

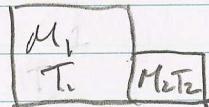
(8)

## Interaction of matter and radiation

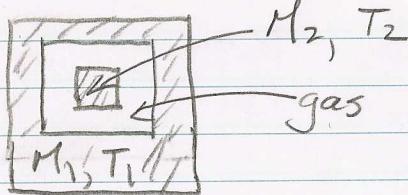
[Reading assignment, Svelto, Ch. 2]

- Thermal radiation. Thermal equilibrium between 2 masses can be achieved by several processes

Conduction

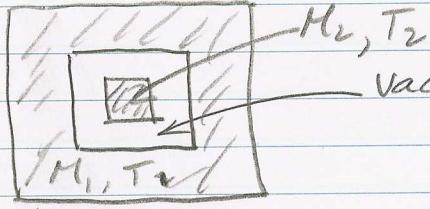


Convection



gas transports  
energy to reach  
equilibrium

Radiation



to reach equilibrium,  
masses must radiate  
energy + absorb  
energy.

Hot bodies radiate light. Think of glowing coals.

Boltzmann's law says that for two energy levels of a system, in thermal equilibrium, the population ratio must be:

$$\frac{N_2}{N_1} = \frac{g_2}{g_1} e^{-(E_2 - E_1)/kT}$$

for a continuous spectrum of energy levels (i.e. a solid)

$$\frac{N_2(E) dE}{N_1(E) dE} = \frac{g_2(E)}{g_1(E)} e^{-(E_2 - E_1)/kT}$$

where here,  $g(E)$  is the density of states function.

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To get visible light ( $\sim 2$  eV photons), consider Boltzmann ratio.

at  $T = 300\text{K}$ ,  $kT = .026\text{eV}$

$$e^{-2/0.026} \approx 4 \times 10^{-34} \quad \text{very small population!}$$

at  $T = 1000\text{K}$ ,  $kT = .087\text{eV}$

$$e^{-2/0.087} \approx 1 \times 10^{-10}$$

at  $T = 500\text{K}$ ,  $kT = 0.43\text{eV}$

$$e^{-2/0.43} \approx 9.6 \times 10^{-3}$$

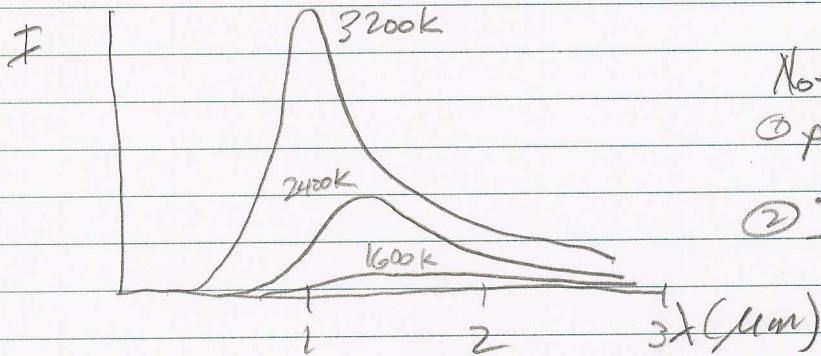
Thermal radiation - total radiation increases sharply with temperature, and peak shifts to shorter  $\lambda$  with  $T$ .

Stefan-Boltzmann law:  $I = \epsilon_m \sigma T^4$

total intensity radiated by surface of a hot body

Stefan-Boltzmann constant  $\sigma = 5.67 \times 10^{-8} \text{W/cm}^2 \cdot \text{K}^4$

emissivity:  $\epsilon_m$  - dimensionless  $0 \leq \epsilon_m \leq 1$   
ability of body to emit + absorb energy.



Notice:

- ① pk  $\lambda$  shifts shorter with  $T$   
 $\lambda_m T = \text{const}$
- ② Intensity increases at all  $\lambda$  with  $T$

If  $M_1$  is initially at higher  $T$  it radiates more and heats up  $M_2$ . But eventually,  $M_2$  must radiate as much as it absorbs to reach equilibrium.

Power/area incident on  $M_2$ :  $I_2$  "irradiance"  
 radiated by  $M_2$ :  $H_2$  "radiance"

$$H_2 = b_2 I_2$$

$b_2$  is fraction of incident power that is absorbed

If several masses are put inside the cavity, the intensity would be the same for all, so

$$I = \frac{H_2}{b_2} = \frac{H_3}{b_3} = \dots$$

This says the ratio of power radiated to fraction absorbed is a constant, independent of material.  
 Since  $H_2 = \epsilon_{M2} T^4$  and in equilibrium, all masses reach the same  $T$ , we can find  $\epsilon_M = b$

Perfect reflector  $\rightarrow \epsilon_M = 0$

Lamp black  $\rightarrow \epsilon_M = 0.95$

For the cavity inside  $I = \sigma T^4$ . If we open a small hole and allow  $I$  to come out, we have a perfect black body radiator at the hole.

### Planck theory of cavity radiation

Calculate the spectral energy density for radiation in thermal equilibrium with walls of cavity.

Total energy density in the e-m field:  $f$   
 Spectral energy density:  $f_\nu$

$$f = \int f_\nu dV$$

Spectral Intensity  $I_\nu = \frac{c}{4\pi} f_\nu$  (for light leaking out of a hole in cavity)

(note: for a beam traveling in a specific direction,  $I_\nu = \frac{c}{n} f_\nu$ .  
 But integrating light coming out of the hole over  $2\pi$  solid angle, taking into account  $\cos\theta$  factor for obliquity, we get the factor of  $\frac{1}{4}$ .)

For cavity radiation,  $f_V$  is calculated as product of  
[ # of modes/unit volume-unit frequency ] [ average energy / mode ]

↑  
Similar to density of states

Mode counting Solve wave equation in a rectangular box

$$E(x, y, z) \sim e^{i(k_x x + k_y y + k_z z)}$$

$$k^2 = k_x^2 + k_y^2 + k_z^2 \quad k = \frac{2\pi}{\lambda} = \frac{2\pi V}{c}$$

$$k_{x,y,z} = \frac{n_{x,y,z} \pi}{L_{x,y,z}} \quad n_{x,y,z} : \text{integer}$$

Total # of modes with frequency from  $0 \rightarrow V$  is  
Volume of one octant of sphere in  $k$ -space

$$N(V) = \frac{\frac{1}{8} \cdot \frac{4}{3} \pi \left(\frac{2\pi V}{c}\right)^3}{\left(\frac{\pi}{L_x}\right)\left(\frac{\pi}{L_y}\right)\left(\frac{\pi}{L_z}\right)} \times 2 = \frac{8\pi V^3}{3c^3} V$$

↑  
2 polarizations  
per mode

We want # modes / freq-vol :

$$\frac{1}{V} \frac{dN(V)}{dV} = \frac{8\pi V^2}{c^3}$$

Average energy / mode =  $\frac{hV}{\text{occupation probability}}$

for photons: Bose-Einstein distribution

$$\langle E \rangle = \frac{hV}{e^{hV/kT} - 1}$$

$$\text{So! } f_V = \frac{8\pi V^2}{c^3} \frac{hV}{e^{hV/kT} - 1}$$

$$I_V = \frac{2\pi hV^3}{c^2} \frac{1}{e^{hV/kT} - 1}$$

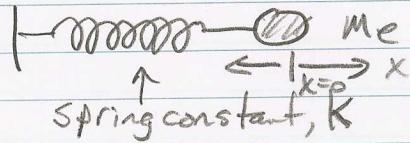
(HW: derive S-B law, find  $T$ )

## Atomic radiative transitions

### - Classical electron oscillator model

radiation from transitions occurs at discrete frequencies.  
Atom appears to behave as a simple oscillator

model - electron on a spring



Assuming no damping, simple spring eqn:

$$m_e \frac{d^2x}{dt^2} + kx = 0$$

$$x = x_0 e^{-j\omega_0 t} \quad \omega_0 = \sqrt{\frac{k}{m_e}} ; \quad k = m_e \omega_0^2$$

Oscillating electron radiates. Total power radiated by a dipole is  $P = \frac{\mu I^2 \omega_0^4}{12\pi \epsilon_0 c^3}$   $\mu$ : dipole moment =  $e x_0$

$$P = \frac{dE}{dt} = -\gamma_0 E \quad E: \text{total energy in the spring system}$$

$$E = E_0 e^{-\gamma_0 t} = E_0 e^{-t/t_{1/2}}$$

Radiative damping - add term  $\gamma_0 \frac{dx}{dt}$  to eqn of motion

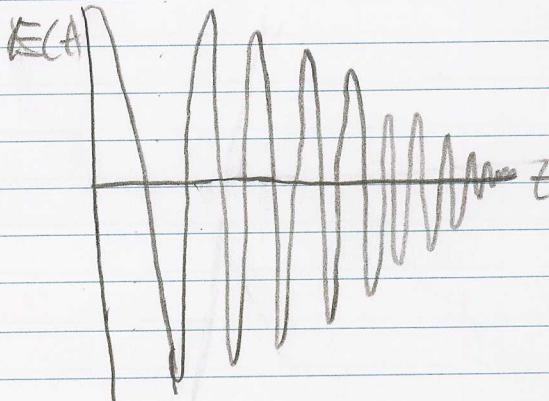
$$m \frac{d^2x}{dt^2} + \gamma_0 \frac{dx}{dt} + m_e \omega_0^2 x = 0$$

$$-t/t_{1/2} - j\omega_0 t$$

$$x \approx x_0 e^{-t/t_{1/2}} e^{-j\omega_0 t}$$

electric field radiated has same time dependence

$$I(t) = |E(t)|^2 = I_0 e^{-2t/t_{1/2}}$$



## Emission spectrum of radiating electron

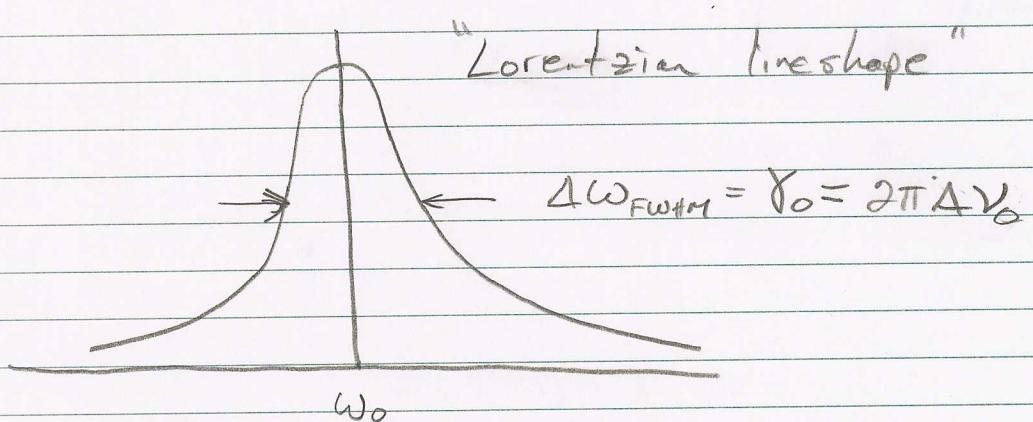
$$E(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} E(t) e^{j\omega t} dt \quad \text{Fourier transform}$$

$$= \frac{E_0}{\sqrt{2\pi}} \int_0^{\infty} e^{j[(\omega - \omega_0) + j\gamma_0/2]t} dt$$

$$= -\frac{E_0}{\sqrt{2\pi}} \frac{1}{j[(\omega - \omega_0) + j\gamma_0/2]}$$

Intensity spectrum  $I(\omega) = |E(\omega)|^2$

$$I(\omega) = I_0 \frac{\gamma_0/2\pi}{[(\omega - \omega_0)^2 + \gamma_0^2/4]} ; \quad I_0 = \int_0^{\infty} I(\omega) d\omega \quad \text{total intensity}$$



## Quantum mechanical treatment

- Rigorous QM treatment is beyond our scope. Need QM theory for radiation field.
- Semiclassical treatment. QM for atoms, classical for field.

Radiation occurs during transition from level 2 to level 1. The atom develops a quantum mechanical dipole moment that radiates.

$$\psi_1(r, t) = u_1(r) e^{-j \frac{E_1}{\hbar} t} \quad \psi_2 = u_2(r) e^{-j \frac{E_2}{\hbar} t}$$

$u_{1,2}$  are stationary eigenfunctions of the atom

During the transition,  $\psi = a_1(t)\psi_1 + a_2(t)\psi_2$

- Electric dipole moment of atom

$$\vec{\mu} = - \int e |\psi|^2 \vec{r} dV$$

$$= -e \int \vec{r} |a_1|^2 |u_1|^2 dV - e \int \vec{r} |a_2|^2 |u_2|^2 dV$$

$$- e \int \vec{r} [a_1 a_2^* u_1 u_2^* e^{j\omega_0 t} + \text{cc}] dV \quad \omega_0 = \frac{E_2 - E_1}{\hbar}$$

oscillating dipole  $\mu_{\text{osc}} = \text{Re} [2 a_1 a_2^* \mu_{21} e^{j\omega_0 t}]$

$$\mu_{21} = \int u_2^* r u_1 dV \quad \text{electric dipole matrix element}$$

If we write the QM power radiated per atom:

$$P_R = \frac{dE}{dt} = A_{21} h V_{21} \quad A_{21}: \text{Spontaneous emission rate}$$

Compare to classical expression:

$$P_R = \frac{\mu^2 \omega_0^4}{12 \pi \epsilon_0 c^3}$$

Identify  $\mu$  with  $2 \mu_{21}$ : dipole matrix element, then

$$A_{21} h \omega_{21} = \frac{4 \mu_{21}^2 \omega_{21}^4}{12 \pi \epsilon_0 c^3}$$

or

$$A_{21} = \frac{\mu_{21}^2 \omega_{21}^3}{3 \pi \epsilon_0 h c^3} = \frac{1}{\tau_{\text{sp}}}$$