EE247
Lecture 4

- Summary last lecture
- Lecture today:
  - High Q high order filters
    - Transmission zero implementation
    - Example
  - Effect of integrator non-idealities on filter behavior
  - Various integrator topologies utilized in monolithic filters
    - Resistor + C based filters
    - Transconductance (gm) + C based filters
    - Switched-capacitor filters

Summary Last Lecture

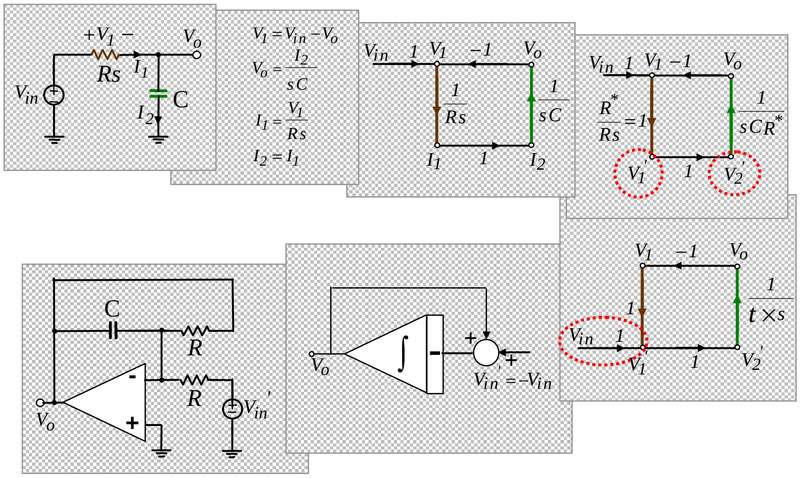
- Integrator based filters
  - Signal flowgraph concept
  - First order integrator based filter
  - Second order integrator based filter & biquads
- High order & high Q filters
  - Cascaded biquads
    - Cascaded biquad sensitivity
  - Ladder type filters

Summary last lecture

- How to convert RLC filters to integrator-based filters?
  - Label V & I for each RLC filter component
  - Derive the state-space description for the network with L & C described as (1/xs)
  - Draw the corresponding SGF with all voltages appearing as nodes and all currents represented by nodes drawn close to their associated voltage-branches are drawn & BMFs defined
  - Convert all current nodes to voltage nodes by multiplying by R* and scale BMFs accordingly
  - Derive the filter block diagram

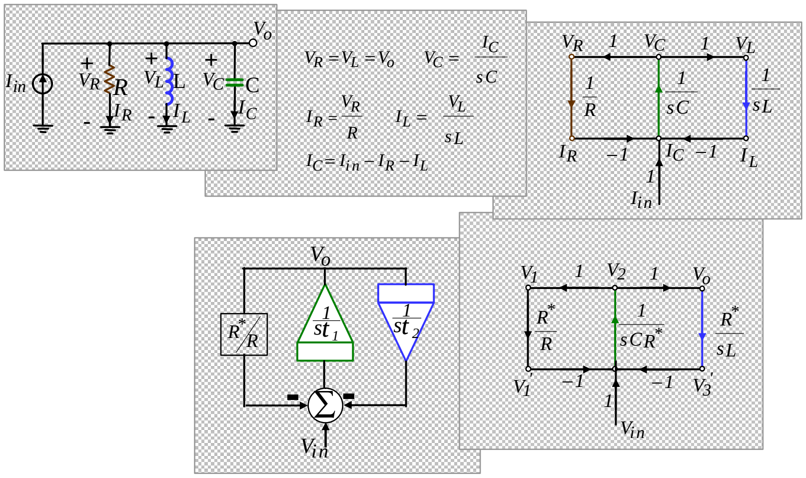
Summary last lecture

First Order RLC Filter Conversion to Integrator Based Active

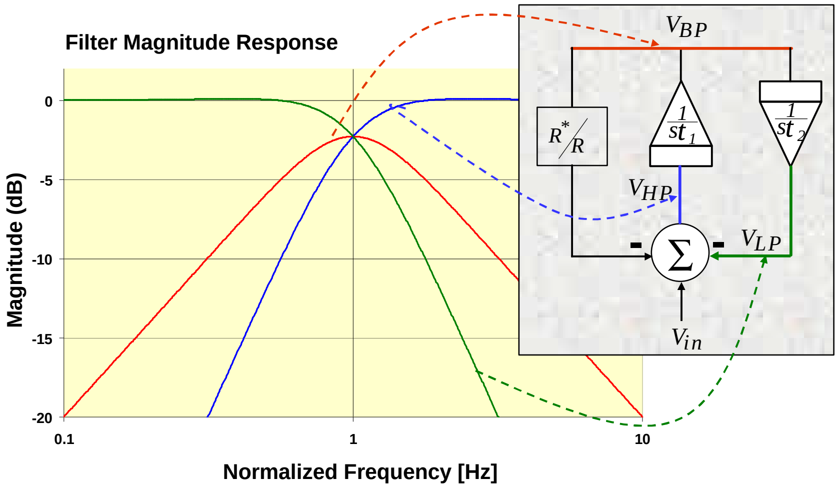


Summary last lecture

Second Order RLC Filter Conversion to Integrator Based Active



Summary Last Session
Second Order Integrator Based Filter



Summary
Integrator Based Monolithic Filters

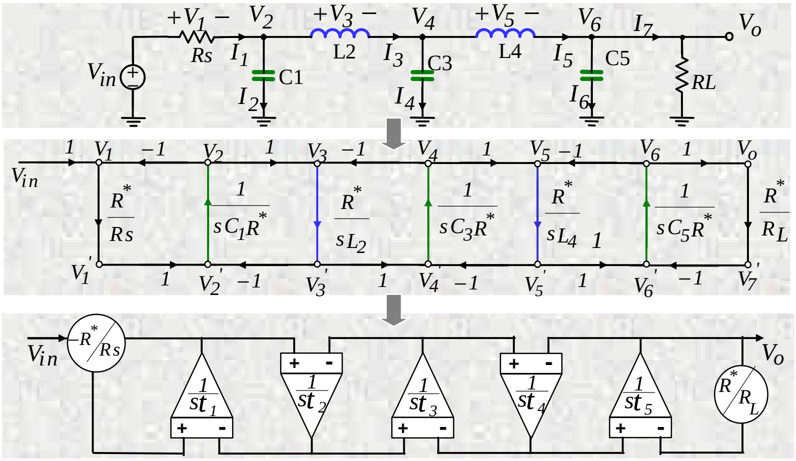
- Signal flowgraph techniques utilized to convert RLC networks to all integrator active filters
- Each reactive element (L& C) replaced by an integrator
- Fundamental noise limitation determined by integrating capacitor:
  - For lowpass filter: $\sqrt{\overline{v_o^2}} = \sqrt{a \frac{k\,T}{C}}$
  - Bandpass filter: $\sqrt{\overline{v_o^2}} = \sqrt{a\;Q \frac{k\,T}{C}}$

where a is a function of filter order and topology

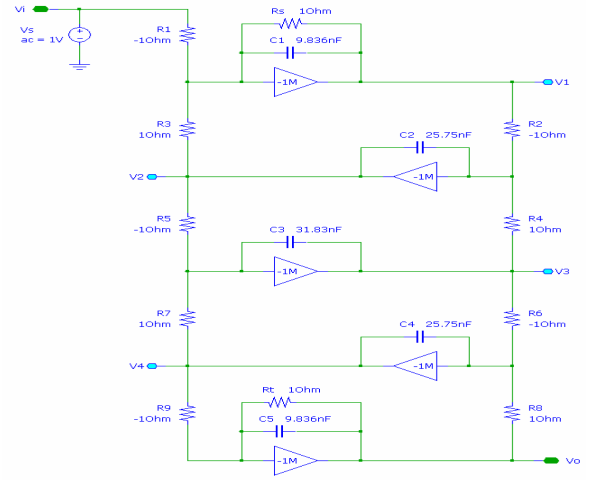
Summary Last Session
High Q & High Order Filters

- Cascade of biquads
  - Highly sensitive to component variations → not suitable for implementation of high Q & high order filters
  - Cascade of biquads only used in cases where required Q for all biquads <4 (e.g. filters for disk drives)
- LC ladder filters more appropriate for high Q & high order filters
  - Less sensitive to component variations

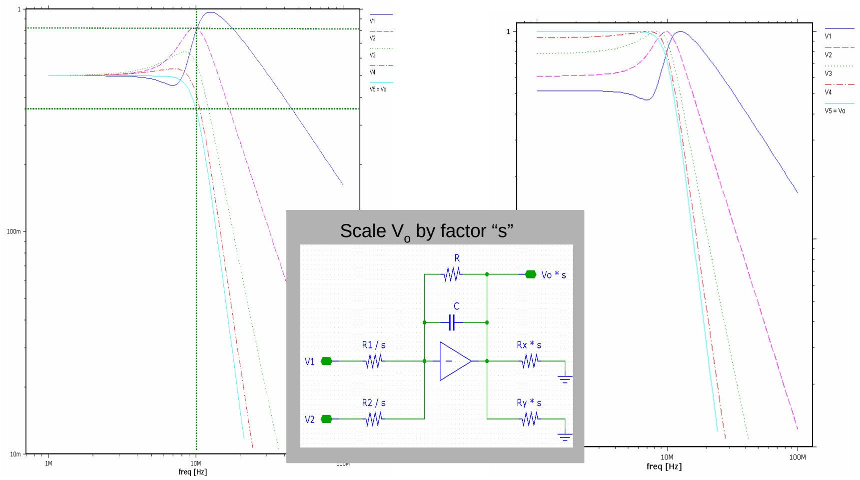
Summary
Integrated High Order Ladder Filters



Summary Last Session
Example 5th Order Butterworth Filter



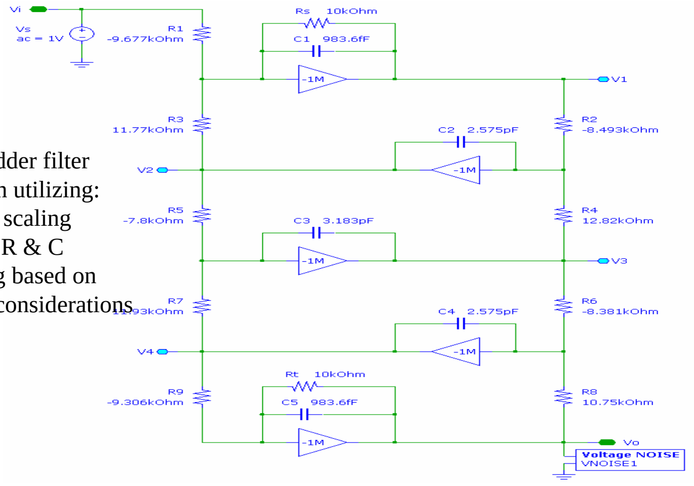
Summary Last Session
Max. Signal Handling by Voltage Node Scaling



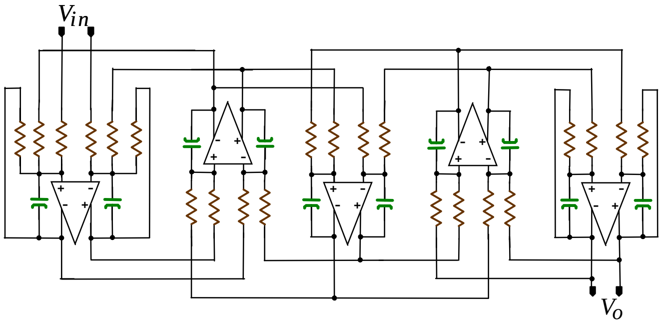
Scale V_o by factor “s”

Summary Last Session

5th order ladder filter
Final design utilizing:
- Node scaling
- Final R & C scaling based on noise considerations

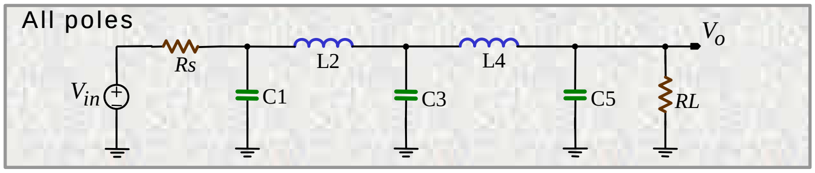


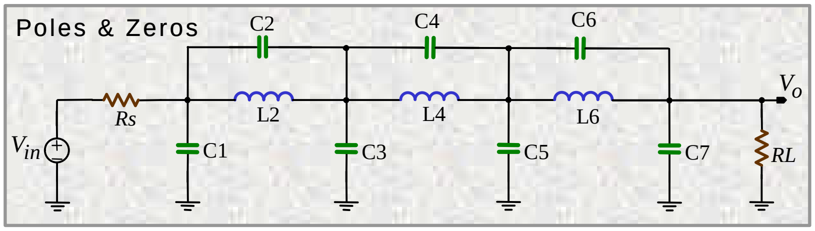
Differential 5th Order Lowpass Filter



- •Since each signal and its inverse readily available, eliminates the need for negative resistors!
- •Differential design has the advantage of even order harmonic distortion and common mode spurious pickup automatically cancels
- •Disadvantage: Double resistor and capacitor area!

RLC Ladder Filters
Including Transmission Zeros





RLC Ladder Filter Design Example

- Design a baseband filter for CDMA IS95 receiver with the following specs.
  - Filter frequency mask shown on the next page
  - Assume any phase impairment can be compensated in the digital domain

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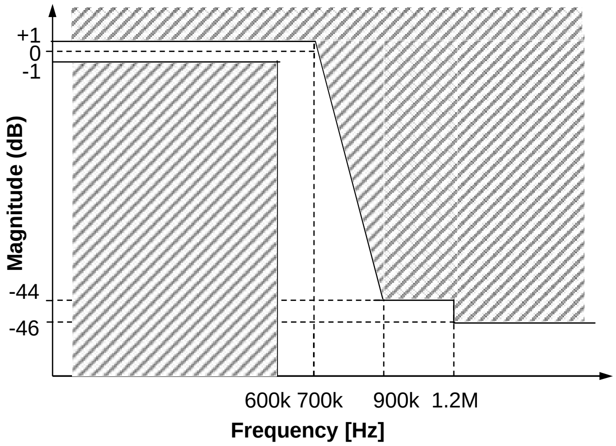
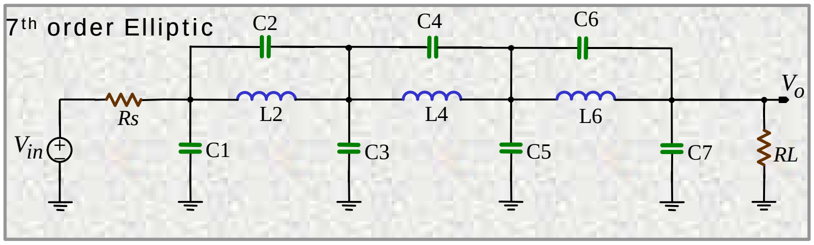
RLC Ladder Filter Design Example CDMA IS95 Receive Filter Frequency Mask



RLC Ladder Filter Design
Example: CDMA IS95 Receive Filter

- Since phase impairment can be corrected for, use filter type with max. cut-off slope/pole
  → Elliptic
- Design filter freq. response to fall well within the freq. mask
  - Allow margin for component variations
- For the passband ripple, allow enough margin for ripple change due to component & temperature variations
  → Passband ripple 0.2dB
- Design to spec.:
  - fpass = 650 kHz Rpass = 0.2 dB
  - fstop = 750 kHz Rstop = 49 dB
- Use Matlab or filter tables to decide the min. order for the filter (same as cascaded biquad example)
  - 7th Order Elliptic

RLC Ladder Filter Design
Example: CDMA IS95 Receive Filter



- Use filter tables to determine LC values
  - Table from A. Zverev, *Handbook of filter synthesis*, Wiley, 1967

RLC Ladder Filter Design
Example: CDMA IS95 Receive Filter

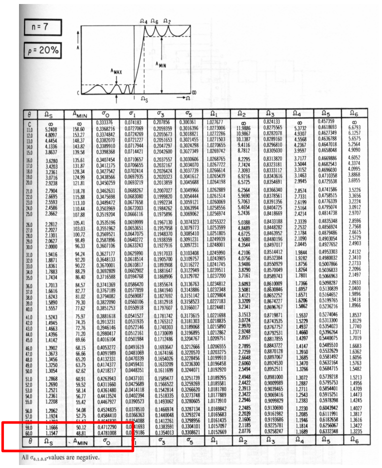
- Spec.
  - fpass = 650 kHz Rpass = 0.2 dB
  - fstop = 750 kHz Rstop = 49 dB
- Use filter tables to determine LC values
  - Table from: A. Zverev, *Handbook of filter synthesis*, Wiley, 1967
  - Elliptic filters tabulated wrt “reflection coeficient r”

$$Rpass = -10 \times \log\left(1 - r^2\right)$$

  - Since *Rpass=0.2dB* → r =20%
  - Use table accordingly

RLC Ladder Filter Design
Example: CDMA IS95 Receive Filter

- Table from Zverev page #281 & 282:
- Since our spec. is Amin=44dB add 5dB margin & design for Amin=49dB

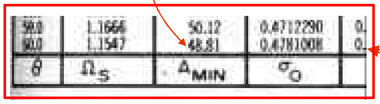


n = 7

ρ = 20%

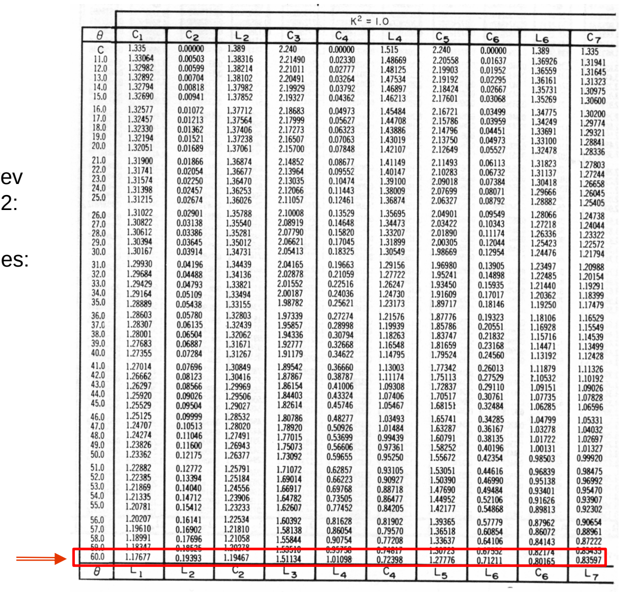
θ	Ω_S	A_{MIN}	σ_0	σ_1	σ_3	σ_5	Ω_1	Ω_2	Ω_3	Ω_4	Ω_5	Ω_6
C	∞	∞	0.333376	0.074183	0.207856	0.300361	1.027677	∞	0.824133	∞	0.457359	∞
11.0	5.2408	158.60	0.3368216	0.0727069	0.2059359	0.3016396	1.0273006	11.9886	0.8275565	5.3732	0.4618693	6.6793
12.0	4.8097	153.27	0.3374842	0.0724269	0.2055673	0.3018821	1.0272286	10.9867	0.8282078	4.9307	0.4627349	6.1257
13.0	4.4454	148.37	0.3382070	0.0721227	0.2051653	0.3021455	1.0271503	10.1387	0.8289160	4.5568	0.4636788	5.6575
14.0	4.1336	143.82	0.3389910	0.0717944	0.2047297	0.3024298	1.0270655	9.4116	0.8296810	4.2367	0.4647018	5.2564
15.0	3.8637	139.58	0.3398368	0.0714421	0.2042600	0.3027349	1.0269742	8.7812	0.8305030	3.9597	0.4658048	4.9090
16.0	3.6280	135.61	0.3407454	0.0710657	0.2037557	0.3030606	1.0268765	8.2295	0.8313820	3.7177	0.4669886	4.6052
17.0	3.4203	131.87	0.3417175	0.0706655	0.2032167	0.3034070	1.0267722	7.7424	0.8323181	3.5044	0.4682543	4.3374
18.0	3.2361	128.34	0.3427542	0.0702414	0.2026424	0.3037739	1.0266614	7.3093	0.8333112	3.3152	0.4696030	4.0995
19.0	3.0716	124.99	0.3438566	0.0697935	0.2020323	0.3041612	1.0265439	6.9216	0.8343616	3.1463	0.4710358	3.8868
20.0	2.9238	121.81	0.3450259	0.0693219	0.2013859	0.3045688	1.0264198	6.5725	0.8354691	2.9945	0.4725538	3.6955
21.0	2.7904	118.78	0.3462631	0.0688267	0.2007027	0.3049966	1.0262889	6.2564	0.8366340	2.8574	0.4741586	3.5226
22.0	2.6695	115.88	0.3475698	0.0683080	0.1999820	0.3054444	1.0261514	5.9690	0.8378561	2.7331	0.4758515	3.3656
23.0	2.5593	113.10	0.3489472	0.0677658	0.1992234	0.3059121	1.0260069	5.7063	0.8391356	2.6199	0.4776339	3.2224
24.0	2.4586	110.44	0.3503969	0.0672003	0.1984262	0.3063994	1.0258556	5.4654	0.8404725	2.5164	0.4795074	3.0912
25.0	2.3662	107.88	0.3519204	0.0666116	0.1975896	0.3069062	1.0256974	5.2436	0.8418669	2.4214	0.4814738	2.9707
26.0	2.2812	105.41	0.3535196	0.0659999	0.1967130	0.3074323	1.0255322	5.0388	0.8433188	2.3339	0.4835348	2.8596
27.0	2.2027	103.03	0.3551962	0.0653651	0.1957958	0.3079773	1.0253599	4.8489	0.8448282	2.2532	0.4856924	2.7568
28.0	2.1301	100.72	0.3569521	0.0647075	0.1948370	0.3085410	1.0251805	4.6725	0.8463952	2.1784	0.4879486	2.6615
29.0	2.0627	98.49	0.3587896	0.0640272	0.1938359	0.3091231	1.0249939	4.5080	0.8480196	2.1090	0.4903054	2.5729
30.0	2.0000	96.33	0.3607106	0.0633243	0.1927916	0.3097231	1.0248001	4.3544	0.8497017	2.0445	0.4927652	2.4903
31.0	1.9416	94.24	0.3627177	0.0625990	0.1917033	0.3103408	1.0245990	4.2106	0.8514412	1.9844	0.4953303	2.4132
32.0	1.8871	92.20	0.3648133	0.0618514	0.1905700	0.3109757	1.0243905	4.0756	0.8532384	1.9282	0.4980032	2.3410
33.0	1.8361	90.22	0.3670001	0.0610817	0.1893908	0.3116272	1.0241745	3.9486	0.8550929	1.8756	0.5007866	2.2733
34.0	1.7883	88.29	0.3692809	0.0602902	0.1881647	0.3122949	1.0239511	3.8290	0.8570049	1.8264	0.5036833	2.2096
35.0	1.7434	86.40	0.3716588	0.0594768	0.1868906	0.3129782	1.0237200	3.7161	0.8589743	1.7801	0.5066963	2.1497
36.0	1.7013	84.57	0.3741369	0.0586420	0.1855674	0.3136763	1.0234812	3.6093	0.8610009	1.7366	0.5098287	2.0933
37.0	1.6616	82.77	0.3767189	0.0577859	0.1841940	0.3143886	1.0232348	3.5081	0.8630846	1.6957	0.5130839	2.0400
38.0	1.6243	81.02	0.3794082	0.0569087	0.1827692	0.3151142	1.0229804	3.4121	0.8652252	1.6571	0.5164652	1.9896
39.0	1.5890	79.30	0.3822090	0.0560106	0.1812918	0.3158523	1.0227183	3.3209	0.8674227	1.6206	0.5199765	1.9418
40.0	1.5557	77.62	0.3851253	0.0550918	0.1797607	0.3166017	1.0224481	3.2341	0.8696767	1.5862	0.5236216	1.8966
41.0	1.5243	75.97	0.3881618	0.0541527	0.1781742	0.3173615	1.0221698	3.1513	0.8719871	1.5537	0.5274046	1.8537
42.0	1.4945	74.35	0.3913231	0.0531935	0.1765312	0.3181303	1.0218835	3.0724	0.8743535	1.5229	0.5313300	1.8129
43.0	1.4663	72.76	0.3946146	0.0522146	0.1748303	0.3189068	1.0215890	2.9970	0.8767757	1.4937	0.5354023	1.7740
44.0	1.4396	71.20	0.3980417	0.0512161	0.1730699	0.3196895	1.0212862	2.9248	0.8792531	1.4660	0.5396264	1.7371
45.0	1.4142	69.66	0.4016104	0.0501984	0.1712486	0.3204767	1.0209751	2.8557	0.8817855	1.4397	0.5440075	1.7019
46.0	1.3902	68.15	0.4053272	0.0491619	0.1693647	0.3212666	1.0206555	2.7895	0.8843722	1.4147	0.5485510	1.6683
47.0	1.3673	66.66	0.4091989	0.0481069	0.1674166	0.3220570	1.0203275	2.7259	0.8870128	1.3910	0.5532629	1.6362
48.0	1.3456	65.20	0.4132331	0.0470339	0.1654026	0.3228456	1.0199910	2.6648	0.8897067	1.3685	0.5581492	1.6056
49.0	1.3250	63.75	0.4174377	0.0459431	0.1633209	0.3236300	1.0196458	2.6060	0.8924530	1.3470	0.5632164	1.5763
50.0	1.3054	62.62	0.4218217	0.0448351	0.1611699	0.3244071	1.0192920	2.5494	0.8952511	1.3266	0.5684715	1.5482
51.0	1.2868	60.91	0.4263943	0.0437101	0.1589477	0.3251739	1.0189295	2.4948	0.8981000	1.3072	0.5739218	1.5213
52.0	1.2690	59.52	0.4311660	0.0425689	0.1566522	0.3259269	1.0185581	2.4422	0.9009989	1.2887	0.5795753	1.4956
53.0	1.2521	58.14	0.4361480	0.0414118	0.1542814	0.3266620	1.0181780	2.3913	0.9039465	1.2711	0.5854401	1.4709
54.0	1.2361	56.77	0.4413524	0.0402394	0.1518335	0.3273748	1.0177889	2.3422	0.9069416	1.2543	0.5915251	1.4473
55.0	1.2208	55.42	0.4467927	0.0390523	0.1493062	0.3280605	1.0173910	2.2946	0.9099829	1.2383	0.5978398	1.4245
56.0	1.2062	54.08	0.4524835	0.0378510	0.1466974	0.3287134	1.0169842	2.2485	0.9130690	1.2230	0.6043942	1.4027
57.0	1.1924	52.75	0.4584410	0.0366363	0.1440048	0.3293274	1.0165683	2.2039	0.9161982	1.2085	0.6111991	1.3817
58.0	1.1793	51.43	0.4646830	0.0354088	0.1412261	0.3298956	1.0161435	2.1606	0.9193686	1.1946	0.6182658	1.3616
59.0	1.1666	50.12	0.4712290	0.0341693	0.1383591	0.3304101	1.0157097	2.1185	0.9225781	1.1814	0.6256067	1.3422
60.0	1.1547	48.81	0.4781008	0.0329186	0.1354013	0.3308621	1.0152669	2.0776	0.9258247	1.1689	0.6332348	1.3235
θ	Ω_S	A_{MIN}	σ_0	σ_1	σ_3	σ_5	Ω_1	Ω_2	Ω_3	Ω_4	Ω_5	Ω_6

All $\sigma_{0,1,3,5}$-values are negative.



59.0	1.1666	50.12	0.4712290
60.0	1.1547	48.81	0.4781008
θ	Ω_S	A_{MIN}	σ_0

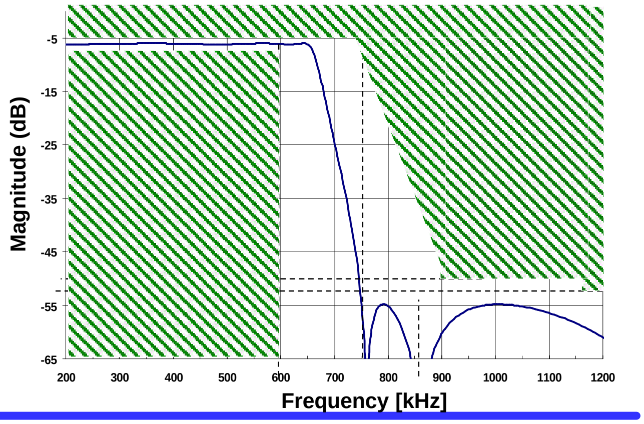
- Table from Zverev page #281 & 282:
- Normalized component values:
  - C1=1.17677
  - C2=0.19393
  - L2=1.19467
  - C3=1.51134
  - C4=1.01098
  - L4=0.72398
  - C5=1.27776
  - C6=0.71211
  - L6=0.80165
  - C7=0.83597



	$K^2 = 1.0$									
θ	C_1	C_2	L_2	C_3	C_4	L_4	C_5	C_6	L_6	C_7
C	1.335	0.00000	1.389	2.240	0.00000	1.515	2.240	0.00000	1.389	1.335
11.0	1.33064	0.00503	1.38316	2.21490	0.02330	1.48669	2.20558	0.01637	1.36926	1.31941
12.0	1.32982	0.00599	1.38214	2.21011	0.02777	1.48125	2.19903	0.01952	1.36559	1.31645
13.0	1.32892	0.00704	1.38102	2.20491	0.03264	1.47534	2.19192	0.02295	1.36161	1.31323
14.0	1.32794	0.00818	1.37982	2.19929	0.03792	1.46897	2.18424	0.02667	1.35731	1.30975
15.0	1.32690	0.00941	1.37852	2.19327	0.04362	1.46213	2.17601	0.03068	1.35269	1.30600
16.0	1.32577	0.01072	1.37712	2.18683	0.04973	1.45484	2.16721	0.03499	1.34775	1.30200
17.0	1.32457	0.01213	1.37564	2.17999	0.05627	1.44708	2.15786	0.03959	1.34249	1.29774
18.0	1.32330	0.01362	1.37406	2.17273	0.06323	1.43886	2.14796	0.04451	1.33691	1.29321
19.0	1.32194	0.01521	1.37238	2.16507	0.07063	1.43019	2.13750	0.04973	1.33100	1.28841
20.0	1.32051	0.01689	1.37061	2.15700	0.07848	1.42107	2.12649	0.05527	1.32478	1.28336
21.0	1.31900	0.01866	1.36874	2.14852	0.08677	1.41149	2.11493	0.06113	1.31823	1.27803
22.0	1.31741	0.02054	1.36677	2.13964	0.09552	1.40147	2.10283	0.06732	1.31137	1.27244
23.0	1.31574	0.02250	1.36470	2.13035	0.10474	1.39100	2.09018	0.07384	1.30418	1.26658
24.0	1.31398	0.02457	1.36253	2.12066	0.11443	1.38009	2.07699	0.08071	1.29666	1.26045
25.0	1.31215	0.02674	1.36026	2.11057	0.12461	1.36874	2.06327	0.08792	1.28882	1.25405
26.0	1.31022	0.02901	1.35788	2.10008	0.13529	1.35695	2.04901	0.09549	1.28066	1.24738
27.0	1.30822	0.03138	1.35540	2.08919	0.14648	1.34473	2.03422	0.10343	1.27218	1.24044
28.0	1.30612	0.03386	1.35281	2.07790	0.15820	1.33207	2.01890	0.11174	1.26336	1.23322
29.0	1.30394	0.03645	1.35012	2.06621	0.17045	1.31899	2.00305	0.12044	1.25423	1.22572
30.0	1.30167	0.03914	1.34731	2.05413	0.18325	1.30549	1.98669	0.12954	1.24476	1.21794
31.0	1.29930	0.04196	1.34439	2.04165	0.19663	1.29156	1.96980	0.13905	1.23497	1.20988
32.0	1.29684	0.04488	1.34136	2.02878	0.21059	1.27722	1.95241	0.14898	1.22485	1.20154
33.0	1.29429	0.04793	1.33821	2.01552	0.22516	1.26247	1.93450	0.15935	1.21440	1.19291
34.0	1.29164	0.05109	1.33494	2.00187	0.24036	1.24730	1.91609	0.17017	1.20362	1.18399
35.0	1.28889	0.05438	1.33155	1.98782	0.25621	1.23173	1.89717	0.18146	1.19250	1.17479
36.0	1.28603	0.05780	1.32803	1.97339	0.27274	1.21576	1.87776	0.19323	1.18106	1.16529
37.0	1.28307	0.06135	1.32439	1.95857	0.28998	1.19939	1.85786	0.20551	1.16928	1.15549
38.0	1.28001	0.06504	1.32062	1.94336	0.30794	1.18263	1.83747	0.21832	1.15716	1.14539
39.0	1.27683	0.06887	1.31671	1.92777	0.32668	1.16548	1.81659	0.23168	1.14471	1.13499
40.0	1.27355	0.07284	1.31267	1.91179	0.34622	1.14795	1.79524	0.24560	1.13192	1.12428
41.0	1.27014	0.07696	1.30849	1.89542	0.36660	1.13003	1.77342	0.26013	1.11879	1.11326
42.0	1.26662	0.08123	1.30416	1.87867	0.38787	1.11174	1.75113	0.27529	1.10532	1.10192
43.0	1.26297	0.08566	1.29969	1.86154	0.41006	1.09308	1.72837	0.29110	1.09151	1.09026
44.0	1.25920	0.09026	1.29506	1.84403	0.43324	1.07406	1.70517	0.30761	1.07735	1.07828
45.0	1.25529	0.09504	1.29027	1.82614	0.45746	1.05467	1.68151	0.32484	1.06285	1.06596
46.0	1.25125	0.09999	1.28532	1.80786	0.48277	1.03493	1.65741	0.34285	1.04799	1.05331
47.0	1.24707	0.10513	1.28020	1.78920	0.50926	1.01484	1.63287	0.36167	1.03278	1.04032
48.0	1.24274	0.11046	1.27491	1.77015	0.53699	0.99439	1.60791	0.38135	1.01722	1.02697
49.0	1.23826	0.11600	1.26943	1.75073	0.56606	0.97361	1.58252	0.40196	1.00131	1.01327
50.0	1.23362	0.12175	1.26377	1.73092	0.59655	0.95250	1.55672	0.42354	0.98503	0.99920
51.0	1.22882	0.12772	1.25791	1.71072	0.62857	0.93105	1.53051	0.44616	0.96839	0.98475
52.0	1.22385	0.13394	1.25184	1.69014	0.66223	0.90927	1.50390	0.46990	0.95138	0.96992
53.0	1.21869	0.14040	1.24556	1.66917	0.69768	0.88718	1.47690	0.49484	0.93401	0.95470
54.0	1.21335	0.14712	1.23906	1.64782	0.73505	0.86477	1.44952	0.52106	0.91626	0.93907
55.0	1.20781	0.15412	1.23233	1.62607	0.77452	0.84205	1.42177	0.54868	0.89813	0.92302
56.0	1.20207	0.16141	1.22534	1.60392	0.81628	0.81902	1.39365	0.57779	0.87962	0.90654
57.0	1.19610	0.16902	1.21810	1.58138	0.86054	0.79570	1.36518	0.60854	0.86072	0.88961
58.0	1.18991	0.17696	1.21058	1.55844	0.90754	0.77208	1.33637	0.64106	0.84143	0.87222
59.0	1.18347	0.18525	1.20278	1.53510	0.95758	0.74817	1.30723	0.67552	0.82174	0.85435
60.0	1.17677	0.19393	1.19467	1.51134	1.01098	0.72398	1.27776	0.71211	0.80165	0.83597
θ	L_1	L_2	C_2	L_3	L_4	C_4	L_5	L_6	C_6	L_7

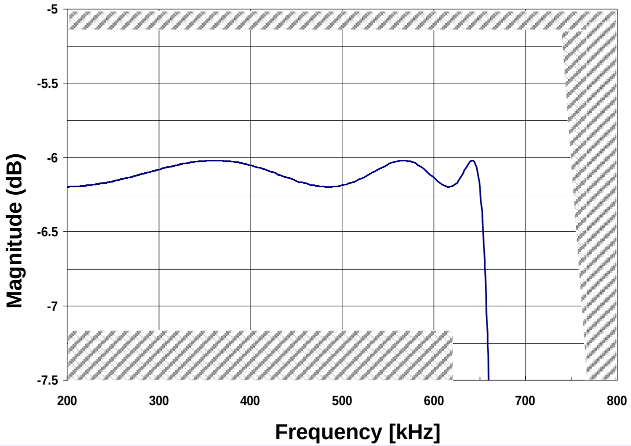
RLC Filter Frequency Response

- •Frequency mask superimposed
- •Frequency response well within spec.



Passband Detail

- Passband well within spec.



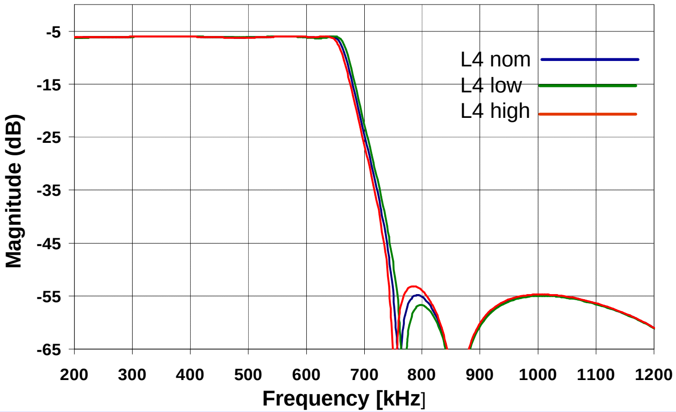
RLC Ladder Filter Sensitivity

- The design has the same spec.s as the previous example implemented with cascaded biquads
- To compare the sensitivity of RLC ladder versus cascaded-biquads:
  - Changed all Ls &Cs by 2% in order to change the pole/zeros by 1% (similar test as for cascaded biquad)
  - Found the frequency response is most sensitive to L4 variations
  - Note that by varying L4 both pole & zeros are varied

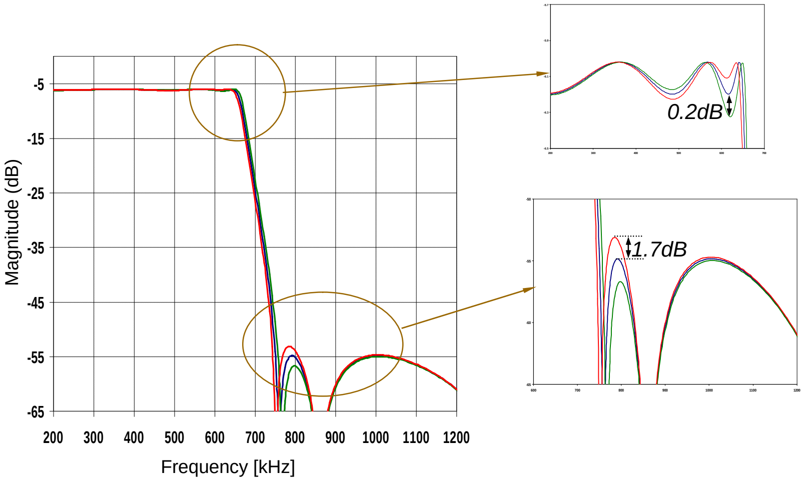
RCL Ladder Filter Sensitivity

Component variation in RLC filter:

- Increase L4 by 2%
- Decrease L4 by 2%



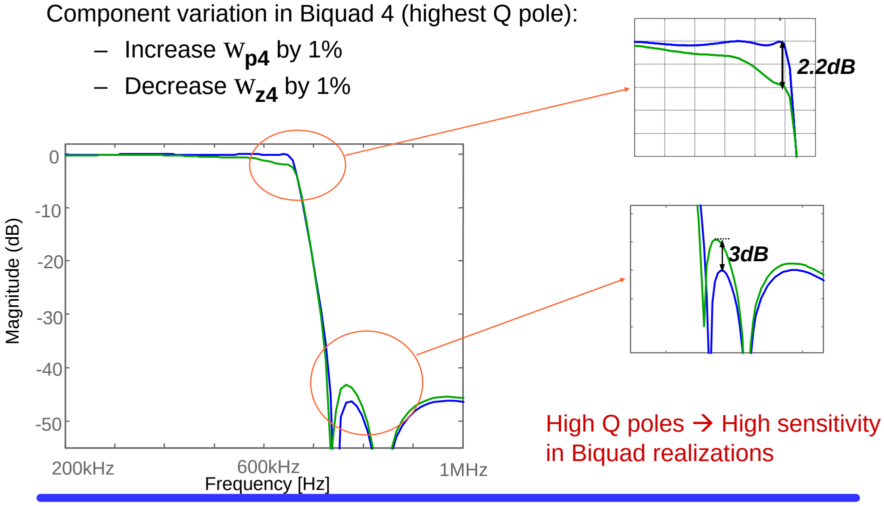
RCL Ladder Filter Sensitivity



Cascade of Biquads Sensitivity

Component variation in Biquad 4 (highest Q pole):

- Increase w_{p4} by 1%
- Decrease w_{z4} by 1%



High Q poles → High sensitivity in Biquad realizations

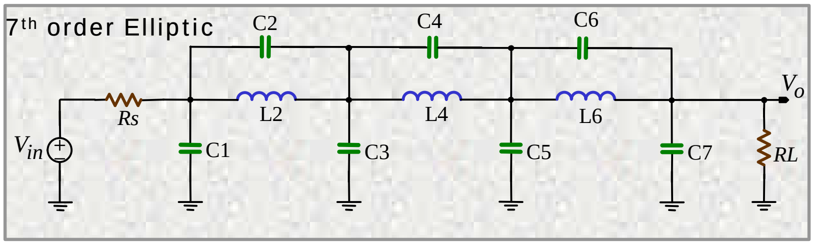
Sensitivity Comparison for Cascaded-Biquads versus RLC Ladder

- 7th Order elliptic filter
  - 1% change in pole & zero pair

	Cascaded Biquad	**RLC Ladder**
Passband deviation	2.2dB (29%)	0.2dB (2%)
Stopband deviation	3dB (40%)	1.7dB (21%)

Doubly terminated LC ladder filters ⇨ Significantly lower sensitivity compared to Cascaded-Biquads particularly within the passband

RLC Ladder Filter Design
Example: CDMA IS95 Receive Filter

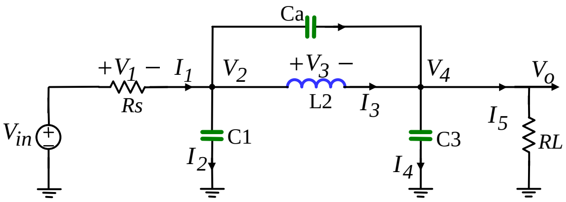


- In lecture 3, designed the integrator based ladder filters without transmission zeros

  → Question:

  How do we implement the transmission zeros in the integrator-based version?

Integrator Based Ladder Filters
How Do We Implement Transmission zeros?



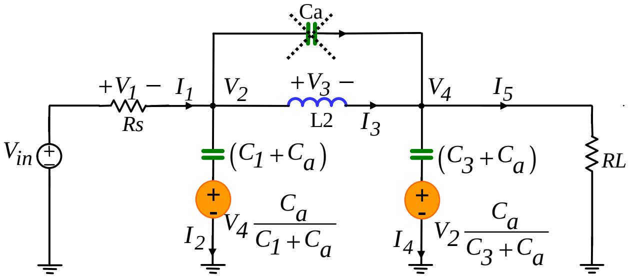
- Use KCL & KVL to derive :

$$V_2 = \frac{I_1 - I_3}{s\left(C_1 + C_a\right)} + V_4 \frac{C_a}{C_1 + C_a}$$

Voltage Controlled Voltage Source!

$$V_4 = \frac{I_3 - I_5}{s\left(C_3 + C_a\right)} + V_2 \frac{C_a}{C_3 + C_a}$$

Integrator Based Ladder Filters
Transmission zeros

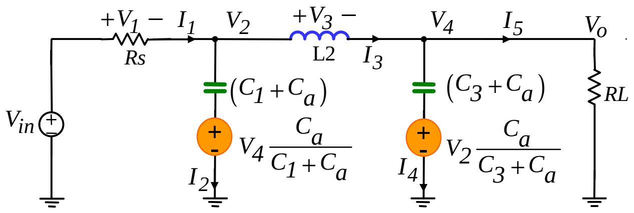


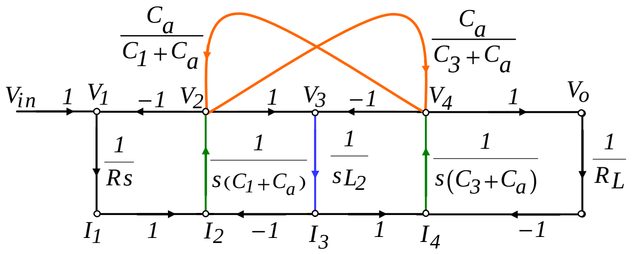
- Replace *shunt capacitor* with *voltage controlled voltage sources*:

$$V_2=\frac{I_1-I_3}{s(C_1+C_a)}+V_4\frac{C_a}{C_1+C_a}$$

$$V_4=\frac{I_3-I_5}{s(C_3+C_a)}+V_2\frac{C_a}{C_3+C_a}$$

LC Ladder Filters
Transmission zeros

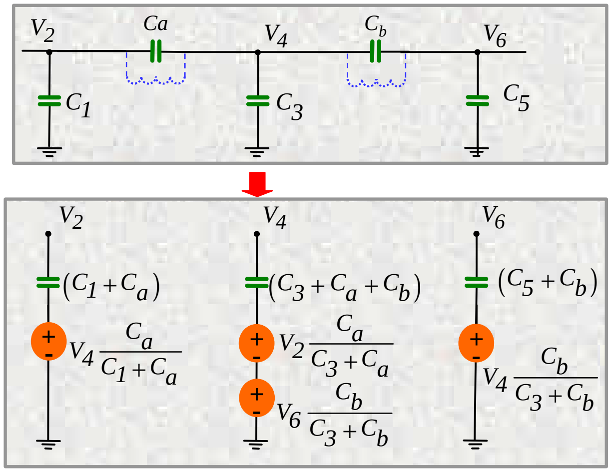




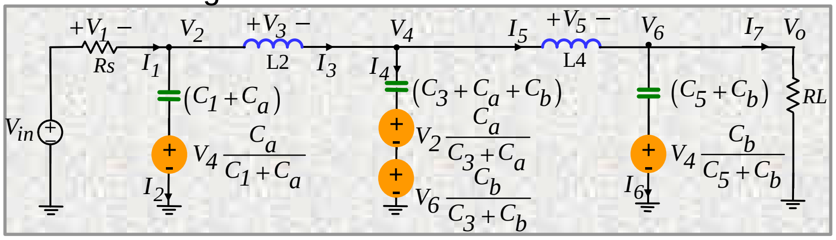
Integrator Based Ladder Filters

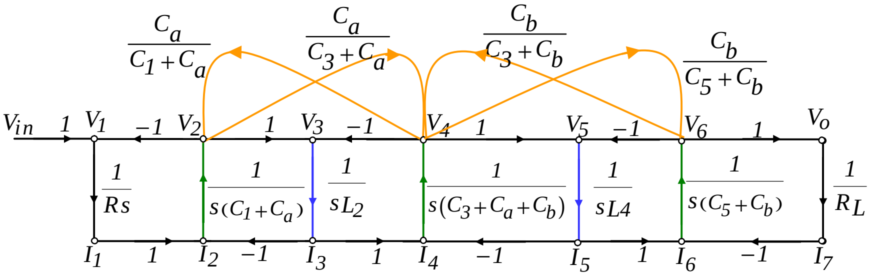
Higher Order Transmission zeros

Convert zero generating Cs in C loops to voltage-controlled voltage sources

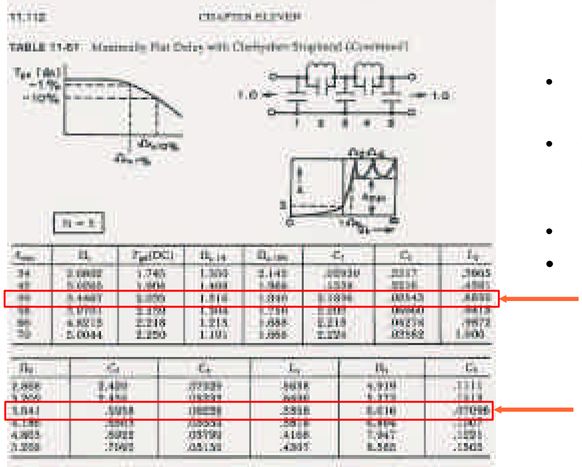


Higher Order Transmission zeros





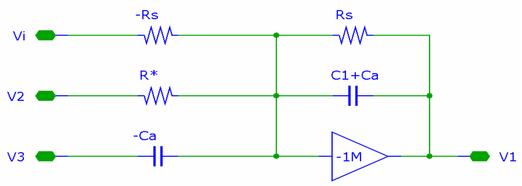
Example:
5^{th} Order Chebyshev II Filter



- 5^{th} order Chebyshev II
- Table from: Williams & Taylor book, p. 11.112
- 50dB stopband attenuation
- f_{-3dB} =10MHz

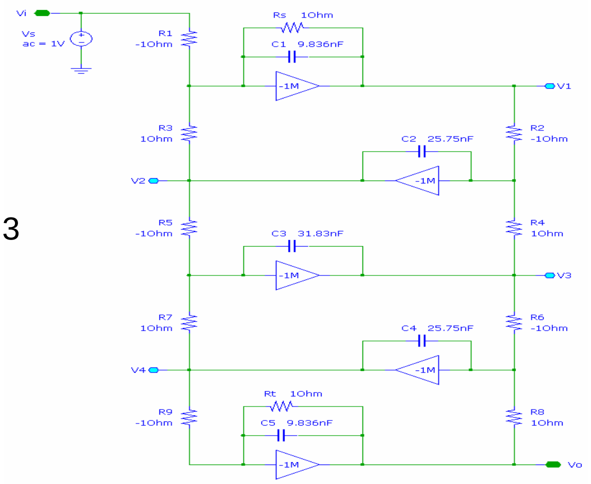
Realization with Integrator

$$V_1 = \frac{1}{s(C_a + C_1)}\left[\frac{V_i - V_1}{R_s} - \frac{V_2}{R^*}\right] + \frac{C_a}{C_a + C_1}V_3$$

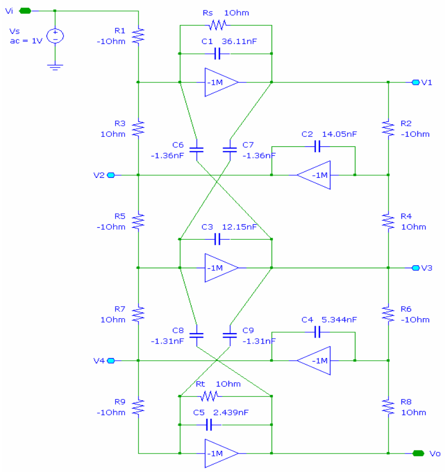


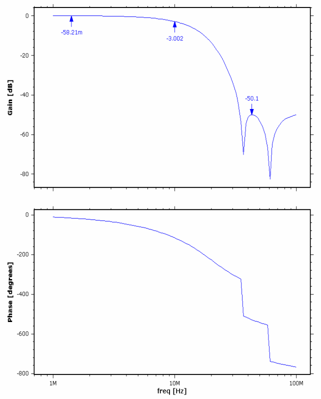
5th Order Butterworth Filter

From:
lecture 3

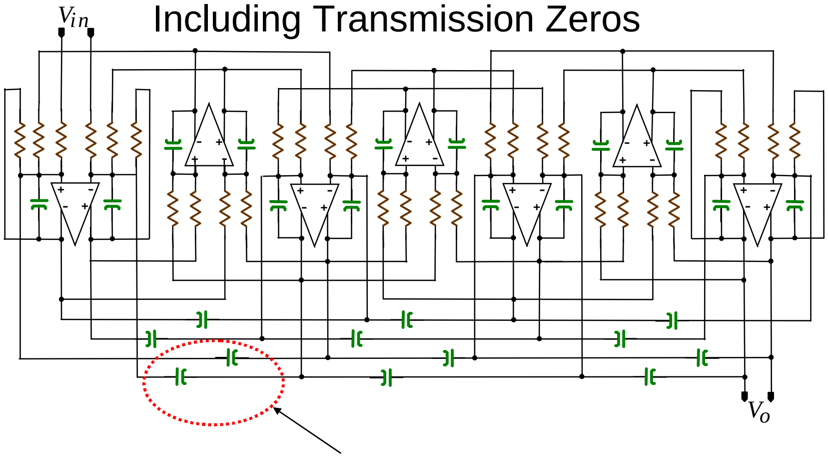


Opamp-RC Simulation



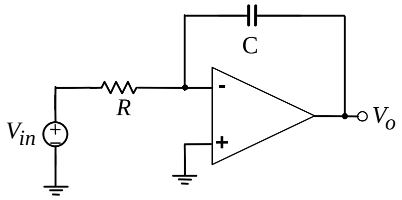


Seventh Order Differential Lowpass Filter Including Transmission Zeros



Transmission zeros implemented with coupling capacitors

Effect of Integrator Non-Idealities on Filter Performance

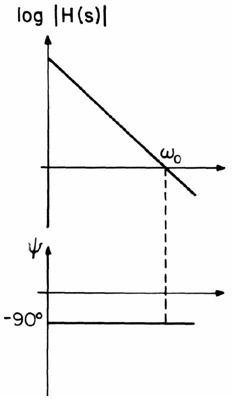


$$opamp\ DC\ \ gain = \infty$$

$$H(s) = \frac{-w_o}{s}$$

$$w_o = 1/RC$$

Ideal Intg.



Ideal Integrator Quality Factor

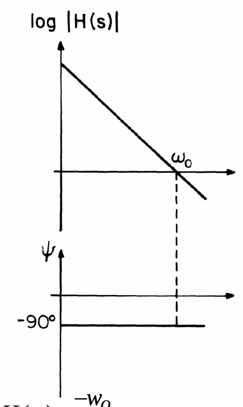
Ideal Intg. $$H(s) = \frac{-w_o}{s} = \frac{-w_o}{jw}$$

Since Q is defined as: $$H(jw) = \frac{1}{R(w) + jX(w)}$$
$$Q = \frac{X(w)}{R(w)}$$

Then: $$\boxed{Q_{ideal}^{intg.} = \infty}$$

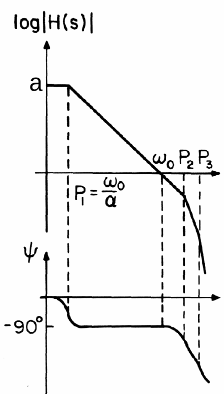
Effect of Integrator Non-Idealities on Filter Performance

Ideal Intg.



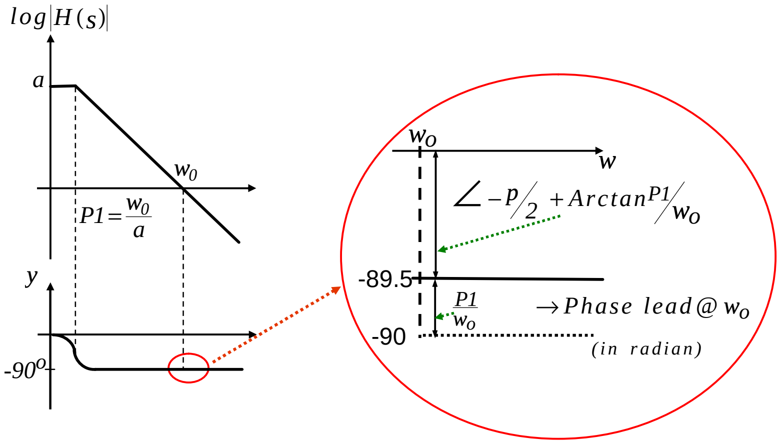
$$H(s) = \frac{-w_o}{s}$$

Real Intg.



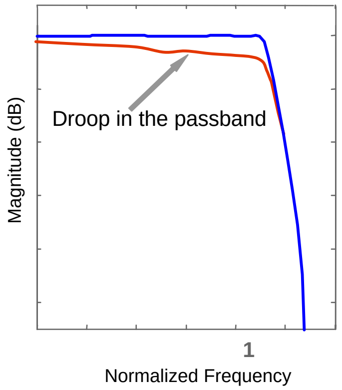
$$H(s) \approx \frac{-a}{\left(1 + s\frac{a}{w_o}\right)\left(1 + \frac{s}{p2}\right)\left(1 + \frac{s}{p3}\right)\ldots}$$

Effect of Integrator Finite DC Gain on Q



Example: $P1/\ \omega_0 = 1/100$

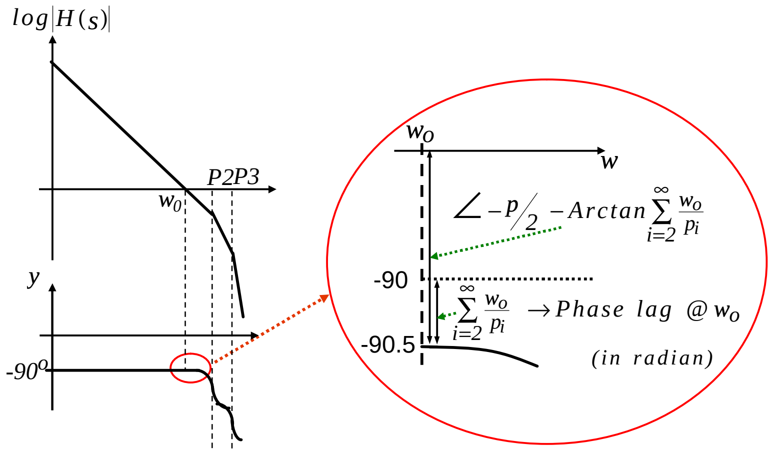
Effect of Integrator Finite DC Gain on Q



- Phase lead @ ω_0

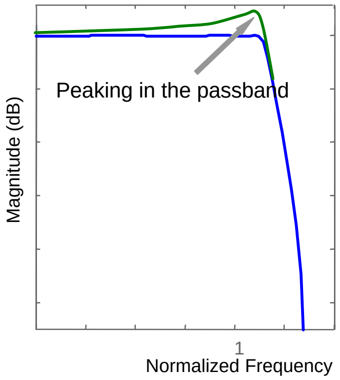
  →Droop in the passband

Effect of Integrator Non-Dominant Poles



Example: ω_0 /P2 =1/100

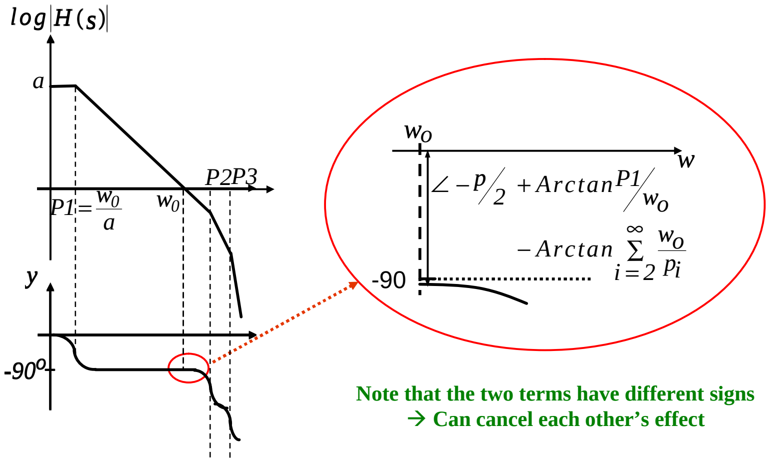
Effect of Integrator Non-Dominant Poles



- Phase lag @ ω_0

→Peaking in the passband
In extreme cases could
result in oscillation!

Effect of Integrator Non-Dominant Poles & Finite DC Gain on Q



Note that the two terms have different signs
→ Can cancel each other's effect

Integrator Quality Factor

Real Intg.

$$H(s) \approx \frac{-a}{\left(1+s\frac{a}{w_o}\right)\left(1+\frac{s}{p2}\right)\left(1+\frac{s}{p3}\right)\ldots}$$

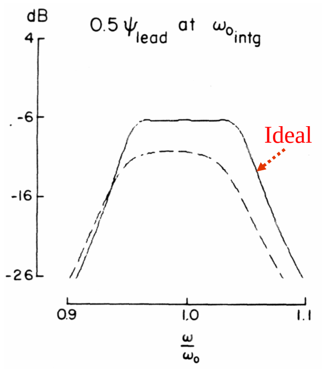
Based on the definition of Q and assuming that:

$$\frac{w_o}{p_{2,3,\ldots\ldots}} \ll 1 \quad \& \quad a \gg 1$$

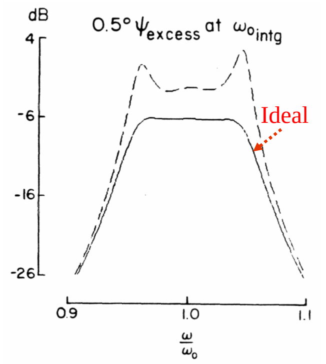
It can be shown that in the vicinity of unity-gain-frequency:

$$Q_{real}^{intg.} \approx \frac{1}{\underbrace{\frac{1}{a}}_{\text{Phase lead @ } w_o} - w_o \underbrace{\sum_{i=2}^{\infty} \frac{1}{p_i}}_{\text{Phase lag @ } w_0}}$$

Example:
Effect of Integrator Finite Q on Bandpass Filter Behavior

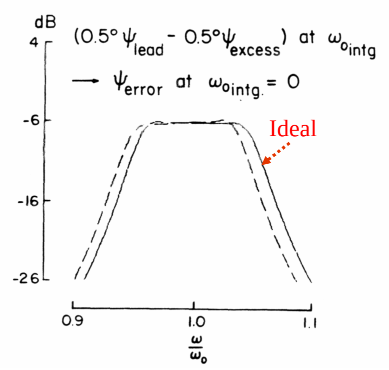


Integrator DC gain=*100*



Integrator *P2 @ 100.*w_o

Example:
Effect of Integrator Finite Q on Filter Behavior



Integrator DC gain=*100* & *P2 @ 100.* w_o

Summary

Effect of Integrator Non-Idealities on Q

$$Q^{intg.}_{ideal} = \infty$$

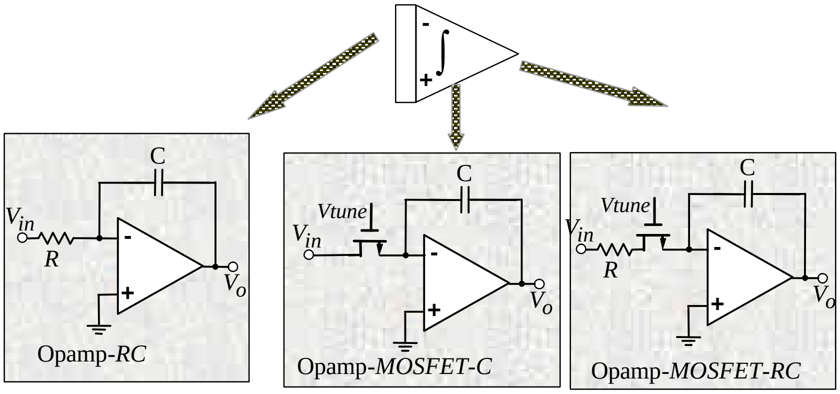
$$Q^{intg.}_{real} \approx \frac{1}{\frac{1}{a} - w_o \sum_{i=2}^{\infty} \frac{1}{p_i}}$$

- Amplifier DC gain reduces the overall Q in the same manner as series/parallel resistance associated with passive elements
- Amplifier poles located above integrator unity-gain frequency enhance the *Q*!
  - If non-dominant poles close to unity-gain freq. → Oscillation
- Depending on the location of unity-gain-frequency, the two terms can cancel each other out!

Various Integrator Based Filters

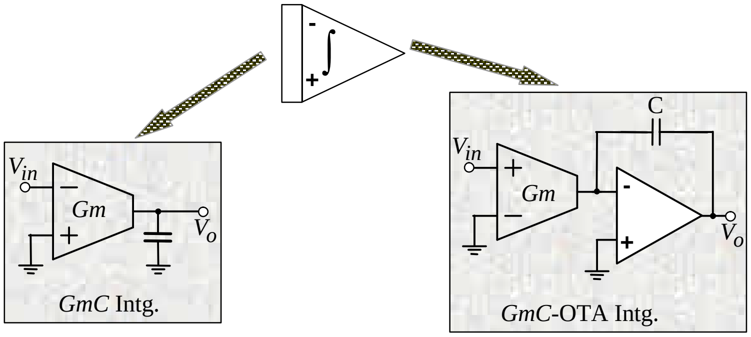
- Continuous Time
  - Resistive element based
    - Opamp-RC
    - Opamp-MOSFET-C
    - Opamp-MOSFET-RC
  - Transconductance (Gm) based
    - Gm-C
    - Opamp-Gm-C
- Sampled Data
  - Switched-capacitor Integrator

Integrator Implementation
Opamp-RC & Opamp-MOSFET-C & Opamp-MOSFET-RC



$$\frac{V_o}{V_{in}} = \frac{-w_o}{s} \quad \text{where} \quad w_o = \frac{1}{R_{eq}C}$$

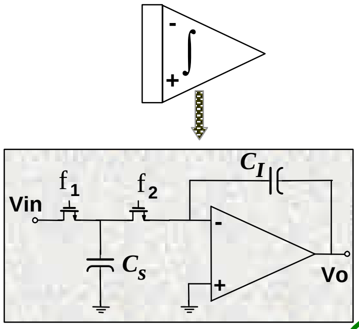
Integrator Implementation
Gm-C & Opamp-Gm-C

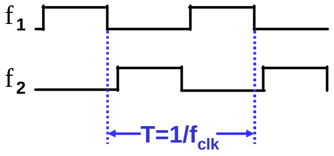


$$\frac{V_o}{V_{in}} = \frac{-w_o}{s} \quad \text{where} \quad w_o = \frac{G_m}{C}$$

Integrator Implementation

Switched-Capacitor





for $f_{signal} \ll f_{clk}$ $V_0 = \frac{f_{clk} \times C_S}{C_I} \int V_{in}\, dt$

$$w_0 = f_{clk} \times \frac{C_S}{C_I}$$

Main advantage: Critical frequency function of ratio of Caps & clock freq.

→ Critical filter frequencies (e.g. LPF -3dB freq.) very accurate