

EE243 Homework #2 Solutions

Problem 1

When the trench is completely filled with oxide, the oxide width will be 1.85 μm :

Soln : 0.46 μm Si gives 1 μm SiO₂. Let width of SiO₂ trench be x μm then $\frac{x-1}{0.46} = x \Rightarrow x = 1.85 \mu\text{m}$

The problem now becomes to solve time t such that $x_{\text{ox}}(110)$ heavily doped + $x_{\text{ox}}(100) = 1.85 \mu\text{m}$

with $x_{\text{ox}}(t) = \frac{1}{2}A \left[\left\{ 1 + \frac{4Bt}{A^2} \right\}^{1/2} - 1 \right]$ (can ignore the 20nm initial oxide correction for D-G model since the two oxide thickness of interest are much larger than that)

From Table 6.2 of PDG, steam oxidation at 1100C,

$$B(100) = 3.86 \times 10^2 \times \exp[-0.78 / (8.62 \times 10^{-5} \times 1373)] = 0.53 \mu\text{m}^2/\text{hr}$$

$$B/A(100) = (1.63 \times 10^8) / (1.68) \times \exp[-2.05 / (8.62 \times 10^{-5} \times 1373)] = 2.91 \mu\text{m}/\text{hr}$$

$$\text{Therefore } A(100) = 0.182 \mu\text{m}$$

$$B(110) \text{ heavily doped} = B(100) = 0.53 \mu\text{m}^2/\text{hr}$$

$$B/A(110) \text{ heavily doped} = 4 \times 1.2 \cdot B/A(100) = 4.8 \cdot B/A(100) = 13.98 \mu\text{m}/\text{hr}$$

$$\text{Therefore } A(110) = 0.038 \mu\text{m}$$

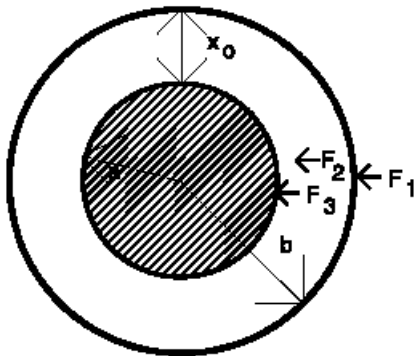
By iteration, oxidation time t is found to be 1.8 hr with $x_{\text{ox}}(100) = 0.89 \mu\text{m}$ and $x_{\text{ox}}(110) = 0.96 \mu\text{m}$

Note: Since the expected x_{ox} is around 1 μm which is $\gg A/2$ (see $\frac{dx_{\text{ox}}}{dt} = \frac{B}{A+2x_{\text{ox}}}$), I am tempted to

approximate oxidation is already in parabolic regime. With $\sqrt{B(100)t} + \sqrt{B(110)t} = 1.85 \mu\text{m}$ or $\sqrt{Bt} = 0.925 \mu\text{m}$, I end up with $t = 1.6 \text{ hr}$. This shows interfacial reaction difference is not a major contribution for the oxide thickness of interest in this particular problem.

Problem 2 2-D and 3-D Deal-Grove Model

(a) Oxidation of a Si cylinder



$$F_1 = h(C^* - C_o)$$

$$F_3 = k_s C_i$$

$$F_2 = D \frac{\partial C}{\partial r}$$

Fluxes continuous at boundaries: $F_1 = F_2$ at $b(t)$ and $F_2 = F_3$ at $a(t)$

From the diffusion equation inside the oxide:

$$D \nabla^2 C = \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial C}{\partial r} \right) = \frac{\partial C}{\partial t} \approx 0 \rightarrow r \frac{\partial C}{\partial r} = K = \text{constant}$$

This quasi-steady state condition shows that the mass transport (flux \times area) is conserved for **all** radius r inside the oxide.

$$\therefore \frac{\partial C}{\partial r} = \frac{K}{r}$$

$$\text{Using } 2\pi r F_2 \text{ (at } a(t)) = 2\pi a(t) F_3 \rightarrow \int_{C_i}^{C_o} dC = \frac{k_s C_i a(t)}{D} \int_{a(t)}^{b(t)} \frac{dr}{r}$$

$$\text{or } C_o - C_i = \frac{k_s C_i a(t)}{D} \ln \left[\frac{b(t)}{a(t)} \right] \quad [1]$$

$$\text{Similarly, using } 2\pi b(t) F_2 \text{ (at } b(t)) = 2\pi b(t) F_1 = 2\pi a(t) F_3 \rightarrow C_o = \frac{b(t) h C_s^* - k_s C_i a(t)}{b(t) h} \quad [2]$$

Combining Equations [1] and [2] gives :

$$N \frac{dx_o}{dt} = k_s C_i = \frac{C^*}{\frac{1}{k_s} + \frac{1}{h b(t)} + \frac{1}{D} a(t) \log \left[\frac{b(t)}{a(t)} \right]}$$

(b) Oxidation of a Si sphere

$$F_1 = h (C^* - C_o)$$

$$F_3 = k_s C_i$$

$$F_2 = D \frac{\partial C}{\partial r}$$

Fluxes continuous at boundaries: $F_1 = F_2$ at $b(t)$ and $F_2 = F_3$ at $a(t)$

From the diffusion equation inside the oxide:

$$D \nabla^2 C = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial C}{\partial r} \right) = \frac{\partial C}{\partial t} \approx 0 \rightarrow r^2 \frac{\partial C}{\partial r} = K = \text{constant}$$

This quasi-steady state condition shows that mass transport (flux \times area) is conserved for **all** radius r inside the oxide.

$$\therefore \frac{\partial C}{\partial r} = \frac{K}{r^2}$$

$$\text{Using } 4\pi r^2 F_2 \text{ (at } a(t)) = 4\pi [a(t)]^2 F_3 \rightarrow \int_{C_i}^{C_o} dC = \frac{k_s C_i [a(t)]^2}{D} \int_{a(t)}^{b(t)} \frac{dr}{r^2}$$

$$\text{or } C_o - C_i = \frac{k_s C_i [a(t)]^2}{D} \left(\frac{1}{a(t)} - \frac{1}{b(t)} \right) \quad [1]$$

Similarly, using $4\pi b^2(t) F_2 \text{ (at } b(t)) = 4\pi b^2(t) F_1 = 4\pi a^2(t) F_3$

$$\rightarrow C_o = \frac{[b(t)]^2 h C_s^* - k_s C_i [a(t)]^2}{b^2(t) h} \quad [2]$$

Combining Equations [1] and [2] gives :

$$N \frac{dx_o}{dt} = k_s C_i = \frac{k_s C^*}{1 + \frac{a^2 k_s}{b^2 h} + \frac{k_s a^2}{D} \left(\frac{1}{a(t)} - \frac{1}{b(t)} \right)}$$

The following show the summary of 1-D(planar) , 2-D (cylindrical) and 3-D (spherical) oxidation fluxes

1-D	$h (C^* - C_o) = \frac{D}{x_o} (C_o - C_i) = k_s C_i$
2-D	$bh (C^* - C_o) = \frac{D}{\ln(b/a)} (C_o - C_i) = a k_s C_i$
3-D	$b^2 h (C^* - C_o) = \frac{D}{(1/a - 1/b)} (C_o - C_i) = a^2 k_s C_i$

Problem 3

Very thin oxide:

Make an estimate for the additional oxide grown by using the following perturbational approach. Assume 1050 degrees centigrade, atmospheric pressure, no HCL, and no doping. Use the values for the thin oxide term in equation 6.37 on pp. 327 PDG.

$$\frac{dx_o}{dt} = \frac{B}{2x_o + A} + C \exp\left(-\frac{x_o}{L}\right) \quad (6.37)$$

where $C = C^0 \exp\left(-\frac{E_A}{kT}\right)$

and $C^0 \approx 3.6 \times 10^8 \mu\text{m hr}^{-1}$, $E_A \approx 2.35 \text{ eV}$, and $L \approx 7 \text{ nm}$. All these numbers apply to

- a) First assume $k=0$ in the thin oxide term and solve for the oxide thickness versus time: (k is the additional term for B/A term in Suprem, see slide 9, Lec #5 notes) By dropping this term, $x_o(t)$ is simply $(B/A)*t$ [linear with t ; The Deal –Grove Model]

$$x_o(t) = 0.826 (\mu\text{m/hr})t$$

- b) $dx_o(t)/dt \sim 0.826 + 0.402 \exp(-0.826t/L)$

This can be integrated to give $x(t)$. The integration of the second term can be approximated as its integral for infinite time. This gives $x_o(t) = 0.826t + 0.402(L/0.826)$.

Here when $L = 7 \text{ nm}$ is plugged into the last term the offset from linear can be evaluated as $3.4 \text{ nm} = 34 \text{ \AA}$. This is quite comparable to the expected small effect of about 25 \AA .