

$$\frac{1}{g_1} = \frac{1}{R_1} - i \frac{\lambda}{\pi w_0^2 n}$$

$$g_n = \frac{A_n g + B_n}{C_n g + D_n}$$

where  $(\begin{matrix} A_n & B_n \\ C_n & D_n \end{matrix})$  is the

$ABCD$  matrix from cross section to cross section  $n$  (same as for rays)

For a resonator, we have

$$g = \frac{Ag + B}{Cg + D}$$

because to construct a solution which satisfies Maxwell's equations everywhere, we need the same shape beam  $\rightarrow$  which means the same spot size and radius of curvature for Gaussian Beams

(2)

solving for  $\frac{1}{g}$  (to make it easier to solve for  $R + \omega$ )

$$I = \frac{A'g + D'g^2}{C + D'g}$$

$$C + D'g - A'g - D\frac{1}{g^2} = 0$$

$$\frac{1}{g} = \frac{(D-A) \pm \sqrt{(D-A)^2 + 4DC}}{2B}$$

But the matrices are always unitary because we end up in the same matrix, so

$$AD - BC = 1$$

$$\frac{1}{g} = \frac{D-A}{2B} \pm i \frac{\sqrt{1 - \left(\frac{D+A}{2}\right)^2}}{B}$$

if we are to have a finite step size the part under the radical must be positive so  $\left(\frac{A+D}{2}\right) \leq 1$

$\Rightarrow$  same as for rays

(3)

The parameters for the Gaussian beam which fit the ABCD's for the resonators can then be found from this value of  $g$ .  
At this location in the resonator

(The one ABCD was calculated from)

so from  $\frac{1}{g} = \frac{1}{R} - i \frac{d}{2\pi n}$

we have

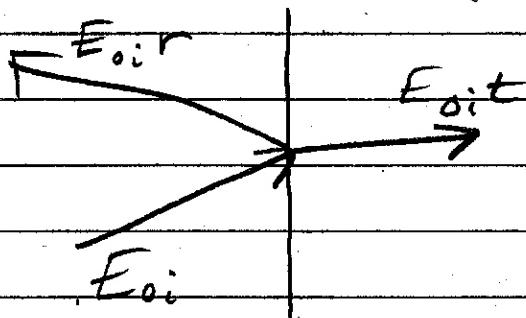
$$R = \frac{(2B)}{D-A}$$

$$\omega = \left(\frac{\Delta}{\pi n}\right)^{1/2} \frac{(B)^{1/2}}{\left[1 - \left(\frac{D+A}{2}\right)^2\right]^{1/4}}$$

(4)

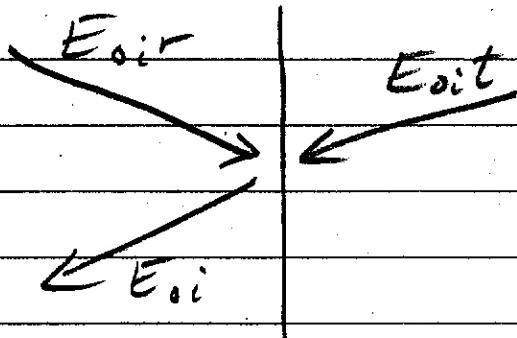
We will consider the lossless case  
 (multilayer dielectric mirrors are  
 nearly lossless)

lets look at the P.C.'s for plane  
 waves hitting a boundary



where  $r + t$  are the reflection +  
 transmission ratio for fields in the  
 forward direction

we can reverse time to see



$$E_{0i}t\epsilon' + E_{0irr}r = E_{0i}$$

and since the reflection to the right is zero

$$E_{0it}r + E_{0irr}r' = 0$$

$$\Rightarrow \epsilon' = 1 - r^2 \quad + r' = -r$$