Discussion 10/11

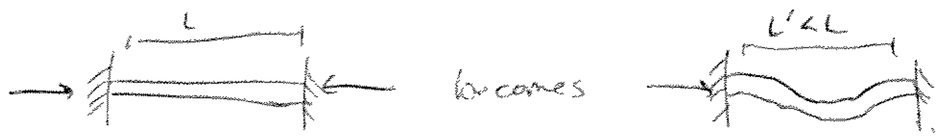
(1) Buckling.

$$\sigma_{crit} = -\frac{\pi^2}{3}\frac{Eh^2}{L^2}.$$

Longer beams buckle easier.
Note: h should be the smaller of the cross-sectional dimensions.

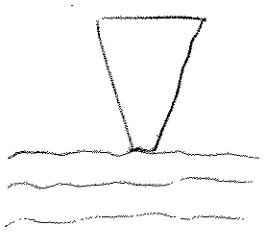
Buckling occurs in fixed-fixed compressive beams.

When compression exceeds this buckling yield strength the beam deforms.



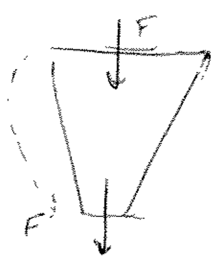
By buckling, the beam relieves compressive stress by reducing its effective length.

For microneedles: Structure needs to withstand compressive stress so that it can pierce the skin!



stratum corneum 330 MPa yield
epidermis 1.7 MPa yield

Consider force F on microneedle:



If we assume the microneedle is rigid, the force moves through the needle to the tip. ⟷ Thus we impart a large stress to the skin $\frac{F}{A_{small}} > 330\ \text{MPa}$.

Since the base of the needle is large, the buckling stress is also large.

Given dimensions $L = 100\,\mu m$, $H_{base} = 10\,\mu m$, $H_{tip} = 1\,\mu m$; $(F = 1\,mN)$

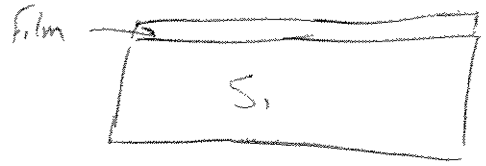
stress on skin $= \frac{F}{A_{tip}} = \frac{1\,mN}{(1\,\mu m)^2} = 1\ GPa$ exceeds skin yield, will pierce

buckling stress $= \frac{\pi^2}{3}\frac{Eh^2}{L^2} \approx \frac{3E}{100} \approx 4.5\ GPa$ we can stay below this!

Discussion 10/11

② Thermal Mismatch

Have a deposited thin film. at some high temp.



As we cool, Si shrinks at will, film cannot shrink at will.

If film shrink rate is different from Si, stress will result.

Heuristics: $\alpha_{poly} = 2.7$. since $\alpha_{poly} > \alpha_{Si}$, film will be in tensile stress.

$\alpha_{oxide} = 0.7$. since $\alpha_{ox} < \alpha_{Si}$, film will be in compressive stress.

$\alpha_{Si} = 2.3$.

Thermal mismatch strain: $\varepsilon = \alpha \Delta T$. α is in units of $1/K$. or $\frac{\text{strain}}{K}$.

most conveniently in $\frac{\mu\text{strain}}{K}$ or 10^{-6} st/K.

So $\varepsilon = (\alpha_{Si} - \alpha_{film})(T_{final} - T_{initial})$.

To find stress, use biaxial modulus. $\sigma_{Film} = \varepsilon_{Film} \cdot \frac{E}{1-\nu}$.

Biggest issue with mismatch problems is the sign.

Use above heuristics + general knowledge (nitride/poly tensile, oxide compressive)

or use formula above. $\alpha_{net} = \alpha_{Si} - \alpha_{Film}$ and $\Delta T = T_{final} - T_{initial}$

room temp

deposition temp.

ΔT will be negative.

③ Keys to watch for in future lectures:

Small angle beam bending: $\boxed{\frac{d^2 w}{dx^2} = \frac{-M}{EI}}$

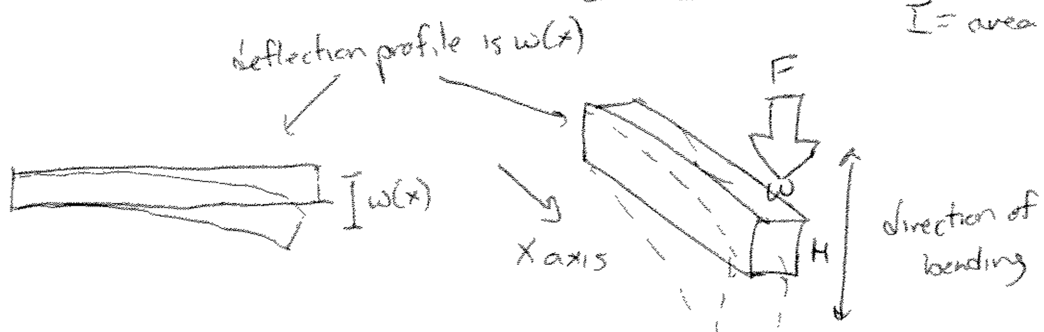
w = deflection

M = moment

I = area of inertia. $I = \frac{1}{12} W H^3$

for rectangular beams.

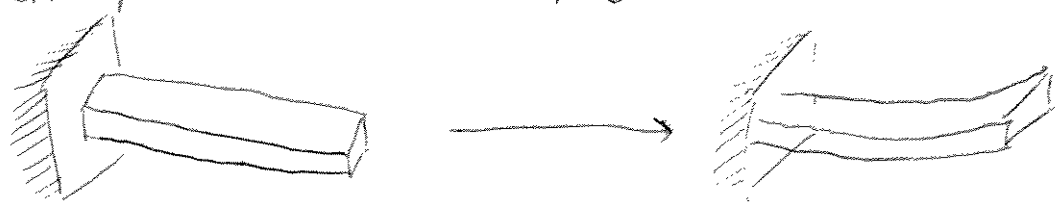
H always in direction of bending.



Discussion 10/11

(X) Strain Gradients (intro)
(4)

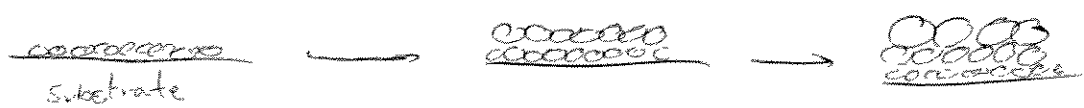
Common phenomenon in MEMS is warping. How does it occur?



During deposition (i.e. LPCVD) grain size is variable.

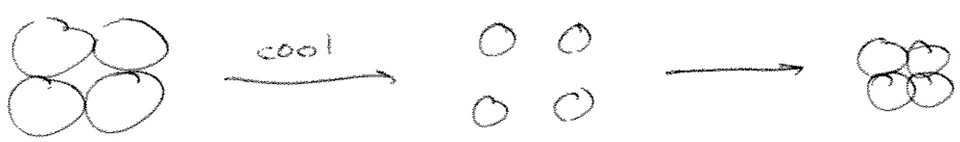
For poly Si, grains start out small and uniform, but grow as time passes.

(Note. not same for all films!)



So when the film cools from the deposition temperature, grains shrink.
As they shrink, the gaps between them grow. As the gaps grow, new grains pack in.

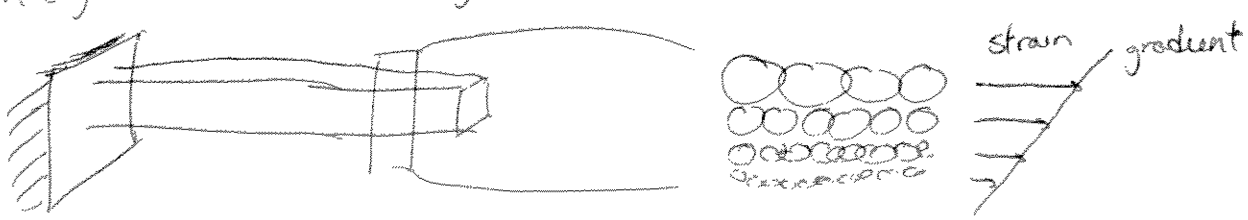
Larger grains move together:



Net result is a shrinking gap.

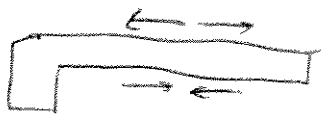
Since grain sizes are non-uniform, we get variable # of gaps closing per layer.

This gives rise to a strain gradient.



Discussion 10/11

(1) Strain Gradient Intuition.



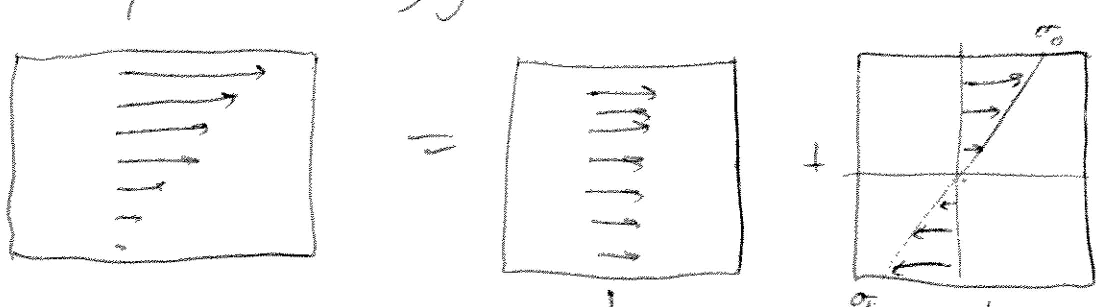
Consider a cantilever under tensile stress at the top and compressive stress at the bottom. Which way will it deflect?

Think of rubber bands. If a rubber band is in tensile stress, it's because you're pulling on it. If you let it go it will shrink back.

Similarly a cantilever once released will shrink. The part of the cantilever under compressive stress will expand by the same argument.

Thus it will deflect up.

In general any strain gradient can be split into a uniaxial component and a symmetric bending gradient. Observe:



However, the uniaxial component does affect the effective stiffness of the beam!

↓ uniaxial tension and compression described by $\sigma = E\varepsilon$.

↓ no effect on bending!

↓ symmetric bending gradient with well defined moment

$$M = \frac{1}{6}\sigma_0 w H^2$$

↓ Can easily calculate tip deflection

$$\Delta z = \frac{1}{2}\Gamma L^2$$

where $\Gamma = \frac{2\varepsilon_0}{H}$ (strain gradient)

↓ no effect on axial loading!