

Propagation in complex materials (lossless)

Anisotropic } crystals

 $E_{ij} = E_{ji}$ } Isotropic with applied field

Optically } quartz

active } sugar solutions

(off diagonals imm) $\begin{matrix} \text{LLL} & \text{LLL} \end{matrix}$

Gyrotropic } materials or plasmas with

 $E_{ij} = -E_{ji}$ } applied magnetic field $M_{ij} = -M_{ji}$ } some crystals $i \neq j$

For the lossless case, the matrix is hermitian

In the case of a symmetric dielectric tensor, $E_{ij} = E_{ji}$, the Matrix can be diagonalized (Hermitian with real eigenvalues)

$$\begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix} = \begin{pmatrix} \epsilon_{xx} & 0 & 0 \\ 0 & \epsilon_{yy} & 0 \\ 0 & 0 & \epsilon_{zz} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

for a proper choice of x, y, z ,If we have a plane wave propagating in the direction of $(\vec{R}) \parallel \hat{a}_i$, where \hat{a}_i is along one of the principle optical axes, then there are two solutions to Maxwell's

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equations, one with $\vec{E} \parallel \hat{a}_z$
 and another with $\vec{E} \parallel \hat{a}_x$
 with $\frac{\omega}{k} = \frac{c}{n} = \omega_p = \frac{1}{\sqrt{\mu_0 \epsilon_{ss}}}$ or ϵ_{zz}

To show this, we start with Maxwell's eq:

$$\nabla \times \vec{H} = i\omega \vec{D} = i\omega (\hat{a}_x \epsilon_{xx} E_x + \hat{a}_y \epsilon_{yy} E_y + \hat{a}_z \epsilon_{zz} E_z) \\ + \nabla \times \vec{E} = -i\omega \mu_0 \vec{H} = -i\omega \mu_0 H_x$$

lets assume that fields are
 indep. of the coordinates in $x + y$
 so the only variation is wrt $t + z$

$$(-\frac{\partial H_y}{\partial z}) \hat{a}_x + (\frac{\partial H_x}{\partial z}) \hat{a}_y = i\omega (\hat{a}_x \epsilon_{xx} E_x + \hat{a}_y \epsilon_{yy} E_y + \hat{a}_z \epsilon_{zz} E_z) \\ - \frac{\partial H_y}{\partial z} = i\omega \epsilon_{xx} E_x \\ \frac{\partial H_x}{\partial z} = i\omega \epsilon_{yy} E_y \quad \text{indep. solns} \\ -\frac{\partial E_y}{\partial z} = -i\omega \mu_0 H_x \\ \frac{\partial E_x}{\partial z} = -i\omega \mu_0 H_y$$

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$$\frac{\partial^2}{\partial z^2} E_x = - \omega \mu_0 (-i \omega \epsilon_{xx} E_x)$$

$$= -\omega^2 \mu_0 \epsilon_{xx} E_x$$

$$E_x = e^{iKz}$$

where

$$K^2 = \omega^2 \mu_0 \epsilon_{xx}$$

$$\frac{\omega}{K} = \frac{1}{\sqrt{\mu_0 \epsilon_{xx}}}$$

$$\vec{E} = E_x \hat{q}_x e^{i(\omega t \pm Kz)}$$

$$+ \vec{H} = \hat{q}_y \left(\frac{1}{-i\omega \mu_0} \right) \frac{\partial E_x}{\partial z}$$

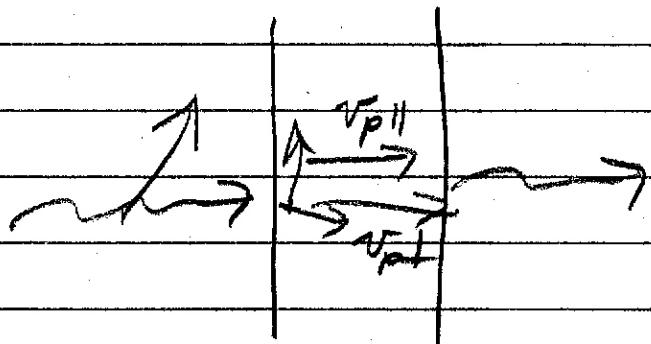
$$= \hat{q}_y E_x e^{i(\omega t \pm Kz)} \left(\frac{iK}{-\delta \omega \mu_0} \right)$$

$$= + \frac{iK}{\omega \mu_0} E_x e^{i(\omega t \pm Kz)}$$

If we can vary one of the ϵ tensor elements relative to the others, then we can independantly vary the phase velocity $\frac{K}{\omega}$ for each component, and this is used to make optical modulators

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or statically, we can make optical quarter wave plates or half wave plates



In a quarter wave plate, one of the polarizations has a phase delay of 90° with respect to the other

→ converts linearly polarized light to circular or vice versa

In a Kerr cell, the entire anisotropy is caused by a low frequency Electric field, & can rotate the polarization by varying amounts

In a Pockels cell, the media is an anisotropic crystal, but the elements E_x , E_y , E_z are varied by applying an Electric field

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We will show later that this means the Pockels effect can be first order (linear) to the applied field, but the Kerr effect is quadratic (α/E^2)

If we propagate at an angle off the principle optical axes, however, we will need to construct the two polarizations which will propagate.

The problem is complex, because the boundary conditions at the surface of a crystal will be in terms of the variations in phase at the surface of the crystal, but this will likely not be \perp to a principle axis. the other B.C. is the frequency ω is the same inside & outside the crystal (for linear cases — almost all B.C.s)

If we have solutions of the form:

$$\vec{E} = E_0 e^{i(\omega t - \vec{k} \cdot \vec{r})}, \quad \vec{D} = D_0 e^{i(\omega t - \vec{k} \cdot \vec{r})}$$

$$\vec{H} = H_0 e^{i(\omega t - \vec{k} \cdot \vec{r})}, \quad \vec{B} = B_0 e^{i(\omega t - \vec{k} \cdot \vec{r})}$$

To satisfy Maxwell's equations over the crystal we must have ω, k be the same.

$$\nabla \times H = \frac{\partial D}{\partial t}$$

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

+ we also have $\nabla \cdot D = 0 \quad \vec{D}_0 \perp$
+ $\nabla \cdot B = 0$

Let, take $\mu = \mu_0$ then $\nabla \cdot H = 0$
 $\Rightarrow H_0 \perp \vec{K} + \vec{B}_0 \perp \vec{K}$

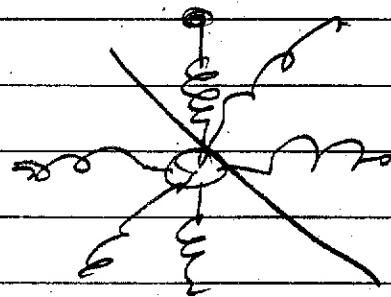
{ but the Electric Field is not necessarily \perp to \vec{K} }

Therefore, if we look at the Poynting vector $\vec{E} \times \vec{H}$ it does not point in the same direction as \vec{K} \rightarrow energy flows in a different direction than the phase variation.

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We also don't know the magnitude of K because $\frac{w}{K} = v_p = C/n$
— changes with direction.—

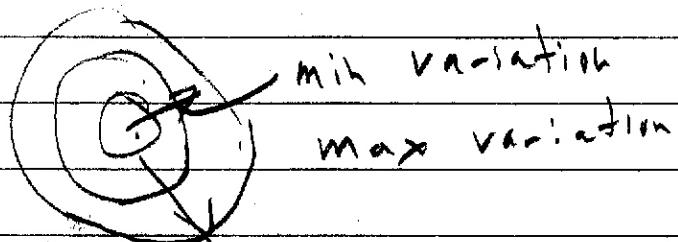
The problem of finding the polarizations which will propagate unchanges in a crystal is equivalent to finding the frequency of oscillations in a 3-D mechanical system which is constrained to oscillate in a particular plane



This is easier to see from an energy point of view

$$P.E. = \frac{1}{2} K_x x^2 + \frac{1}{2} K_y y^2 + \frac{1}{2} K_z z^2$$

as long as the potential energy is at a minimum (quadratic) then there is a coordinate system in which the Energy can be diagonalized in this form



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If we look at the total stored energy in the electric field and the polarization of the material

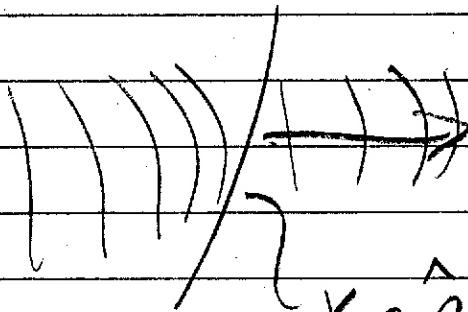
$$W_e = \frac{1}{2} \vec{E} \cdot \vec{D}$$

+ use the principle optical axes,

$$W_e = \frac{1}{2} \epsilon_{xx} E_x^2 + \frac{1}{2} \epsilon_{yy} E_y^2 + \frac{1}{2} \epsilon_{zz} E_z^2$$

So the constant Energy contours are ellipsoids

We now have to translate our B.C.s into a constraint on the solutions. They might be of a form



$$\vec{K} \cdot \hat{\vec{a}}_1 = \text{some value}$$

$$\vec{K} \cdot \hat{\vec{a}}_2 = \text{some value}$$

$$\text{or } \vec{K} \times \hat{\vec{n}} = \text{constant vector}$$