Homework #3
EE 232 – Lightwave devices (Spring 2019)

DUE: 3/07/2019 (Please hand-in homework prior to the beginning of lecture)

(1) In this problem, you will investigate bulk absorption and gain in indium phosphide (InP) at T=4K and T=300K. You can neglect the temperature dependence on the bandgap and assume $E_g = 1.344\text{eV}$ for both temperatures. Further, you may assume $m_e = 0.077m_0$, $m_h = 0.6m_0$, $n_r = 3.5$ and $E_p = 20.7\text{eV}$.

a. Estimate the transparency carrier density ($n_{tr}$) for bulk InP at T=300K. Assume only the heavy-hole band is populated in the valence band.

b. Plot the gain spectrum (gain as a function of photon energy) for injected carrier density of $n = 0.2n_{tr}$, $n = n_{tr}$, $n = 2n_{tr}$, and $n = 4n_{tr}$ at T=4K and T=300K. Assume only transitions between the conduction band and heavy-hole band. You will have to calculate the quasi-Fermi levels using the Fermi-Dirac integral. Refer to equation (38) in the paper by Blakemore for an approximation to the Fermi-Dirac integral.

(2) In this problem, you will investigate bulk absorption and gain in an AlGaAs/GaAs quantum well at T=0K. You can assume the GaAs bandgap to be $E_g = 1.519\text{eV}$ (at T=0K) and the quantum well thickness to be $L_o = 8\text{nm}$. Further, the material parameters for GaAs are $m_e = 0.067m_0$, $m_{hh} = 0.5m_0$, $m_{lh} = 0.087m_0$, $n_r = 3.5$, and $E_p = 25.7\text{eV}$.

Assume the light traveling in the quantum well is transverse electric (TE).

a. Calculate the energy at the bottom of the first conduction subband ($E_{c1}$), and top of the light-hole ($E_{lh1}$) and heavy-hole subbands ($E_{hh1}$). As a simplification, you may use the infinite barrier model.

b. Write a general expression Fermi level splitting ($F_c-F_v$) as a function of carrier density for a quantum well at T=0K. Assume only the heavy-hole band is populated.

c. Plot the gain spectrum of the quantum well (gain as a function of photon energy) for Fermi level splitting of $F_c-F_v = 0$, and $F_c-F_v = 1.02(E_g + E_{c1} + |E_{hh1}|)$. Plot the spectra such that the transitions between each band can be observed on the chosen x-axis scale. Assume only transitions between the first quantized conduction, heavy-hole, and light-hole subbands (i.e. $n=m=1$) and assume the overlap integral is equal to unity.

(3) Show that, for a cube-like quantum dot with infinite barriers, the momentum matrix element is given by

$$\hat{\mathbf{e}} \cdot \mathbf{p}_{cv} = \langle \psi_c | \hat{\mathbf{e}} \cdot \mathbf{p} | \psi_v \rangle = \delta_{m,m} \delta_{n,n} \delta_{l,l} \langle u_c | \hat{\mathbf{e}} \cdot \mathbf{p} | u_v \rangle$$

where $\psi_c$ and $\psi_v$ are the Bloch states in the conduction and valence band, $u_c$ and $u_v$ are the Bloch functions, $\mathbf{p}$ is the momentum operator, $\delta$ is the Kronecker delta function where the indices correspond to the quantum number of the confined state in the $x,y,$ and $z$ directions.