

Recombination Processes
 (reading in supplementary references: Singh, Pankove)

Radiative recombination

Radiative recombination is the reverse of absorption. If excess carriers are present, the system is not in equilibrium. This can lead to the emission of photons which is sometimes referred to as luminescence.

Luminescence is sometimes specified based on the method of excitation as follows:

Excitation Process	Emission Process
Current Injection	Electroluminescence
Optical Excitation	Photoluminescence
Electron Beam Excitation	Cathodoluminescence
Ultrasonic Acoustic Excitation	Sonoluminescence

We can calculate the emission rate and spectrum from absorption rate and spectrum. The method is similar to the Einstein A & B coefficient analysis for atoms.

We put the system in thermal equilibrium. Then emission rate = absorption rate. The thermodynamic principle of detailed balance holds that the rates must balance for each frequency ν .

$$R(\nu) d\nu = P(\nu) \rho(\nu) d\nu$$

↑

emission
rate per unit
volume

↑

probability
per unit time
of absorbing
photon ν

↑

density of pho-
tons of freq ν

We also can relate the absorption probability to the absorption coefficient:



where n is the index of refraction. In thermal equilibrium, the photon field is described by the Planck distribution:

$$\rho(\nu) = \frac{8\pi\nu^2 n^3}{c^3} \frac{1}{e^{h\nu/kT} - 1}$$

Plugging these in, we find:

$$R(\nu) = \frac{\alpha(\nu)8\pi\nu^2 n^2}{c^2(e^{h\nu/kT} - 1)}$$

The total recombination rate in thermal equilibrium is obtained by integrating over frequency:

$$R_0 = \int_0^\infty R(\nu)d\nu = \frac{8\pi(kT)^3}{c^2 h^3} \int_0^\infty \frac{\alpha\left(\frac{kTx}{h}\right) n^2 \left(\frac{kTx}{h}\right)^2 dx}{e^x - 1}$$

since $\alpha = 0$ for $h\nu < E_g$, the exponential falls off fast and n usually varies slowly

$$R_0 \cong \frac{8\pi(kT)^3 n^2}{ch^3} \int_0^\infty \frac{\alpha x^2 dx}{e^x - 1}$$

This is known as the “Van Roosbroeck - Shockley relation”
if $\alpha(\nu)$ is known, then $R(\nu)$ and R_0 can be calculated directly.

Out of equilibrium

Since one electron recombines with one hole, the recombination rate goes like:



In thermal equilibrium $n_0 p_0 = n_i^2$, so the thermal equilibrium recombination rate is:

$$R_0 = A n_0 p_0 = A n_i^2$$

Thus $R = R_0 \frac{n p}{n_i^2}$, so if we know α , we can calculate R_0 and R .

The deviation of the carrier densities from equilibrium is given by $n = n_0 + \Delta n$,
 $p = p_0 + \Delta p$.

Charge neutrality demands that .

Strong injection case $\Delta n \gg n_0, p_0$

(easily achieved by pulsed laser excitation, for example)



solution $\Delta n(t) = \frac{\Delta n(0)}{1 + \frac{R\Delta n(0)}{n_i^2}t}$ non-exponential decay

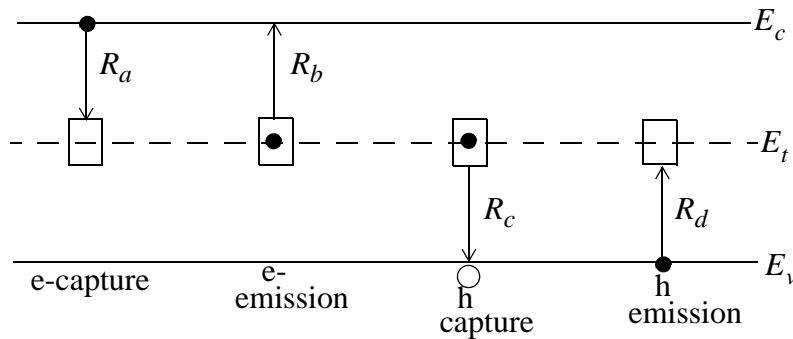
more typically - consider p-type $p_0 > n_0$

and $\Delta n \ll p_0$ - low injection

$$\frac{d\Delta n}{dt} = -\frac{\Delta n p_0}{n_0 p_0} R_0 = -\frac{\Delta n}{n_0} R_0 = -\frac{\Delta n}{\tau}$$

"minority carrier lifetime"

Indirect Recombination (SRH: Shockley-Read-Hall theory)



recombination center, deep level

example : acceptor-like deep level - neutral empty, negative when occupied

let N_t = total density of traps

F = occupation probability of trap

$$R_a = \sigma_n v_{th} n N_t (1 - F)$$

The probability that the trap is empty = $1 - F$. Then the electron capture rate is written:

$$R_a = \sigma_n v_{th} n N_t (1 - F)$$

v_{th} - thermal velocity

σ_n - capture cross section

The electron emission rate is written:

$$R_b = e_n N_t F$$

e_n - emission probability

For thermal equilibrium $R_a = R_b$

$$v_{th} \sigma_n n N_t (1 - F) = e_n N_t F$$

$$\therefore e_n = \frac{v_{th} \sigma_n n (1 - F)}{F}$$

In equilibrium, $n = n_i e^{(E_F - E_i)/kT}$, where E_i is the intrinsic Fermi-level, given by:

$$E_i = \frac{E_c + E_v}{2} + \frac{3}{4} kT \ln \frac{m_h^*}{m_e^*}$$

$$R_b = e_n N_t F$$

$$\therefore e_n = v_{th} \sigma_n n_i e^{(E_i - E_i)/kT}$$

which shows how the emission probability is related to the capture cross section. Similarly for hole capture:

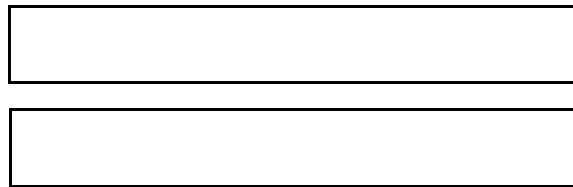
$$R_c = v_{th} \sigma_p p N_t F$$

For hole emission:

$$R_d = e_p N_t (1 - F)$$

$$e_p = v_{th} \sigma_p n_i e^{(E_i - E_t)/kT}$$

Now consider the non-equilibrium case. We assume some free carrier generation rate G . In steady state: $R_a \neq R_b$, $R_c \neq R_d$.



$$\text{then } R_a - R_b = R_c - R_d$$

$$v_{th} \sigma_n N_t [n(1 - F) - n_i e^{(E_t - E_i)/kT} F] = v_{th} \sigma_p N_t [pF - n_i e^{(E_i - E_t)/kT} (1 - F)]$$

solving for F:

$$F [n \sigma_n + n_i \sigma_n e^{(E_t - E_i)/kT} + p \sigma_p + n_i \sigma_p e^{(E_i - E_t)/kT}] = n \sigma_n + n_i \sigma_p e^{(E_i - E_t)/kT}$$

$$F = \frac{n \sigma_n + n_i \sigma_p e^{(E_i - E_t)/kT}}{n \sigma_n + n_i \sigma_n e^{(E_t - E_i)/kT} + p \sigma_p + n_i \sigma_p e^{(E_i - E_t)/kT}}$$

then the net recombination rate is:

$$U = R_a - R_b = v_{th} \sigma_n N_t [n - (n + n_i e^{(E_t - E_i)/kT}) F]$$

$$= v_{th} \sigma_n \sigma_p N_t \frac{[pn - n_i^2]}{\sigma_n (n + n_i e^{(E_t - E_i)/kT}) + \sigma_p (p + n_i e^{(E_i - E_t)/kT})}$$

example: n-type, low injection $n \gg p$ $n = n_o + \Delta n$ $p = p_o + \Delta p$

For a trap located near mid-gap: $n \gg n_i e^{(E_t - E_i)/kT}$

$$\text{then } U \cong v_{th} \sigma_p N_t \frac{\sigma_n (pn - n_o p_o)}{\sigma_n n}$$

$$= v_{th}\sigma_p N_t \frac{[(p_o + \Delta p)(n_o + \Delta n) - n_o p_o]}{n}$$

$$\cong v_{th}\sigma_p N_t \left[\frac{\Delta n p_o + \Delta p n_o}{n_o + \Delta n} \right]$$



Since U is the recombination rate, we can write the following differential equation:

$$\frac{d\Delta p}{dt} = -v_{th}\sigma_p N_t \Delta p = -\frac{\Delta p}{\tau_p}$$

where we define the minority carrier lifetime τ_p :



Dependence of τ on E_t

Assume $\sigma_n \cong \sigma_p = \sigma_o$

$$U \cong v_{th}\sigma_o N_t \frac{(pn - n_i^2)}{p + n + 2n_i \cosh\left(\frac{E_i - E_t}{kT}\right)}$$

take $n = n_o + \Delta n$ $p = p_o + \Delta p$

recall that $\Delta n = \Delta p$

$$U = v_{th}\sigma_o N_t \frac{\Delta p(n_o + p_o)}{n_o + p_o + 2n_i \cosh\left(\frac{E_i - E_t}{kT}\right)}$$

so the minority carrier lifetime is more generally:

$$\tau = \frac{1 + \frac{2n_i}{n_o + p_o} \cosh\left(\frac{E_i - E_t}{kT}\right)}{v_{th}\sigma_o N_t}$$

Consider the case of depletion (reverse bias p n junction) $n \ll n_i$ $p \ll n_i$, so we have car-

rier generation instead of recombination.



→ leads to excess reverse leakage current in diodes

