

EE236
9/24/04

Parity operations (1)
Semi Classical States
Start EdM in matter (ch.5)

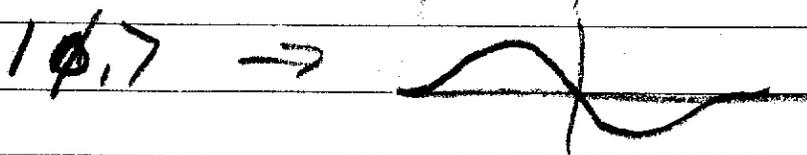
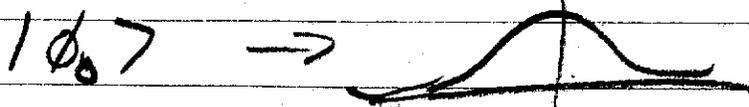
Note: We will cover higher order perturbation theory later (nonlinear interactions) and we will skip chapter 4

The solutions we have found for QM systems can now be applied to our problem, the interaction of light with matter,

But first, let's look at some of the properties of the QM solutions give us behavior which we can interpret classically

Consider the harmonic oscillator at a time ($t=z$)

$$|\Psi(z)\rangle = \sum_{n=0}^{\infty} a_n e^{i(n+1/2)\omega_0 t} |\phi_n\rangle$$



how does this give us a classical like result?

(2)

Consider the parity operator

$$\hat{P} |\psi(r)\rangle = |\psi(-r)\rangle$$

if the Hamiltonian commutes with the parity operator, then (from Homework) there exists a set of eigenstates which diagonalize both \hat{H} and \hat{P} . If there are no degenerate eigenvalues of \hat{H} , then all of the eigenstates of \hat{H} are eigenstates of \hat{P} but $\hat{P}^2 = \hat{I}$

so:

$$\hat{P} |\psi_e(r)\rangle = |\psi_e(r)\rangle$$

$$\hat{P} |\psi_o(r)\rangle = -|\psi_o(r)\rangle$$

So the eigenstates of the harmonic oscillator (or any symmetric Hamiltonian) can be divided into even and odd solutions

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if we look at the action of the parity operator on the raising & lowering operator

$$\hat{P} \hat{a} |\phi_n\rangle = \hat{P} |\phi_{n-1}\rangle$$

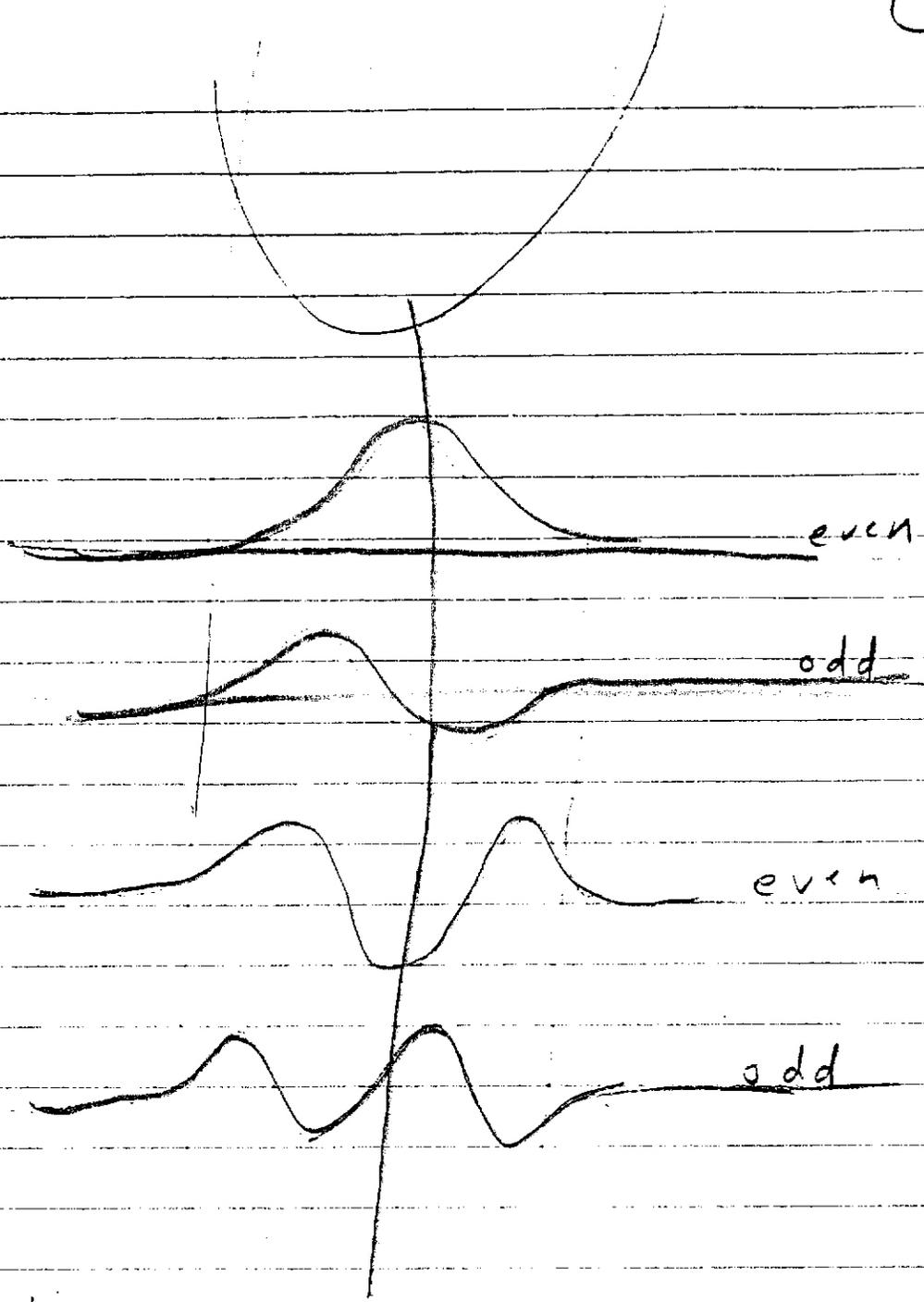
$$\hat{P} \left(\frac{\alpha}{\sqrt{2}} x - \frac{i}{\sqrt{2}\hbar\alpha} p_x \right) |\phi_n\rangle$$

$$= \hat{P} |\phi_{n-1}\rangle$$

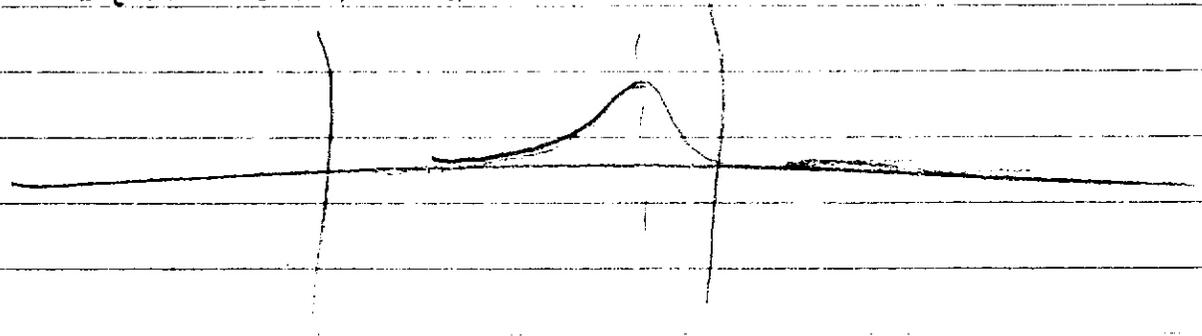
$$\left(-\frac{\alpha}{\sqrt{2}} x - \frac{i}{\sqrt{2}\hbar\alpha} p_x \right) \hat{P} |\phi_n\rangle = \hat{P} |\phi_{n-1}\rangle$$

$$\underbrace{(-1)}_{(\pm 1)} \hat{a} \hat{P} |\phi_n\rangle = \hat{P} \underbrace{|\phi_{n-1}\rangle}_{(\pm 1)}$$

\Rightarrow so the states of the harmonic oscillator must have alternating parity



⇒ it is possible to be in a state where voltage (position) is reasonably well defined



if we then separate this into even and odd parts, we see

$$\Psi(V) = \sum_{\substack{n=1 \\ n \text{ odd}}}^{\infty} a_n e^{i\omega_n t} \phi_n(V) + \sum_{\substack{n=0 \\ n \text{ even}}}^{\infty} a_n e^{i\omega_n t} \phi_n(V)$$

$$\Psi(V) = \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} a_n e^{i(n+1/2)\omega_0 t} \phi_n(V) + \sum_{\substack{n=0 \\ n \text{ even}}}^{\infty} a_n e^{i(n+1/2)\omega_0 t} \phi_n(V)$$

and so if we look at time $\tau = \frac{\pi}{\omega_0}$ (1/2 the period)

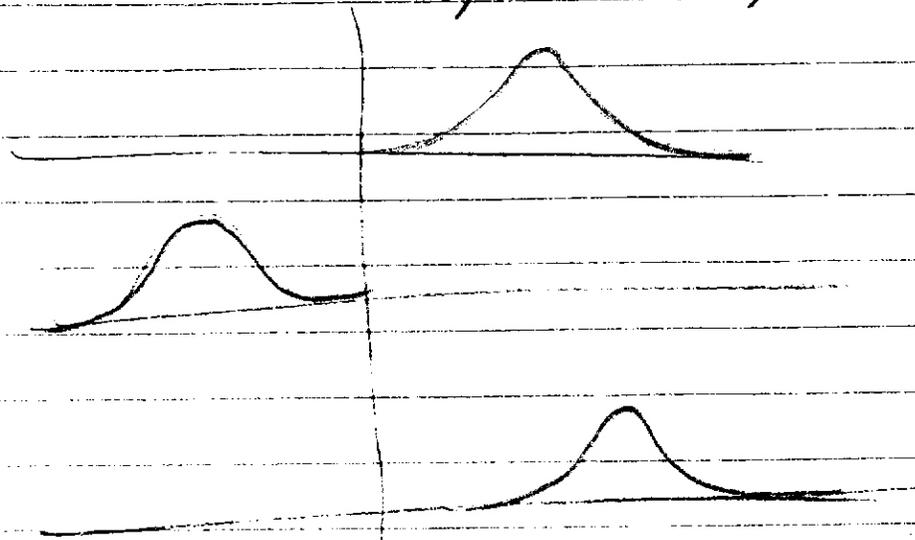
$$\begin{aligned} \Psi(V) &= \sum_{\substack{n=1 \\ n \text{ odd}}}^{\infty} a_n e^{i(n+1/2)\pi} \phi_n(V) + \sum_{n=1}^{\infty} a_n e^{i(n+1/2)\pi} \phi_n(V) \\ &= i e_{\substack{\omega \\ -1 \rightarrow \beta}} \sum_{\substack{n=1 \\ n \text{ odd}}}^{\infty} a_n \phi_n(V) + i \sum_{n=0}^{\infty} a_n \phi_n(V) \end{aligned}$$

(6)

So the wave function

$$\Psi(t = \pi/\omega_0) = \hat{P} \Psi(t=0)$$

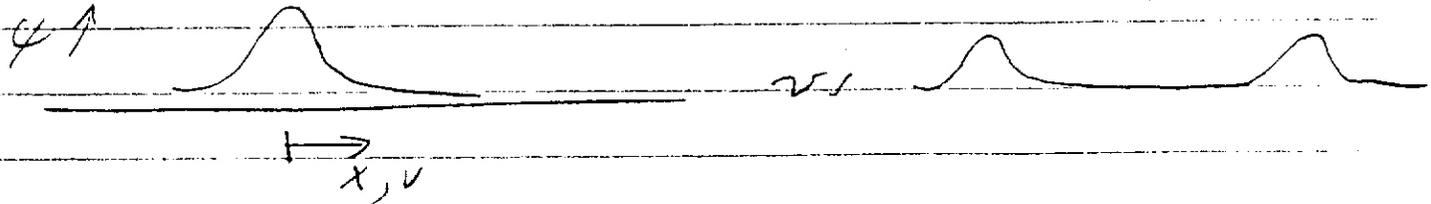
$$\text{or } \Psi(t = \pi/\omega_0, r) = \Psi(t=0, -r)$$



→ this is called a semiclassical state
or more elegantly a squeezed state

in this particular squeezed state,
you can know the phase of the
voltage to a high degree of
accuracy for an indefinite period
(only limited to how well the
H.O. model works)

For states which are semiclassical the notion of a standard deviation is useful



The standard deviation is the R.M.S deviation of a value from the mean of a random variable j

$$\sigma^2 \equiv \langle (\Delta j)^2 \rangle$$

$$\sigma^2 = \sum (\Delta j)^2 P(j) = \sum (j - \langle j \rangle)^2 P(j)$$

$$= \sum (j^2 - 2j\langle j \rangle + \langle j \rangle^2) P(j)$$

$$= \sum j^2 P(j) - 2\langle j \rangle \sum j P(j) + \langle j \rangle^2 \sum P(j)$$

$$= \langle j^2 \rangle - 2\langle j \rangle \langle j \rangle + \langle j \rangle^2$$

$$= \langle j^2 \rangle - (\langle j \rangle)^2$$

if we find the standard deviation for an observable \hat{A} of an arbitrary state

$$\sigma_A^2 = \langle \psi | (\hat{A} - \langle A \rangle)^2 | \psi \rangle$$

because A is Hermitian

$$\sigma_A^2 = \langle (\hat{A} - \langle A \rangle) \psi | (\hat{A} - \langle A \rangle) \psi \rangle$$

or $\langle g | g \rangle$ where $|g\rangle = (\hat{A} - \langle A \rangle) | \psi \rangle$

for any other observable \hat{B}

$$\sigma_B^2 = \langle f | f \rangle \text{ where } |f\rangle = (\hat{B} - \langle B \rangle) | \psi \rangle$$

but by the Schwartz inequality

$$|\langle g | f \rangle|^2 \leq \langle g | g \rangle \langle f | f \rangle$$

For any complex number z

$$|z|^2 = (\text{Re}(z))^2 + (\text{Im}(z))^2$$

$$\geq \text{Im}(z^2) = \left[\frac{1}{2i} (z - z^*) \right]^2$$

therefore

$$\sigma_A^2 \sigma_B^2 \geq \left(\frac{1}{2i} [\langle F|g\rangle - \langle g|F\rangle] \right)^2$$

plugging back in the values of $|f\rangle$ & $|g\rangle$

$$\langle f|g\rangle = \langle \hat{A} - \langle A \rangle \psi | (\hat{B} - \langle B \rangle) \psi \rangle$$

$$= \langle \psi | (\hat{A} - \langle \hat{A} \rangle) (\hat{B} - \langle B \rangle) | \psi \rangle$$

$$= \langle \psi | \hat{A} \hat{B} - \hat{A} \langle B \rangle - \hat{B} \langle A \rangle + \langle A \rangle \langle B \rangle | \psi \rangle$$

$$= \langle \hat{A} \hat{B} \rangle - \langle B \rangle \langle A \rangle - \langle A \rangle \langle B \rangle + \langle A \rangle \langle B \rangle$$

$$= \langle \hat{A} \hat{B} \rangle - \langle A \rangle \langle B \rangle$$

$$\downarrow \langle g|f\rangle = \langle \hat{B} \hat{A} \rangle - \langle B \rangle \langle A \rangle$$

$$\begin{aligned} \langle f|g\rangle - \langle g|f\rangle &= \langle \hat{A} \hat{B} \rangle - \langle \hat{B} \hat{A} \rangle \\ &= \langle \hat{A} \hat{B} - \hat{B} \hat{A} \rangle \\ &= \langle [\hat{A}, \hat{B}] \rangle \end{aligned}$$

$$\text{so } \sigma_A^2 \sigma_B^2 \geq \left(\frac{1}{2i} \langle [\hat{A}, \hat{B}] \rangle \right)^2$$

\Rightarrow So any operators which do not commute have standard deviations limited by this result \leftarrow

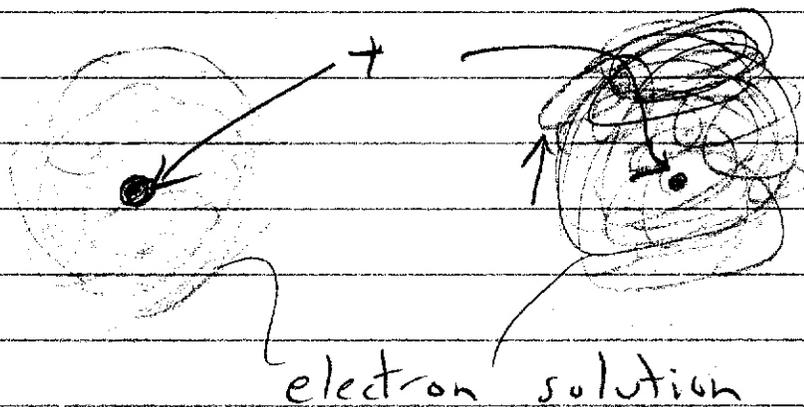
We will come back to Q.M. when we treat specific systems -
 We now need to look at the propagation of Electromagnetic fields
 (Yariv chapter 5)

Starting with Maxwell's equations:

$$\nabla \times H = I + \frac{\partial D}{\partial t}$$

$$\nabla \times E = - \frac{\partial B}{\partial t}$$

we will start with cases where there are no free charges or currents, but where there are polarizable atoms



this produces a dipole moment which can

- absorb power from EM field
- change its propagation (n, ϵ)

(1)

We will find later that the dipole moment will have frequency components proportional to \vec{E} , and we define a new vector field

where \vec{P} is the dipole moment per unit volume

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

$$\vec{\nabla} \cdot \vec{D} = \epsilon_0 \vec{\nabla} \cdot \vec{E} + \vec{\nabla} \cdot \vec{P}$$

$$\epsilon_0 \vec{\nabla} \cdot \vec{E} = \vec{\nabla} \cdot \vec{D} - \vec{\nabla} \cdot \vec{P}$$

$$\underbrace{\epsilon_0 \vec{\nabla} \cdot \vec{E}} = \underbrace{\vec{\nabla} \cdot \vec{D}} - \underbrace{\vec{\nabla} \cdot \vec{P}}$$

$$\epsilon_0 \vec{\nabla} \cdot \vec{E} = \rho \quad \rho_{\text{free}} \quad \rho_{\text{bound}}$$

↑
total charge density

leaving $\vec{\nabla} \cdot \vec{D} = \rho_{\text{free}} = 0$

we will generally take

$$\vec{B} = (\mu_0 \vec{H} + \vec{M})$$

↑
mag

(12)

The polarization $\vec{P} = Nq \overline{\langle \hat{x} \rangle}$

+ other polarizations

will depend on:

- the Q.M. of the material
- the history of the perturbing \vec{E} field
- damping mechanisms

- it can be

- nonlinear
- anisotropic
- otherwise messy

often hidden in $\vec{D} = \underset{\uparrow}{\epsilon} \vec{E}$