_{D1} = \mathbf{m}C_{ox} \frac{W}{L} \left(V_{gs} - V_{th} - \frac{V_i}{4} \right) \frac{V_i}{2}$$

$$I_{D2} = -\mathbf{m}C_{ox} \frac{W}{L} \left(V_{gs} - V_{th} + \frac{V_i}{4} \right) \frac{V_i}{2}$$

$$I_{D1} - I_{D2} = \mathbf{m}C_{ox} \frac{W}{L} (V_{gs} - V_{th}) V_i$$

$$G = \frac{\partial (I_{D1} - I_{D2})}{\partial V_i} = \mathbf{m}C_{ox} \frac{W}{L} (V_{gs} - V_{th})$$



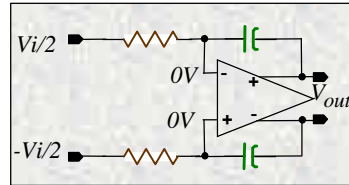
Opamp-MOSFET-C

- Non-linear term cancelled!
- Admittance independent of Vi

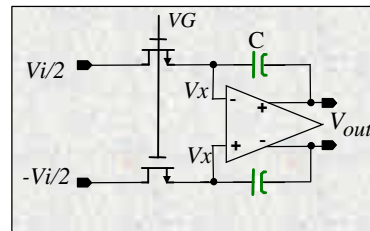
Problem: Threshold voltage dependence

MOSFET-C Integrator

•For the Opamp-RC integrator, opamp input stays at $0V$ (virtual gnd.)



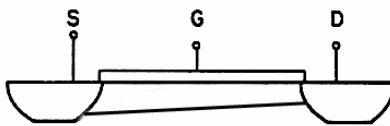
•For the MOSFET-C integrator, opamp input stays at the voltage V_x which is a function of 2nd order MOSFET non-linearities



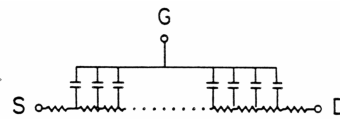
→ **Common-mode voltage sensitivity**

Use of MOSFET as Resistor Issues

MOS xtor operating in triode region
Cross section view



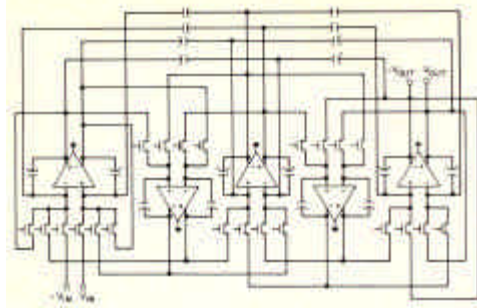
Distributed channel resistance & gate capacitance



- Distributed nature of gate capacitance & channel resistance results in infinite no. of high-frequency poles → excess phase
- Filter performance mandates well-matched MOSFETs → long channel devices
- Excess phase increases with L^2
 - Tradeoff between matching and integrator Q
 - This type of filter limited to low frequencies

Example: Opamp MOSFET-C Filter

- Suitable for low frequency applications
- Issues with linearity
- Linearity achieved ~40-50dB
- Needs tuning



5th Order Elliptic MOSFET-C LPF with 4kHz Bandwidth

Ref: Y. Tsvividis, M. Banu, and J. Khoury, "Continuous-Time MOSFET-C Filters in VLSI", *IEEE Journal of Solid State Circuits* Vol. SC-21, No.1 Feb. 1986, pp. 15-30

Improved MOSFET-C Integrator

$$I_D = \mu C_{ox} \frac{W}{L} \left(V_{gs} - V_{th} - \frac{V_{ds}}{2} \right) V_{ds}$$

$$I_{D1} = \mu C_{ox} \frac{W}{L} \left(V_{gs1} - V_{th} - \frac{V_i}{4} \right) \frac{V_i}{2}$$

$$I_{D3} = -\mu C_{ox} \frac{W}{L} \left(V_{gs2} - V_{th} + \frac{V_i}{4} \right) \frac{V_i}{2}$$

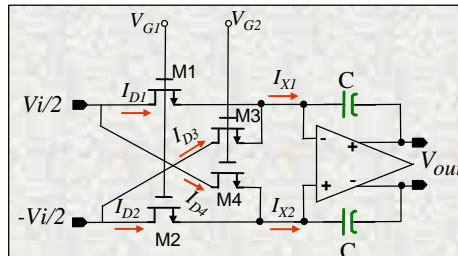
$$I_{X1} = I_{D1} + I_{D3}$$

$$= \mu C_{ox} \frac{W}{L} \left(V_{gs1} - V_{gs2} - \frac{V_i}{2} \right) \frac{V_i}{2}$$

$$I_{X2} = \mu C_{ox} \frac{W}{L} \left(V_{gs2} - V_{gs1} - \frac{V_i}{2} \right) \frac{V_i}{2}$$

$$I_{X1} - I_{X2} = \mu C_{ox} \frac{W}{L} (V_{gs1} - V_{gs2}) V_i$$

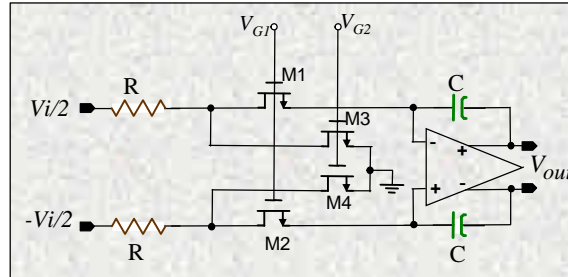
$$G = \frac{\partial (I_{D1} - I_{D2})}{\partial V_i} = \mu C_{ox} \frac{W}{L} (V_{gs1} - V_{gs2})$$



- No threshold dependence**
- First order Common-mode non-linearity cancelled**
- Linearity achieved in the order of 60-70dB**

Ref: Z. Czarnul, "Modification of the Banu-Tsvividis Continuous-Time Integrator Structure," *IEEE Transactions on Circuits and Systems*, Vol. CAS-33, No. 7, pp. 714-716, July 1986.

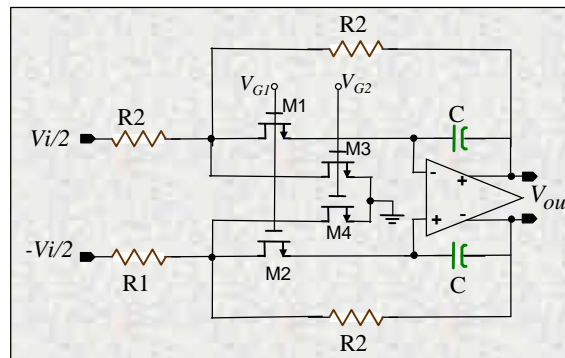
R-MOSFET-C Integrator



Improvement over MOSFET-C by adding resistor in series with MOSFET
Voltage drop primarily across resistor \rightarrow small MOSFET V_{ds} \rightarrow improved linearity
Linearity in the order of 90dB possible
Generally low frequency applications

Ref: U-K Moon, and B-S Song, "Design of a Low-Distortion 22-kHz Fifth Order Bessel Filter," *IEEE Journal of Solid State Circuits*, Vol. 28, No. 12, pp. 1254-1264, Dec. 1993.

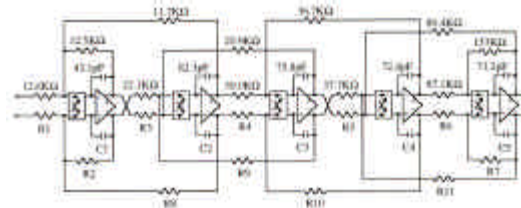
R-MOSFET-C Lossy Integrator



Negative feedback around the non-linear MOSFETs improves linearity
Reduced frequency response accuracy

Ref: U-K Moon, and B-S Song, "Design of a Low-Distortion 22-kHz Fifth Order Bessel Filter," *IEEE Journal of Solid State Circuits*, Vol. 28, No. 12, pp. 1254-1264, Dec. 1993.

Example: Opamp MOSFET-RC Filter



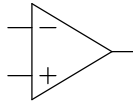
5th Order Bessel MOSFET-RC LPF -22kHz bandwidth
THD \rightarrow -90dB for 4Vp-p 2kHz input signal

- Suitable for low frequency applications
- Significant improvement in linearity compared to MOSFET-C
- Needs tuning

Ref: U-K Moon, and B-S Song, "Design of a Low-Distortion 22-kHz Fifth Order Bessel Filter," *IEEE Journal of Solid State Circuits*, Vol. 28, No. 12, pp. 1254-1264, Dec. 1993.

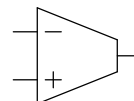
Operational Amplifiers (Opamps) versus Operational Transconductance Amplifiers (OTA)

Opamp
Voltage controlled
voltage source



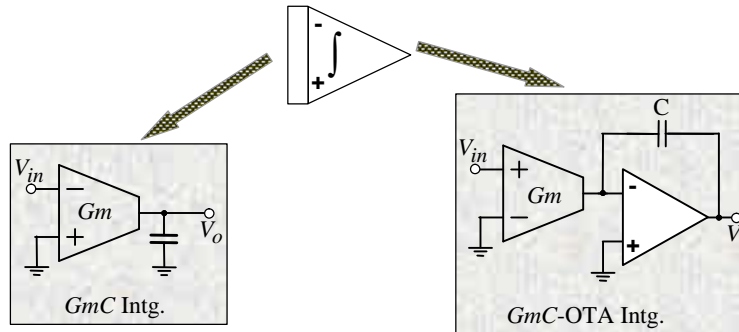
- Low output impedance
- Output in the form of voltage
- Can drive R-loads
- Good for RC filters, OK for SC filters
- Extra buffer adds complexity, power dissipation

OTA
Voltage controlled
current source



- High output impedance
- In the context of filter design called gm-cells
- Output in the form of current
- Cannot drive R-loads
- Good for SC & gm-C filters
- Typically, less complex compared to opamp \rightarrow higher freq. potential
- Typically lower power

Integrator Implementation Gm-C & Opamp-Gm-C



$$\frac{V_o}{V_{in}} = \frac{-w_o}{s} \quad \text{where} \quad w_o = \frac{G_m}{C}$$

Gm-C Filters Simplest Form of CMOS Gm-C Integrator

- MOSFET in saturation region:

$$I_d = \frac{mC_{ox} W}{2L} (V_{gs} - V_{th})^2$$

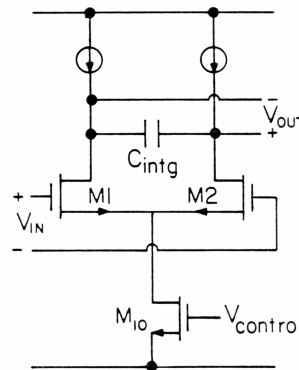
- Gm is given by:

$$g_m = \frac{\partial I_d}{\partial V_{gs}} = mC_{ox} \frac{W}{L} (V_{gs} - V_{th})$$

$$= 2 \frac{I_d}{(V_{gs} - V_{th})}$$

$$= 2 \left(\frac{1}{2} mC_{ox} \frac{W}{L} I_d \right)^{1/2}$$

*I_d varied via V_{control}
→ g_m tunable via V_{control}*



Gm-C Filters

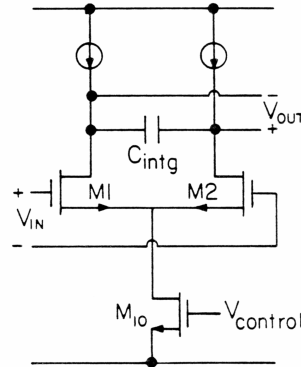
Simplest Form of CMOS Gm

- Integrator behavior:

$$\frac{V_{out}}{V_{in}} = \frac{-w_o}{s}$$

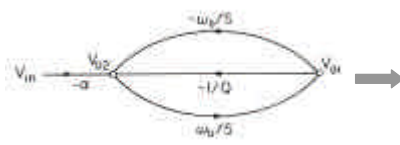
$$\text{where } w_o = \frac{g_m^{M1,2}}{2 \times C_{intg}}$$

- Critical frequency continuously tunable via $V_{control}$



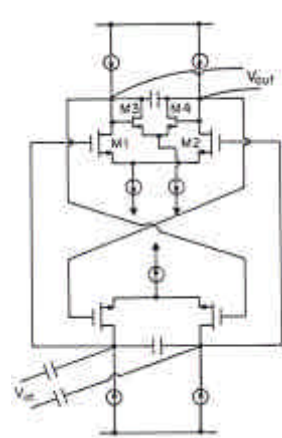
Ref: H. Khorramabadi and P.R. Gray, "High Frequency CMOS continuous-time filters," IEEE Journal of Solid-State Circuits, Vol.-SC-19, No. 6, pp.939-948, Dec. 1984.

Second Order Gm-C Filter



- Simple design
- Tunable
- Q function of device ratios:

$$Q = \frac{g_m^{M1,2}}{g_m^{M3,4}}$$



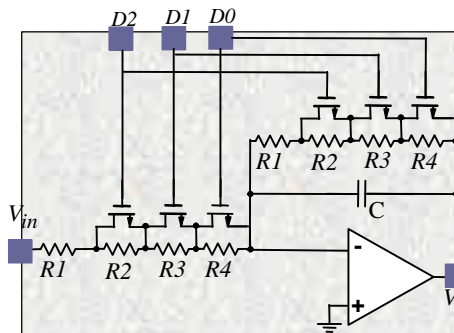
Filter Frequency Tuning Techniques

- Component trimming
- Automatic on-chip filter tuning
 - Continuous tuning
 - Master-slave tuning
 - Periodic off-line tuning
 - Systems where filter is followed by ADC & DSP, existing hardware can be used to periodically update filter freq. response

Example: Tunable Opamp-RC Filter

Post manufacturing:

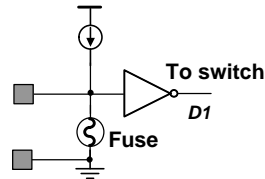
- Usually at wafer-sort tuning performed
- Measure -3dB frequency
 - If frequency too high decrement D to D-1
 - If frequency too low increment D to D+1
 - If frequency within 10% of the desired corner freq. stop



Not practical to require end-user to tune the filter
→ Need to fix the adjustment at the factory

Trimming

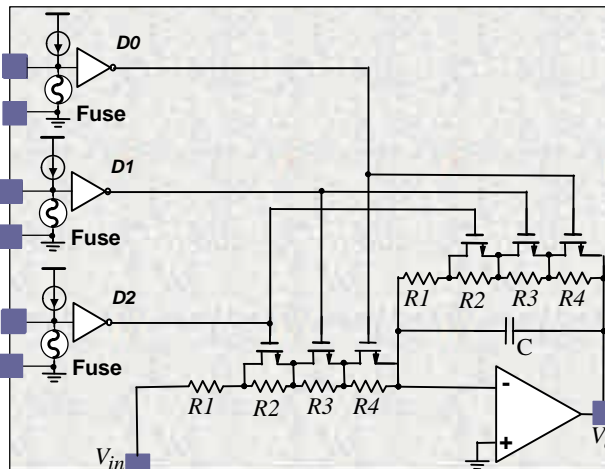
- Component trimming
 - Build fuses on-chip,
 - Based on measurements @ wafer-sort blow fuses by applying high current to the fuse
 - Expensive
 - Fuse regrowth problems!
 - Does not account for temp. variations & aging
 - Laser trimming
 - Trim components or cut fuses by laser
 - Even more expensive
 - Does not account for temp. variations & aging



Fuse not blown → $D1=1$
Fuse blown → $D1=0$

Example: Tunable/Trimmable Opamp-RC Filter

D2	D1	D0	Rtotal
1	1	1	6.5K
1	1	0	7.5K
1	0	1	8.5K
.....			
.....			
0	0	0	13.5K



Automatic Frequency Tuning

- By adding additional circuitry to the main filter circuit
 - Have the filter critical frequency automatically tuned
 - Expensive trimming avoided
 - Accounts for critical frequency variations due to temp. and voltage changes

Master-Slave Automatic Frequency Tuning

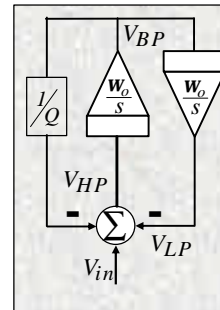
- Following facts used in this scheme:
 - Use a replica (master) of the main filter (called the slave) in the tuning circuitry
 - Place the replica in close proximity of the main filter
 - Use the tuning signal generated to tune the replica, to also tune the main filter
 - In the literature, this scheme is called master-slave tuning!

Master-Slave Frequency Tuning Reference Filter (VCF)

- Use a biquad for master filter (VCF)
- Utilize the fact that @ the frequency f_0 the lowpass (or highpass) outputs are 90 degree out of phase wrt to input

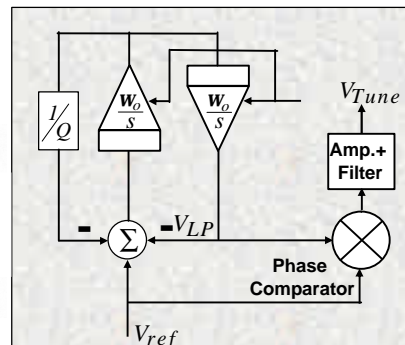
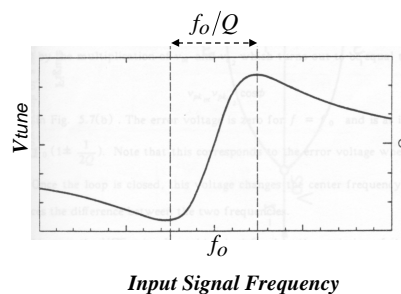
$$\frac{V_{LP}}{V_{in}} = \frac{1}{\frac{s^2}{\omega_0^2} + \frac{s}{Q\omega_0} + 1} \quad @ \quad \omega = \omega_0 \quad \angle = -90^\circ$$

- Apply a sinusoid at the desired f_0
- Compare the LP output phase to the input
- Based on the phase difference
 - Increase or decrease filter critical freq.



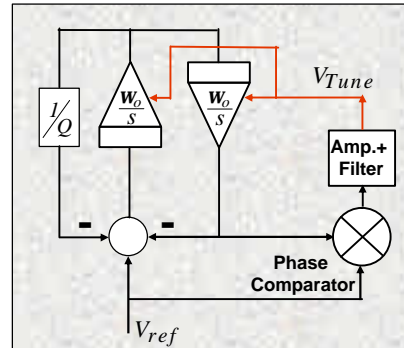
Master-Slave Frequency Tuning Reference Filter (VCF)

$$V_{tune} \approx -K \times V_{ref}^{rms} \times V_{LP}^{rms} \times \cos \phi$$

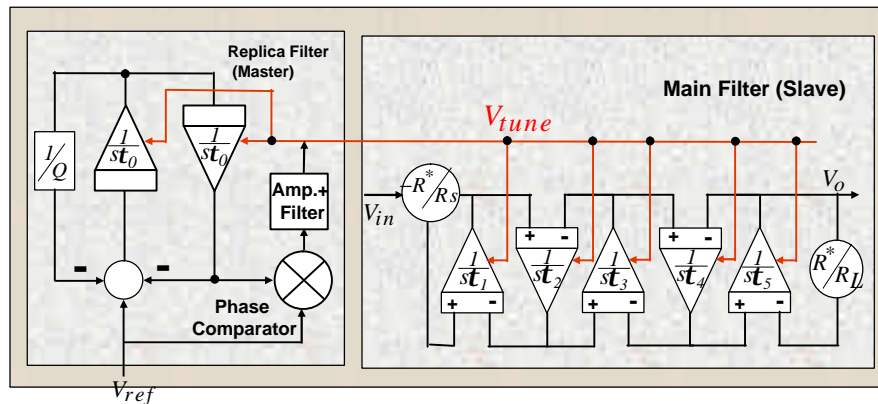


Master-Slave Frequency Tuning Reference Filter (VCF)

- By closing the loop, feedback tends to drive the error voltage to zero.
 - Locks f_o , the critical frequency of the filter to the accurate reference frequency
- Typically the reference frequency is provided by a crystal oscillator with accuracies in the order of few ppm



Master-Slave Frequency Tuning Reference Filter (VCF)



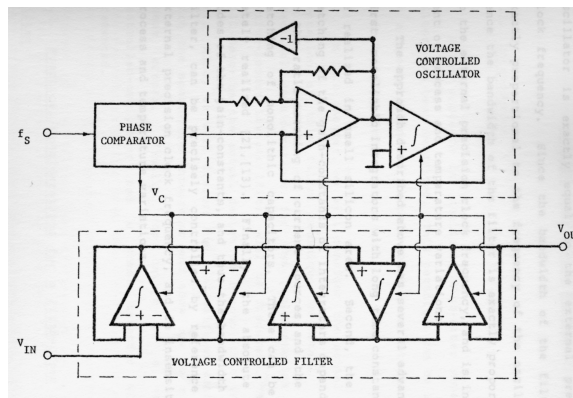
Ref: H. Khorrabadi and P.R. Gray, "High Frequency CMOS continuous-time filters," IEEE Journal of Solid-State Circuits, Vol.-SC-19, No. 6, pp.939-948, Dec. 1984.

Master-Slave Frequency Tuning Reference Filter (VCF)

- Issues to be aware of:
 - Input reference tuning signal needs to be sinusoid
→ disadvantage since clocks are usually available as square waveform
 - Reference signal feed-through to the output of the filter can limit filter dynamic range (reported levels or about 100uVrms)
 - Ref. signal feed-through is a function of:
 - Reference signal frequency wrt filter passband
 - Filter topology
 - Care in the layout
 - Fully differential topologies beneficial

Master-Slave Frequency Tuning Reference Voltage-Controlled-Oscillator (VCO)

- Instead of VCF a voltage-controlled-oscillator (VCO) is used
- VCO made or replica integrator used in main filter
- Tuning circuit operates exactly as a conventional phase-locked loop (PLL)
- Tuning signal used to tune main filter

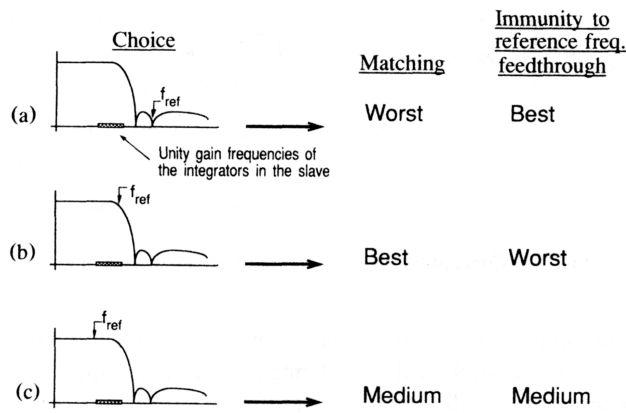


Ref: K.S. Tan and P.R. Gray, "Fully integrated analog filters using bipolar FET technology," IEEE, J. Solid-State Circuits, vol. SC-13, no.6, pp. 814-821, December 1978..

Master-Slave Frequency Tuning Reference Voltage-Controlled-Oscillator (VCO)

- Issues to be aware of:
 - Design of stable & repeatable oscillator challenging
 - VCO operation should be limited to the linear region or else the operation loses accuracy
 - Limiting the VCO signal range to the linear region not a trivial design issue
 - In the case of VCF based tuning ckt there was only ref. signal feedthrough. In this case, there is also the feedthrough of the VCO signal!!
 - Advantage over VCF based tuning → Reference input signal square wave (not sin.)

Master-Slave Frequency Tuning Choice of Ref. Frequency wrt Feedthrough Immunity



Ref: V. Gopinathan, et. al, "Design Considerations for High-Frequency Continuous-Time Filters and Implementation of an Antialiasing Filter for Digital Video," *IEEE JSSC*, Vol. SC-25, no. 6 pp. 1368-1378, Dec. 1990.