University of California
Department of Electrical Engineering
and Computer Sciences

Prof. J. S. Smith
Fall 2004
EECS 236A

Final Exam

December 17, 2004

NAME: Solutions
Last, First

Student ID:

- Open book (Yariv), open notes.
- Calculators are allowed.
- Show all of your work and reasoning to receive full or partial credit.

Problem	Possible points	Score
1	25	
2	25	
3	25	
4	25	
Total	100	

1) Find the spontaneous lifetime of an electron in an infinite square well potential in the shape of a cube, with a side length a. The starting state is the first excited state with an extra null in the z direction. You do not have to evaluate any integrals.

a) For photons traveling in a direction (θ,φ) where θ is the angle from the z axis, and φ is the angle around the z axis from the x axis, find the polarization of the photons.

b) For photons traveling in a direction (θ,φ) where θ is the angle from the z axis, find the probability distribution as a function of the direction a single photon would be emitted.

the spontaneous lifetime of a state is: $t_{spont} = \left[\frac{e^2 \omega_0^3 n^3}{3\pi c^3 \hbar \varepsilon} |r_{12}|^2 \right]^{-1}$

(Yariv 8,3-9) where, in this case

$$|r_{12}|^2 = |z_{12}|^2 = |\langle 0 | z | 1 \rangle|^2$$

$$= K^{-1} \iiint_{-a/2}^{a/2} \cos\left(\frac{\pi}{2a}x\right) \cos\left(\frac{\pi}{2a}y\right) \cos\left(\frac{\pi}{2a}z\right) \cdot z \cdot \cos\left(\frac{\pi}{2a}x\right) \cos\left(\frac{\pi}{2a}y\right) \sin\left(\frac{\pi}{a}z\right) d^3r$$

where $K = \iiint_{-a/2}^{a/2} \cos^2\left(\frac{\pi}{2a}x\right) \cos^2\left(\frac{\pi}{2a}y\right) \cos^2\left(\frac{\pi}{2a}z\right) d^3r$

$$K = \left(\frac{a}{2}\right)^3$$

$$|r_{12}|^2 = \left(\frac{a}{2}\right)^{-1} \int_{-a/2}^{a/2} \cos\left(\frac{\pi}{2a}z\right) z \sin\left(\frac{\pi}{a}z\right) dz$$

(b) for each mode, $W_{spont} \propto |\hat{e}_{k\lambda} \cdot \hat{z}|^2$
$\rightarrow$ spontaneous polarization in $\hat{\theta}$ direction (in $\hat{z}$ $\hat{k}$ plane)

(c) $W_{spont} \propto |\hat{e}_{k\lambda} \cdot \hat{z}|^2 \propto (\sin\theta)^2$

2) When we solved for the Gaussian beam shape reproduced in a resonator from the ABCD matrices, we assumed that the beam would have to reproduce itself in one round trip. Could the beam reproduce itself in two, three or more round trips? Explain thoroughly, and give examples or show why it is not possible.

for 1 round trip, we have $q = \frac{Aq+B}{Cq+D}$

$$Cq^2 + (D-A)q - B = 0$$

for two round trips,

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix}^2 = \begin{pmatrix} A^2+BC & AB+DB \\ AC+CD & D^2+CB \end{pmatrix}$$

$$(AC+CD)q^2 + (A^2-D^2)q - AB+DB = 0$$

$$- B(A+D) = 0$$

$$C(A+D)q^2 + (A-D)(A+B)q$$

$$Cq^2 + (A-D)q - B = 0$$

$\Rightarrow$ same equation Gaussian beam repeats after 1 round trip as well

for the general case use Yariv 6.8-2 for n passes

$$A_T = \frac{A\sin(s\theta) - \sin[(s-1)\theta]}{\sin\theta}$$

where $\theta = \frac{1}{2}(A+D)$

$$B_T = \frac{B\sin(s\theta)}{\sin\theta}$$

$$C_T = \frac{C\sin(s\theta)}{\sin\theta}$$

$$D_T = \frac{D\sin(s\theta) - \sin[(s-1)\theta]}{\sin\theta}$$

$$C_T q^2 + (D_T - A_T)q - B_T = 0$$

$$\frac{C\sin(s\theta)}{\sin\theta}q^2 + (A-D)\frac{\sin(s\theta)}{\sin\theta}q + \frac{D\sin s\theta}{\sin\theta}$$

$Cq^2 + (A-D)q + B = 0 \Rightarrow$ $\boxed{\text{if it works for N it works for 1}}$

3) The index of refraction of a semiconductor changes as the number of carriers is changed. Find an expression for the small change in the index of refraction with a small change in the number of carriers, for light which has a frequency **just below** that corresponding to the bandgap energy, for a semiconductor at zero temperature. Use the notation:

$N_{elec} = N_{holes}$

$\Delta N = \Delta N_{elec} = \Delta N_{holes}$

$n \Rightarrow n + \Delta n$

m_c

m_h

x_{vc}

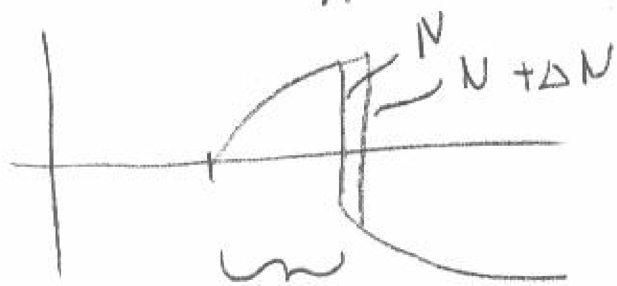
E_g

$T_2 = \infty$

We have $\alpha_0(\omega) = C(\hbar\omega - E_g)^{1/2}$

where $C = \dfrac{\omega e^2 x_{vc}^2 (2m_r)^{3/2}}{2\pi \varepsilon_0 n c \hbar^3}$

$$\gamma = -\frac{k\chi''(\omega)}{n^2}$$



$$\Delta\gamma = 2C(\hbar\omega - E_g)^{1/2} \cdot \Delta\omega \, \delta(\omega - \omega(N))$$

$$\omega(N) = \frac{E}{\hbar} = \frac{\hbar}{2m_r}[3\pi^2 N]^{2/3} + E_g$$

$$\Delta\omega = \frac{\partial\omega}{\partial N}\Delta N = \frac{\hbar}{2m_r}\frac{2}{3}(3\pi^2)^{2/3} N^{-1/3}$$

$$\Delta\chi'' = -\frac{n^2}{k} 2C(\hbar\omega - E_g)^{1/2}\Delta\omega\,\delta(\omega - \omega(N))$$

$$\Delta\chi' = \frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{\chi''(\omega')}{\omega' - \omega}\, d\omega'$$

$$\Delta\chi' = \frac{1}{\pi}\frac{1}{\omega(N) - E_g/\hbar} - \frac{n^2}{k} C(\hbar\omega - E_g)^{1/2}\Delta\omega$$

$$n = \frac{\omega}{k} \approx \left(1 - \frac{\varepsilon_0}{2\varepsilon}\chi'\right)$$

$$\Delta n \approx \frac{\varepsilon_0}{2\varepsilon}\Delta\chi'$$

4) When spontaneous emission is included, the gain is just smaller than the loss in a lasing mode. For a cavity of length L and mirror reflectivities both R, find the difference between the actual gain needed and that given by the total loss. Assume a lasing intensity I, and a nearly uniform mode 1 mm in radius, and a lasing frequency ν.

start with the rate equation for photons

$$\frac{dP}{dt} = P\left(\gamma \frac{c}{n} - \frac{1}{t_c}\right) + \text{Spont.}$$

where $\gamma = \dfrac{(N_2 - N_1 \frac{g_2}{g_1})\lambda^2 n}{8\pi n^2 t_{spont}} g(\nu)$

the spontaneous emission rate is $\frac{1}{P}$ of the stimulated emission rate, which corresponds to the N_2 above

So we have

$$\frac{dP}{dt} = P\left(\gamma \frac{c}{n} - \frac{1}{t_c}\right) + \frac{N_2 \gamma}{(N_2 - N_1 \frac{g_2}{g_1})}$$

in steady state, $\frac{dP}{dt} = 0$

$$\gamma\left(P\frac{c}{n} + \frac{N_2}{(N_2 - N_1 \frac{g_2}{g_1})}\right) = \frac{P}{t_c}$$

$$\gamma = \frac{P/t_c}{P\frac{c}{n} + \frac{N_2}{(N_2 - N_1)\frac{g_2}{g_1}}}$$

$$\gamma = \frac{\frac{1}{t_c}\frac{n}{c}}{1 + \frac{n}{c}P^{-1}\left(\frac{N_2}{N_2 - N_1 \frac{g_2}{g_1}}\right)} \approx \boxed{\frac{1}{t_c}\frac{n}{c}\left(1 - \frac{n}{c}P^{-1}\frac{N_2}{N_2 - N_1\frac{g_2}{g_1}}\right)}$$

$$\boxed{P = I\pi r^2 2L \frac{n}{c}}$$