Optimal Routing Control: Repeated Game Approach

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**Motivation**
- Bandwidth demand → efficient routing schemes
- Network access providers compete to offer services and to minimize their costs
- Providers interact several times before game changes and they should have an incentive to cooperate
- Nash equilibrium point (NEP)
- Fixed demand: total system cost measures network efficiency
- Goal: move towards the system minimum and stay in NEP's
- reduce the costs for some, others do not want to deviate

**Repeated Games**
- Interaction occurs several times
  - strategy: history-based plan of actions
  - perfect information games (all actions taken known)
  - infinite games (game end is unknown)
- Discounted games:
  \[ r_i = (1-\delta) \sum \delta^n r_i(s_i(n),...,s_f(n)) \]
- NEPs
  - not only repeating a stage game NEP
  - problem with NEP's: an action may be irrational
  **SPNEP**: NEP for every possible game continuation

**Games - Results**
- Simultaneous strategy:
  \[ f \in F^1 \times \cdots \times F^f \]
- **Rosen's theorem** (static game)
  \[ j^i \text{ is continuous in } f \text{ and convex in } F^i \]
  \[ \text{NEP existence} \]
- **NEP uniqueness**
- **i-th player's reservation cost**
  \[ V_i = \max_{f \in F^f} \min_{f^i \in F^i} J^i(f^i, f^{-i}) \]
- **folk theorem** (repeated game)
  - For each vector \[ v_i \leq v_f \], there is \[ \delta < 1 \text{ s.t. } \delta \in (\delta, 1) \]
  - there is a NEP with the cost vector \[ v \]

**Cooperation in Repeated Games**
- \[ \delta \to 0: \text{ static game} \]
- \[ \delta \to 1: \text{ average rewards per unit time} \]
  - rational (SPNEP) strategies that enforce cooperation
- Main idea: any gain from a deviation will be outweighed by the penalty in the stages after the deviation
- Prisoners' Dilemma example:
  - credible threat strategy: "I will choose to mum as long as you do. If you choose fink once, I will fink thereafter."
  - if \[ \delta \] is sufficiently large this strategy is SPNEP and enforces the decisions (mum,mum) forever

**Games – Results (2)**
- **Friedman's theorem** (repeated)
  Let a stage game NEP has cost \( e \)
  - For each vector \( v_i \leq e_f \) there is \( \delta < 1 \text{ s.t. } \delta \in (\delta, 1) \)
  - there is a SPNEP with the cost vector \( v \)
- **Fudenberg-Maskin's theorem** (repeated game)
  Let \( v \) be the reservation cost
  - If the set \( \{ v_i \leq v_f, 1 \leq i \leq f \} \) is fully dimensional
  - there is a SPNEP with the cost vector \( v \)
### Main Problem
- Existence of SPNEPs that cost less than single stage NEPs
- Two-node parallel link networks
- General networks
  - single source-destination pair
  - multiple source-destination pairs

### Parallel Links – Game Model
- $I$ users with demand rates $r'_i, 1 \leq i \leq I$
- $L$ links with capacities $C'_l, 1 \leq l \leq L$
- $R = \sum_{l=1}^{L} r'_l \leq \sum_{l=1}^{L} C'_l$ (stability constraint)
- $f'_i$: user $i$ sends over link $l$
  - $f'_i > 0$ (nonnegativity), $\sum_{i=1}^{I} f'_i = r'_i$ (demand constraint)
  - $f'_i$: $i$-th user flow configuration, $f = (f'_1, \ldots, f'_I)$: flow configuration
  - $f'_i$: total flow on $i$-th link

### Parallel Links – Cost Function
- Type-B cost functions
  - $V_i(l) = \sum_{l=1}^{L} f'_l (t'_l f'_l) L_i(l)$
  - $L_i(l)$ cost per unit flow is a function of the residual capacity $C'_l - f'_l$
  - $T_i(f'_i)$ is positive, strictly increasing in $f'_i$, and convex
  - $T_i(f'_i) \rightarrow \infty$, as $f'_i \rightarrow C'_l$

### Parallel Links – Static Game
- Competitive Routing in Multiuser Communication Networks
  - [Orda, Rom, Shimkin]
- Rosen’s theorem
- Total operation cost $C(f) = \sum_{l=1}^{L} V_i(l)$
- Convex optimization
- NEPs are not efficient: can be far from optimum

### Parallel Links – Repeated Game
- $T_1$: If $\delta$ is close to one there is an NEP $\tilde{f}$ in the repeated game that achieves the minimum system cost $C'$
  - $\tilde{f} = (\tilde{f'}_1, \ldots, \tilde{f'}_I)$, $\tilde{f'}_i = \frac{C'_l f'_i}{R}$
  - Fairness: the same cost per unit flow for every user
  - If $\tilde{f}$ is a stage game NEP then possibly $J'(\tilde{f}) > J'_{t}(\tilde{f})$

- $T_2$: If $\delta$ is close to one there is an SPNEP $\tilde{f}$ in the repeated game that achieves the minimum system cost $C'$ and the cost of each user is at most equal to its cost in the stage game NEP (for each $i$, $J'(\tilde{f}) \leq J'(\tilde{f}))$
  - If $\overline{C} > C'$ then $J'(\tilde{f}) < J'(\tilde{f})$
  - Decrease in the total system cost benefits all users

### Single Source-Destination Pair
- Network model
  - topology: $G = (V, L)$, $E \subseteq V \times V$
  - same: demand, capacity model
    - Type-B cost function
    - different: paths share links
- Unique flow $f'$ configuration with minimum cost $C'$
  - Assumption: two paths with different cost per unit flow

- $T_3$: If $\delta$ is close to one there is an NEP in the repeated game that achieves the minimum system cost $C'$
  - users with small demands use the paths with smallest cost per unit
- $T_4$: Any optimal NEP flow configuration is SPNEP
Multiple Source-Destination Pairs

- Different users have different SD pairs
- Uniqueness of the minimal cost configuration: open
- Negative result: uniqueness holds, but no matter how close is, NEP does not exist

\[ C_1 = 8, C_2 = 8, C_3 = 50, C_4 = C_5 = 2000 \]

\[ r_1 = 40, r_2 = 4.857 \]

Summary

<table>
<thead>
<tr>
<th>Unique optimal configuration</th>
<th>SPNEP</th>
<th>SPNEP cost less than in stage game</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parallel links</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Single SD pair</td>
<td>YES</td>
<td>YES</td>
</tr>
<tr>
<td>Multiple SD pairs</td>
<td>?</td>
<td>NO</td>
</tr>
</tbody>
</table>

Discussion