EE C245 Discussion 11/22/10

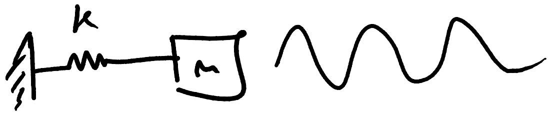
Monday, November 22, 2010
2:18 PM

Today: Monday 11/22/10

★ Review of important concepts:
  - Resonance frequency
  - Electrical force
  - Pull in voltage
  - Electrical Stiffness

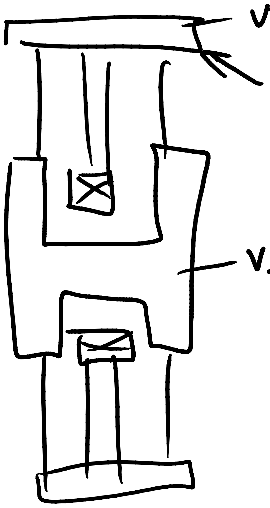
★ How to use SPICE (circuit simulation tool)

Resonant frequency (Natural frequency)



☆ $\omega_0 = \sqrt{\frac{k}{m}} = 2\pi f_0$, stiffness k, mass m

For a folded flexure resonator,



$V_{truss} \sim \frac{1}{2} V_{shuttle} \Rightarrow \frac{KE_t}{M_t} = \frac{KE_s}{4M_s}$

truss has a $\frac{1}{4}$ contribution to equivalent mass because it moves slower than the shuttle.

$V_{shuttle}$

$$M_{eq} = M_1 + \frac{1}{4} M_t + \frac{12}{35} M_b$$

Assumes all beams are the same (L, w, h, etc.)

$$k = k_c = \frac{1}{4} EW\left(\frac{h}{L/2}\right)^3$$
$$= 2k_b = 2 \cdot EW\left(\frac{h}{2L}\right)^3$$

Electrical Force

"p" → polarizing voltage

$F_e = \frac{1}{2} \frac{\partial C}{\partial x} V_p^2$ (For a fixed voltage) V_p ↙

Q: How did we get this?

Recall: The stored energy in a capacitor is given by

$W = \frac{1}{2} C V^2$

$$\left(W = \int_0^{V_F} Q \, dV = \int_0^{V_F} C V \, dV = \frac{1}{2} C V^2 \Big|_{V=0}^{V=V_F} \right)$$

Coulombs ↗ ↖ Joules / coulomb

Intuition:

The work needed to charge a capacitor by an additional volt increases as the capacitor voltage increases.
(Harder to add charge)

So we just proved $W = \frac{1}{2} C V^2$...

Recall: Work = Force × Distance = Energy

$\Rightarrow \Delta W = F \Delta x \Rightarrow F = \frac{\Delta W}{\Delta x} \quad \left(dW = \vec{F} \cdot \underline{d\ell} \right)$

$$\Rightarrow \frac{\partial W}{\partial x}\Big|_{V \text{ is fixed}} = F_e = \frac{1}{2} \frac{\partial C}{\partial x} V^2 \; \star$$

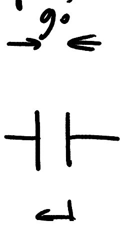
For a folded beam resonator...

If the shuttle has a $\underline{\underline{\text{net}}}$ force on it, it will move (accelerate).

~~Pullin voltage~~

Pull in occurs when the electrical force exceeds the restoring force over the entire displacement range.

Consider a parallel plate capacitor with a variable gap



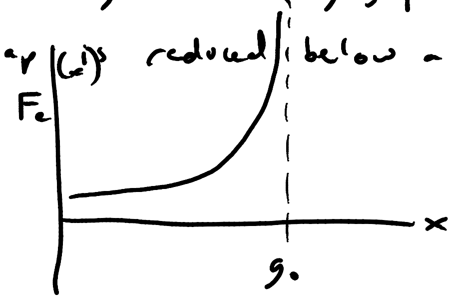
$C(x) = \frac{\varepsilon_0 A}{g_0 - x}$

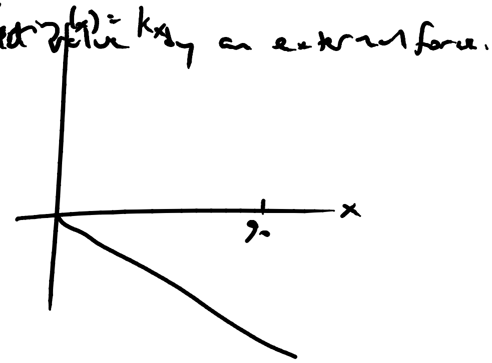
$\Rightarrow \frac{\partial C}{\;} = \frac{\varepsilon_0 A}{\;}$

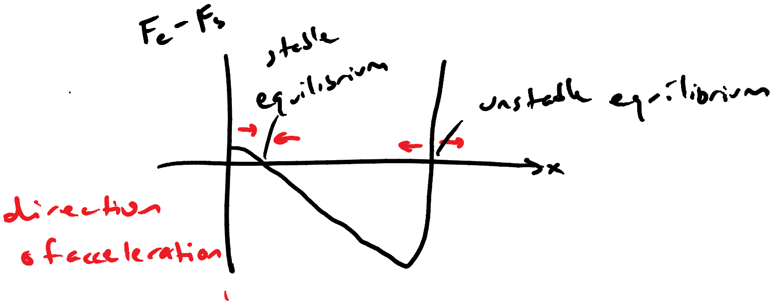
$(g_0-x)^2$ x^{1}

$\Rightarrow$ As $x \rightarrow g_0$, $\frac{\partial C}{\partial x}$ increases without bound.

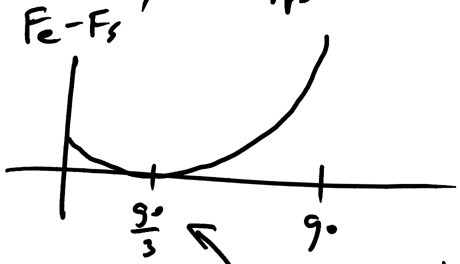
$\Rightarrow$ For a given voltage, pull-in also occurs when the gap is reduced below a threshold value by an external force.







Since $F_e \propto V_p^2$, As V_p increases F_e increases.
eventually at V_{pI}, $F_e > F_s$ for $0 \leq x \leq g_0$



$\Rightarrow$ @ V_{pI}, pull in occurs

Minimum at $g_0/3 \rightarrow V = V_{pI}$

Electrical Stiffness

A force in phase with and proportional to displacement that opposes the spring force. (thus lowering resonant frequency)

Derivation.

$$F = \frac{1}{2}\frac{\partial C}{\partial x}\Delta V^2$$

★ $\frac{\partial C}{\partial x} \cong A + \boxed{Bx}$ — origin of electrical stiffness.

We can get $\frac{\partial C}{\partial x}$ into this form using a Taylor expansion.

Reminder: $f(x-a) \cong f(a) + \overset{\text{1st order term.}}{f'(a)(x-a)} + f''(a)\frac{(x-a)^2}{2!} + f'''(a)\frac{(x-a)^3}{3!} + \dots$

Often we pick $\overset{\text{center point}}{a} = 0$.

$$F = \frac{1}{2}(A+Bx)\Delta V^2 = \frac{1}{2}A\Delta V^2 + \overbrace{\frac{1}{2}B\Delta V^2}^{k_e} x \; ☆ \qquad x << g_0$$

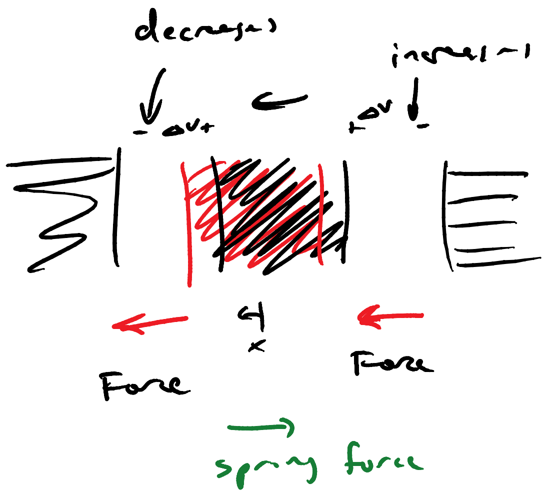
☆ The force acts to increase x (because B>0)

⇒ Opposes the spring force!

Therefore $k_{eq} = \underset{\text{mechanical}}{k_m} - \underset{\text{electrical.}}{k_e}$

☆ And $f_0' = f_0\sqrt{1-\frac{k_e}{k_m}}$

(resonant frequency shift.)

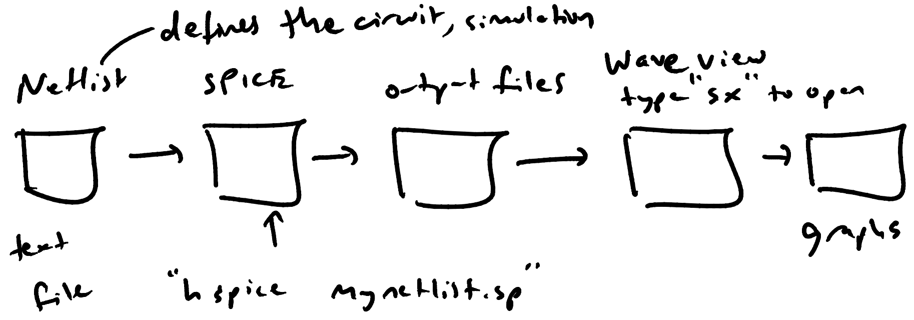


⇒ The point: electrical stiffnesses add.

SPICE

Circuit simulation software.

☆ Developed @ UC Berkeley!!



Tutorial and example netlist will be put online.

A short SPICE problem is on HW7.