MEMS-Based Tuning Fork Gyroscope

- Drive and sense axes must be stable or at least track one another to avoid output drift.

**Problem:** if drive frequency changes relative to sense frequency, output changes ⇒ bias drift.

**Need:** small or matched drive and sense axis temperature coefficients to suppress drift.

Mode Matching for Higher Resolution

- For higher resolution, can try to match drive and sense axis resonance frequencies and benefit from Q amplification.

**Problem:** mismatch between drive and sense frequencies ⇒ even larger drift!

**Need:** small or matched drive and sense axis temperature coefficients to make this work.

Issue: Zero Rate Bias Error

- Imbalances in the system can lead to zero rate bias error.

**Mass imbalance** ⇒ off-axis motion of the proof mass.

**Output signal** in phase with the Coriolis acceleration.

**Quadrature output signal** that can be confused with the Coriolis acceleration.

Nuclear Magnetic Res. Gyroscope

- The ultimate in miniaturized spinning gyroscopes?

**Solution:** Spin polarize Xe¹²⁹ nuclei by first polarizing e⁻ of Rb⁸⁷ (a la CSAC), then allowing spin exchange.

**Better if this is a noble gas nucleus (rather than e⁻), since nuclei are heavier ⇒ less susceptible to B field.**

**Challenge:** suppressing the effects of B field.
MEMS-Based Tuning Fork Gyroscope

Drive Voltage Signal

(-) Sense Output Current

(+) Sense Output Current

Drive Oscillation Sustaining Amplifier

Differential TransR Sense Amplifier

MEMS-Based Tuning Fork Gyroscope

[Zaman, Ayazi, et al, MEMS'06]

Determining Sensor Resolution

Drive Axis Equivalent Circuit

* Generates drive displacement velocity \( x_d \) to which the Coriolis force is proportional

\[ \eta \cdot i \]

\[ C_{11} \]

\[ C_{12} \]

\[ C_{22} \]

\[ V_{RF} \]

\[ VCO \]

\[ \text{Digital PLL} \]

\[ \text{Differential TransR Sense Amplifier} \]

\[ \text{To Sense Amplifier (for synchronization)} \]

\[ \text{Drive Voltage Signal} \]

\[ 180^\circ \]

\[ 180^\circ \]
**Drive-to-Sense Transfer Function**

\[
\begin{align*}
\dot{x}_d &= \omega f x_d \\
\dot{x}_s &= \omega f x_s
\end{align*}
\]

**Gyro Readout Equivalent Circuit** (for a single tine)

\[
\begin{align*}
\bar{F}_e &= m\ddot{\alpha}_c = m \cdot (2\dddot{x}_d \times \Omega) \\
F_c &= \frac{c x f_{r}}{r_x} \\
i_f^2 &= \eta_i^2 + i_{av}^2 + i_{ai}^2 \\
V_0 &= \left(\frac{R_f}{r_c}\right)^2 \frac{i_{f}}{r_x}
\end{align*}
\]

**Minimum Detectable Signal (MDS)**

*Minimum Detectable Signal (MDS):* Input signal level when the signal-to-noise ratio (SNR) is equal to unity

\[
\begin{align*}
\text{Sensed Signal} &\rightarrow \text{Sensor} \\
\text{Sensor Scale Factor} &\rightarrow \text{Circuit Gain} \\
\text{Sensor Noise} &\rightarrow \text{Circuit Output Noise} \\
\text{Signal Conditioning Circuit} &\rightarrow \text{Output}
\end{align*}
\]

*Includes desired output plus noise

*The sensor scale factor is governed by the sensor type

*The effect of noise is best determined via analysis of the equivalent circuit for the system

**Move Noise Sources to a Common Point**

*Move noise sources so that all sum at the input to the amplifier circuit (i.e., at the output of the sense element)

*Then, can compare the output of the sensed signal directly to the noise at this node to get the MDS

\[
\begin{align*}
\text{Sensed Signal} &\rightarrow \text{Sensor} \\
\text{Sensor Scale Factor} &\rightarrow \text{Circuit Gain} \\
\text{Sensor Noise} &\rightarrow \text{Circuit Input-Reflected Noise} \\
\text{Signal Conditioning Circuit} &\rightarrow \text{Output}
\end{align*}
\]

*Includes desired output plus noise
Gyro Readout Equivalent Circuit
(for a single tine)

\[ F_c = m \ddot{a}_c = m \cdot (2 \dddot{x}_d \times \dddot{\Omega}) \]

Gyro Sense Element
Output Circuit

Noise Sources

\[ I_x, C_x, F_{i_{sa}} \]
\[ i_a, V_{i_{sa}} \]

\[ \eta : 1 \]

\[ R_f \]

Noiseless

Signal Conditioning Circuit
(Transresistance Amplifier)

- Easiest to analyze if all noise sources are summed at a common node

Noise Sources

\[ \text{Gyro Sense Element} \]
\[ \text{Output Circuit} \]
\[ \text{Signal Conditioning Circuit} \]
\[ \text{(Transresistance Amplifier)} \]

- Here, \( v_{eq}^2 \) and \( i_{eq}^2 \) are equivalent input-referred voltage and current noise sources

Noise

- Noise: Random fluctuation of a given parameter \( I(t) \)
- In addition, a noise waveform has a zero average value
- We can't handle noise at instantaneous times
- But we can handle some of the averaged effects of random fluctuations by giving noise a power spectral density representation
- Thus, represent noise by its mean-square value:

\[ \bar{I}^2 = (I - I_D)^2 = \lim_{T \to \infty} \frac{1}{T} \int_0^T (I - I_D)^2 \, dt \]
**Noise Spectral Density**

- We can plot the spectral density of this mean-square value:

\[
\frac{1}{\Delta f} [\text{units}^2/\text{Hz}]
\]

One-sided spectral density
- used in circuits
- measured by spectrum analyzers

Two-sided spectral density (1/2 the one-sided)

\[
\frac{1}{\pi} \int_{-\infty}^{\infty} S_i(\omega) d\omega = \int_{0}^{\infty} S_i(\omega) d\omega
\]

Integrated mean-square noise spectral density over all frequencies (area under the curve)

**Circuit Noise Calculations**

- **Deterministic:**
  \[ v_o(j\omega) = H(j\omega)v_i(j\omega) \]

- **Random:**
  \[ S_o(\omega) = |H(j\omega)H^*(j\omega)|S_i(\omega) = |H(j\omega)|^2 S_i(\omega) \]

**Systematic Noise Calculation Procedure**

- Assume noise sources are uncorrelated
  1. For \( i_n \), replace w/ a deterministic source of value
  \[ i_n = \sqrt{\frac{i_{n1}}{\Delta f}} (1 \text{ Hz}) \]

**Handling Noise Deterministically**

- Can do this for noise in a tiny bandwidth (e.g., 1 Hz)

\[ \frac{v_n^2}{\Delta f} = S_i(f) \rightarrow v_n = \sqrt{S_i(f)} \cdot B \]

Can approximate this by a sinusoidal voltage generator (especially for small B, say 1 Hz)

\[ S_o(j\omega) = \int_{0}^{\infty} S_i(\omega) d\omega \]

Root mean square amplitudes
Systematic Noise Calculation Procedure

1. Calculate \( v_{on1}(\omega) = i_{n1}(\omega)H(j\omega) \) (treating it like a deterministic signal)
2. Determine \( v_{on1} = i_{n1}^2 |H(j\omega)|^2 \)
3. Repeat for each noise source: \( i_{n2}^2, i_{n3}^2 \)
4. Add noise power (mean square values)
   \[
   v_{onTOT}^2 = v_{on1}^2 + v_{on2}^2 + v_{on3}^2 + \cdots
   \]
   \[
   V_{onTOT} = \sqrt{v_{on1}^2 + v_{on2}^2 + v_{on3}^2 + \cdots}
   \]
   Total rms value

Example: Gyro MDS Calculation

\[ F_{c} = m\alpha_{c} = m \cdot (2\dot{x}_d \times \Omega) \]

\[ \eta_{r}:1 \]

\[ i_{e} = i_{eq} \]

\[ R_{f} \]

Noiseless

\[ v_{eq} \]

\[ v_{0} \]

\[ x_{d} \]

\[ C_{p} \]

* The gyro sense presents a large effective source impedance
  * Currents are the important variable; voltages are “opened” out
  * Must compare \( i_{e} \) with the total current noise \( i_{eqTOT} \) going into the amplifier circuit

Example: Gyro MDS Calculation (cont)

\[ F_{c} = m\alpha_{c} = m \cdot (2\dot{x}_d \times \Omega) \]

\[ \eta_{r}:1 \]

\[ i_{e} = i_{eq} \]

\[ R_{f} \]

Noiseless

\[ v_{eq} \]

\[ x_{d} \]

\[ C_{p} \]

\[ i_{eq} \]

\[ v_{0} \]

\[ \dot{x}_{d} \]

\[ \Omega \]

\[ \Theta(\omega) \]

* First, find the rotation to \( i_{e} \) transfer function:
  \[
  \dot{x}_{d} = \frac{\omega_{\Omega}Q}{k_{s}} \Theta(\omega) \]
Example: Gyro MDS Calculation (cont)

Now, find the $i_{eqTOT}$ entering the amplifier input:

$$i_{eqTOT} = i_x + i_{vT} = \frac{e_{2x}}{r_x} + \frac{e_{rvT}}{r_v}$$

Brownian motion noise of the sensor element is determined entirely by the noise in $e_x$, $e_{vT}$.

Easiest to convert to an all-electrical equiv ckt.

Example: Gyro MDS Calculation (cont)

In amplifier input:

$$\frac{v_{CUT}}{R_f}$$

Learning to get these from EE245.

Example: Gyro MDS Calculation (cont)

### LF356 Op Amp Data Sheet

**LF155/LF156/LF256/LF257/LF355/LF356/LF357 JFET Input Operational Amplifiers**

- **General Description**
  - JFET input operational amplifiers for incorporating high-matched, high-voltage JFETs in high-speed, high-input impedance, low-drift, low-power, low-noise operational amplifiers. They feature low input bias current and offset voltage, low drift, and low noise.

- **Features**
  - High-speed, high-input impedance operational amplifiers
  - Low input offset voltage
  - Low input bias current
  - Low drift

- **Applications**
  - High-speed, high-input impedance operational amplifiers

- **Uncommon Features**
  - 0.01 pF input capacitance
  - 0.8 pF output capacitance
  - 0.05 pF input capacitance

**Specifications**

- **Input Offset Voltage**
  - LF155: 0.1 mV
  - LF156: 0.1 mV
  - LF256: 0.1 mV
  - LF257: 0.1 mV
  - LF355: 0.1 mV
  - LF356: 0.1 mV
  - LF357: 0.1 mV

- **Input Bias Current**
  - LF155: 10 nA
  - LF156: 10 nA
  - LF256: 10 nA
  - LF257: 10 nA
  - LF355: 10 nA
  - LF356: 10 nA
  - LF357: 10 nA

- **Input Current**
  - LF155: 0.1 nA
  - LF156: 0.1 nA
  - LF256: 0.1 nA
  - LF257: 0.1 nA
  - LF355: 0.1 nA
  - LF356: 0.1 nA
  - LF357: 0.1 nA

- **Gain bandwidth product**
  - LF155: 5 MHz
  - LF156: 5 MHz
  - LF256: 5 MHz
  - LF257: 5 MHz
  - LF355: 5 MHz
  - LF356: 5 MHz
  - LF357: 5 MHz

**LF356 Op Amp Data Sheet**

EE 245: Introduction to MEMS Design  Lecture 27: Gyros, Noise & MDS
Example ARW Calculation

- **Example Design:**
  - **Sensor Element:**
    - \( m = (100\mu m)(100\mu m)(20\mu m)(2300kg/m^3) = 4.6x10^{-10}kg \)
    - \( \omega_s = 2\pi(15kHz) \)
    - \( \omega_d = 2\pi(10kHz) \)
    - \( k_s = \frac{\omega_s}{2}m = 4.09 N/m \)
    - \( \Delta x = 20 \mu m \)
    - \( Q_s = 50,000 \)
    - \( V_p = 5V \)
    - \( h = 20 \mu m \)
    - \( d = 1 \mu m \)
  - **Sensing Circuitry:**
    - \( R_f = 100k\Omega \)
    - \( i_{io} = 0.01 pA/\sqrt{Hz} \)
    - \( \nu_{io} = 12 nV/\sqrt{Hz} \)

Example ARW Calculation (cont)

- **Sensing Circuitry:**
  - \( R_f = 100k\Omega \)
  - \( i_{io} = 0.01 pA/\sqrt{Hz} \)
  - \( \nu_{io} = 12 nV/\sqrt{Hz} \)

What if \( \omega_d = \omega_s \)?

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  - **Sensor Element:**
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Example ARW Calculation (cont)

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