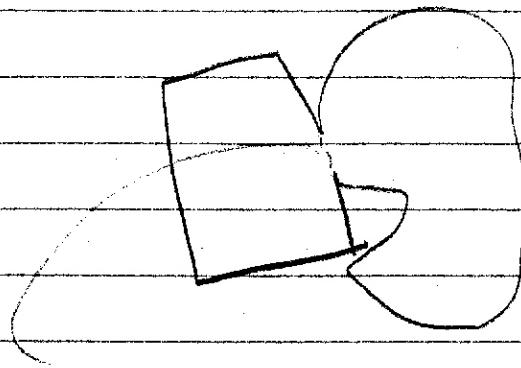


EE 236
10/11/04
①

Blackbody radiation



the principle of detailed balance states
that any flux in a thermal
equilibrium must be exactly balanced
by the reverse process

emission = absorption

radiation in at one angle
& frequency
= radiation out at the
same angle & frequency
etc

(2)

So if we find a distribution of radiation in any cavity the flux will be the same as that for any other cavity at thermal equilibrium.

The energy of a mode is given by

$$E_n = (n + \frac{1}{2}) \hbar \omega$$

because photons are identical Bose-Einstein particles, there is only one microscopic state associated with each of these energy levels, so the probability of finding a mode in a state n is $\alpha e^{-E_n/kT}$

since the mode must be in some state;

$$P(n) = \frac{e^{-E_n/kT}}{\sum_{s=0}^{\infty} e^{-E_s/kT}}$$

where k is the Boltzmann constant
we can now find the average energy of the mode

$$\bar{E} = \sum_{n=0}^{\infty} E_n P(n)$$

(3)

finding the sum of this series,
we have

$$\boxed{\overline{E} = \frac{\hbar\omega}{2} + \frac{\hbar\omega}{e^{\hbar\omega/kT} - 1}}$$

notice that this is independent
of the volume of the cavity,
so for the same temperature,
a large box has the same
energy per mode as a small box,
with a lower energy density.

how is this compatible with
detailed balance?

→ there are more modes in the
bigger cavity

so we need to count the modes
in a box

the modes are the solutions to
the equations

$$\nabla^2 E_a = -k_a^2 E_a$$

+ there are an ∞ number of them
that meet the B.C.'s

(4)

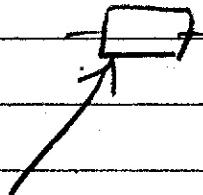
for periodic B.C.'s in a rectangular region; we have a mode for each combination ℓ, M, N integers

$$\vec{E} = E_0 e^{i\vec{k}\cdot\vec{r}}$$

$$K_x = \frac{2\pi\ell}{L_x} \quad K_y = \frac{2\pi m}{L_y} \quad K_z = \frac{2\pi n}{L_z}$$

so the density of states in K space is $\left(\frac{(2\pi)^3}{L_x L_y L_z}\right)^{\frac{m}{n}}$

that is, in a volume



$$K_x \rightarrow K_x + dK_x$$

$$K_y \rightarrow K_y + dK_y$$

$$K_z \rightarrow K_z + dK_z$$

we have $\left(\frac{2\pi}{L_x L_y L_z}\right)^3 dK_x dK_y dK_z$ states

to find the number of states with $K \leq K_{max}$
we can just divide

$$\frac{4/3 \pi K^3}{\left(\frac{(2\pi)^3}{L_x L_y L_z}\right)}$$

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so we have

$$N(\vec{k} < k) = \frac{k^3 L_x L_y L_z}{3\pi^2}$$

If the box is not too extreme
 L_x, L_y, L_z all $\gg \lambda$, this
 only depends on the volume,
 & we have

$$\boxed{\frac{N_k}{V} = \frac{k^3}{3\pi^2}}$$

To see how many modes there are
 as a function of frequency $\nu = \frac{\omega}{2\pi}$
 we use $K = \frac{2\pi\nu n}{c}$

$$\boxed{\frac{N(\nu)}{V} = \frac{8\pi\nu^3 n^3}{3c^3}}$$

To convert this into a density,
 we differentiate to find

$$p(\nu) = \frac{1}{V} \frac{dN_\nu}{d\nu} = \frac{8\pi\nu^2 n^3}{c^3}$$

which is the density of modes in a
 volume V (number between ν + $(\nu + d\nu)$)

⑥

We now multiply by the average energy for mode to find the energy density

$$\rho(v) = p(v) \bar{E} = \frac{8\pi h n^3 v^3}{c^3} \left(\frac{1}{e^{hv/kt} - 1} \right)$$

it is customary to leave out the factor of $1/2$, which does not change,

so we have

$$\rho(v) dv = \frac{8\pi h n^3 v^3}{c^3} \left(\frac{1}{e^{hv/kt} - 1} \right) dv$$