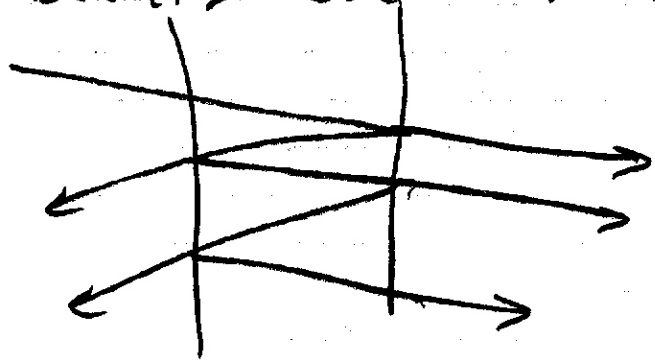
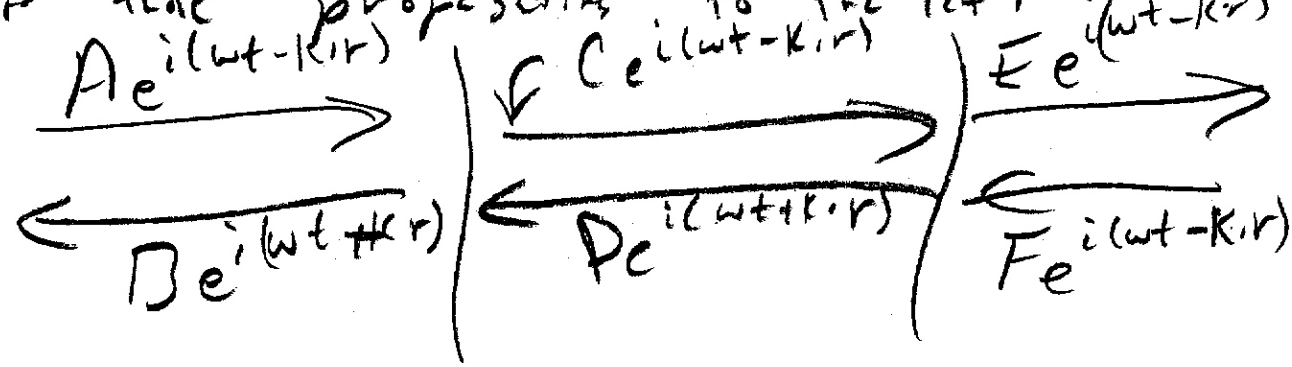
Two mirror resonators

Two mirror resonators are used to make lasers & very narrow filters called Fabry-Perot filters. We can also use them to model resonators with more complex cavities.

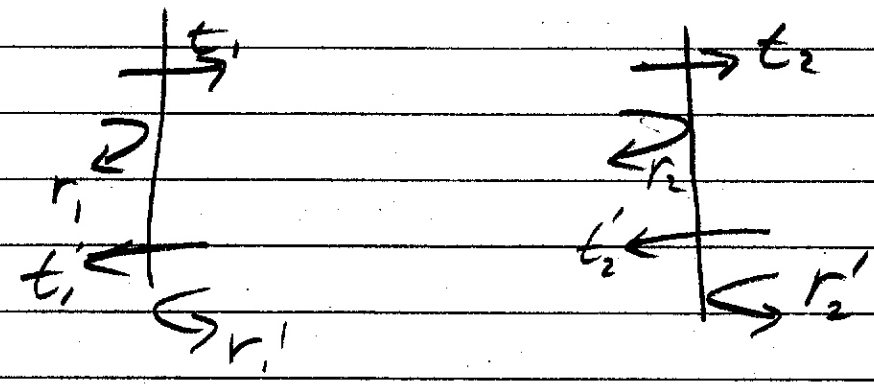
if we have two high reflectivity mirrors, we can consider putting a beam of monochromatic light through one of the mirrors. Some light would go through the mirror, and start bouncing back and forth



lets restrict the problem to plane mirrors and plane waves, in which case we can sum all of the plane waves in each region, and the same for those propagating to the left



and we will define the reflectivities



we then have

$$B = r_1 A + t_1' D$$

$$C = t_1 A + r_1' D$$

$$E = C e^{-iK_z z} t_2 \quad (F=0)$$

$$D = C e^{-iK_z z} r_2 e^{+iK_z(-z)}$$

we can now solve for $C(A)$

$$C = t_1 A + r_1' \left(C e^{-iK_z z} r_2 e^{-iKz} \right)$$

$$A = \left(1 - r_1' r_2 e^{-2iK_z z} \right) \frac{1}{t_1} C$$

$$\boxed{C = \frac{A t_1}{1 - r_1' r_2 e^{-2iK_z z}}}$$

$$E = \frac{A t_1 t_2 e^{-iK_z z}}{1 - r_1' r_2 e^{-2iK_z z}}$$

So we can see that the amount of energy inside the cavity is a strong function of $e^{-2ik_z z}$, especially where $r_1' r_2 \approx 1$

this equation has maxima where

$$e^{-2ik_z z} = +1$$

$$\Rightarrow \quad 2k_z z = n2\pi$$

Note that $2k_z z$ is just the phase change in one round trip in the cavity.

$$\boxed{\phi_{\text{one round trip}} = n2\pi}$$

Yariv (7.3)

the round trip phase found for our gaussian beam solution was just

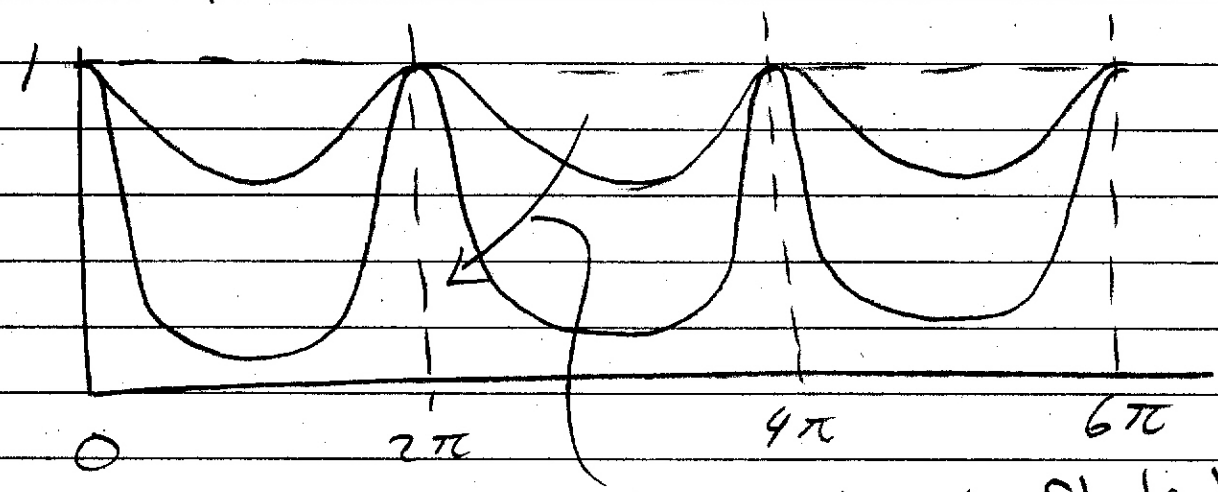
$$2\left(k_{gl} - (m+n+1)\left(\tan^{-1}\frac{z_2}{z_1} - \tan^{-1}\frac{z_1}{z_0}\right)\right) = n2\pi$$

(Yariv 7.3-3, but for one round trip)

if we let $K_z z = \delta$

& find the ratio of the intensities $\left(\frac{E}{A}\right)\left(\frac{E}{A}\right)^* = \frac{I_i (tt')^2}{(1+r^4) - 2r^2\cos\delta}$

where we take $r_1 = r_2 = r$
& then plot vs δ



increasing reflectivity

if r is very close to 1, this becomes a comb of spikes at $n(2\pi)$

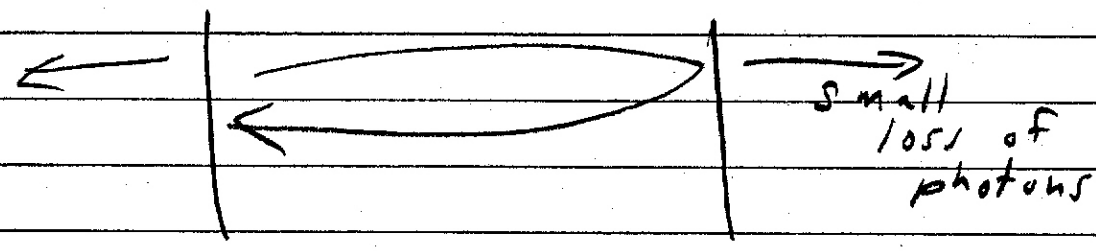
A useful measure of the Fabry Perot is the Finesse (F)

$$F \equiv \frac{\text{Free spectral range}}{\text{Full width at } 1/2 \text{ maximum}}$$

for the asymetric Fabry Perot $F = \frac{\pi (R_1 R_2)^{1/4}}{1-(R_1 R_2)^{1/2}}$
which will be a homework problem

The narrowness of a resonance is related to the lossyness of a resonator, $\Rightarrow$ concept of Q & photon lifetime

Photon lifetime will be the first of several <u>rate equations</u> which will be important for solving laser resonators



If we reverse time, our equations can represent light trapped in a cavity & slowly leaking out

the total ratio of photons lost after hitting both mirrors is

$$(1 - R_1 R_2)$$

We can convert that into a loss of photons per unit time

by dividing by the time per pass (should be group velocity, but assume slow $n(\omega)$ for now)

$$\text{rate of photons lost/time} = \frac{1-R_1R_2}{2\frac{n}{c}d}$$

we can then write a <u>rate equation</u> for photons

$$\frac{dN_p}{dt} = \frac{1-R_1R_2}{2\frac{n}{c}d} N_p$$

which has the solution

$$N_p(t) = N_{p0}\, e^{-t/\tau_p}$$

where τ_p is the photon lifetime

$$\tau_p = \frac{2nd/c}{1-R_1R_2} = \frac{\text{round trip time}}{\text{fractional loss per pass}}$$

Remember the definition of Q:

$$Q = \omega_0 \left(\frac{\text{energy stored}}{\text{power lost}}\right)$$

$$Q = \omega_0 \left(\frac{W}{dW/dt}\right)$$

$$\tau_p = Q/\omega_0 \quad \text{or} \quad Q = \omega_0 \tau_p$$

Note that for optical frequencies Q can become very large (10^8 or larger) τ can easily be made larger by simply lengthening the cavity — but the free spectral range goes down — so Finesse is the more useful quantity.

If we have a pair of curved mirrors, then the modes are the Hermite gaussian modes, so we must take our input to be the same H-G mode. What if it isn't?

$\Rightarrow$ the H-G modes of the resonator are a complete set

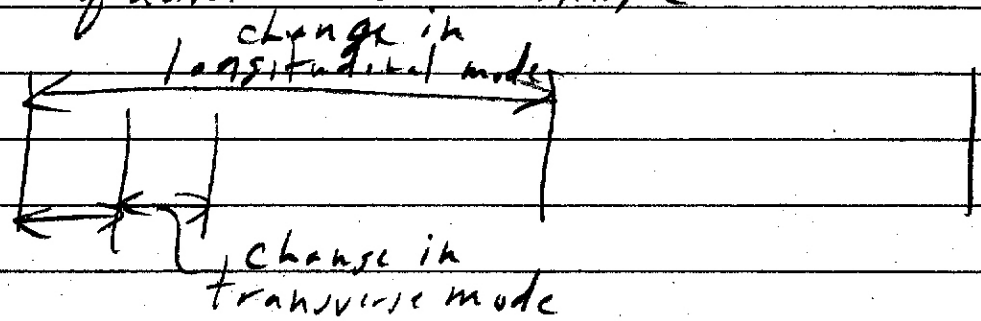
1) expand the input over the H-G modes
2) propagate each of them through the cavity
3) recombine at exit

Going back to the G.B. phase condition

$$K_1 l - (m+n+1)\left(\tan^{-1}\frac{z_2}{z_0} - \tan\frac{z_1}{z_0}\right) = q\pi$$

The resonances of the cavity will occur at frequencies which correspond to all three indices q, m, + n

+ the relative spacing depends on the gaussian beam shape



For a confocal resonator, the mode spacing for the transverse modes is 1/2 of that for the longitudinal modes

Reading for next lecture:
Yariv Chapter 8: 8.0 → 8.2