

(1)

In the last lecture, we found a polarization of an atomic media in response to an \vec{E} field

$$\vec{E}(\omega) \rightarrow H' \rightarrow \begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix} \rightarrow \text{steady state}$$

↑
radiating
collisions

$$\begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix} \rightarrow \vec{P}(\omega) \quad \text{where}$$

$$\vec{P}(t) = R_c (\epsilon_0 \chi(\omega) E_0 e^{i\omega t})$$

where $\chi(\omega)$ is the susceptibility

& we found that $\chi(\omega)$ has an imaginary part which is large only close to the frequency ω_0 where $\hbar\omega_0 = E_2 - E_1$, and whose breadth is determined by the inelastic collision rate & the electric field strength:

$$\chi(\omega) = \frac{\mu^2 T_2 D N_0}{\epsilon_0 \hbar} \left[\frac{(\omega_0 - \omega) T_2 - i}{1 + (\omega - \omega_0)^2 T_2^2 + 4\pi^2 T_2^2} \right]$$

where, of course, the real & imm. parts of $\chi(\omega)$ are related by the Kramers-Kronig relationship

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for low \vec{E} , the $\propto \mu E_0 / \hbar \kappa$ term is negligible & we have a line shape which is Lorentzian

$$g(\nu) = \frac{2 T_1}{1 + 4\pi^2 (\nu - \nu_0)^2 T_2^2} = \frac{(\Delta\nu/2\pi)}{(\nu - \nu_0)^2 - (\Delta\nu/2)^2}$$

where the full width at half maximum is $\Delta\nu = (\pi T_2)^{-1}$

If the field strength becomes stronger, the third term in the denominator is no longer negligible, and the absorption line shape broadens - this is called power broadening. It due to the fact that the population will saturate ($\Delta N \rightarrow 0$) more near the line center.

Saturation caused by the growth of the term $4\omega^2 T_2 \tau$ in the denominator, is important in the behavior of laser oscillators.

We now want to connect the next link in the chain

$$\vec{E}(r, t) \rightarrow H' \rightarrow \begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix}$$

$$\rightarrow \vec{P}_{cr, t} \rightarrow \vec{E}(r, t)$$

This is accomplished, as discussed before, by defining a new field — \vec{D}

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

Since \vec{P} is proportional to \vec{E} at optical frequencies, we can define

$$\vec{P} = \epsilon_0 \chi_1 \vec{E} + \epsilon_0 \chi_2 \vec{E} + \epsilon_0 \chi_3 \vec{E} + \dots$$

where χ_n is a slowly varying function (wrt ω) of $|\vec{E}|$ and a function of frequency for each transition. Assuming only one transition is near, we then define a dielectric constant ϵ

$$\vec{D} = \epsilon \left[1 + \frac{\epsilon_0}{\epsilon} \chi(\omega) \right] \vec{E} = \epsilon'(\omega) \vec{E}$$

where $\epsilon'(\omega) = \epsilon \left[1 + \frac{\epsilon_0}{\epsilon} \chi(\omega) \right]$

ϵ' takes into account all other polarizations

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This is then plugged back into Maxwell's Equation

$$\nabla \times H = \epsilon \frac{\partial E}{\partial t}$$

$$\nabla \times E = -\mu \frac{\partial H}{\partial t}$$

$$\nabla \cdot E = \rho_{\text{total}}$$

$$\Rightarrow \nabla \cdot D = 0 \quad (\text{no free charge})$$

$$\nabla \cdot E = \frac{\rho_{\text{total}}}{\epsilon_0}$$

$$D = \epsilon_0 E + P$$

$$\nabla \cdot D = \epsilon_0 \nabla \cdot E + \nabla \cdot P$$

$$\nabla \cdot D = \rho_{\text{total}} - \rho_{\text{pol}} = \rho_{\text{free}}$$

when we take the curl $\nabla \times \nabla \times E = -\mu \nabla \times \frac{\partial H}{\partial t}$

we get $\nabla^2 E - \mu \epsilon \frac{\partial^2}{\partial t^2} E = -\nabla \left(\frac{1}{\epsilon} E \cdot \nabla \epsilon \right)$

if ϵ is slowly varying with \vec{r} ($+E$)
 we get $\nabla^2 E - \mu \epsilon \frac{\partial^2}{\partial t^2} E = 0$

(5)

If we try solution of the form

$$E(z, t) = \text{Re} [E e^{i(\omega t - Kz)}]$$

we get

$$K' = \omega \sqrt{\mu \epsilon'}$$

$$K' = \omega \sqrt{\mu} \sqrt{\epsilon + \epsilon_0 \chi(\omega)}$$

where $\chi(\omega)$ is complex

Expanding the square root, and

keeping only the first terms

$$\sqrt{1+\epsilon} = 1 + \frac{1}{2}\epsilon + \dots$$

$$K' = \omega \sqrt{\mu \epsilon} \left\{ 1 + \frac{\epsilon_0}{2\epsilon} \chi'(\omega) - i \frac{\epsilon_0}{2\epsilon} \chi''(\omega) \right\}$$

we then define $\gamma(v) = 2 \text{Im}(K')$

as the gain (loss) $\alpha = -\gamma$ of the medium so:

$$\gamma(v) = -\chi''(v) \omega(v) \sqrt{\mu \epsilon} \left(\frac{\epsilon_0}{\epsilon} \right)$$

where of course $\omega = \frac{v}{2\pi}$ and $\frac{\epsilon_0}{\epsilon} = \frac{1}{n^2}$
where n is the index of the media, not counting this resistor