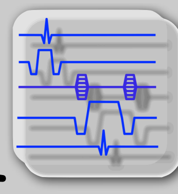


Principles of MRI

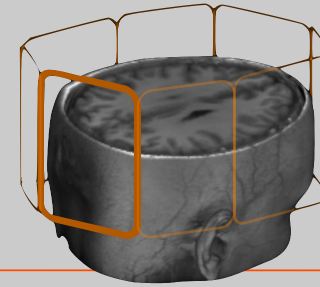
EE225E / BIO265

Lecture 21

Instructor: Miki Lustig
UC Berkeley, EECS

Today

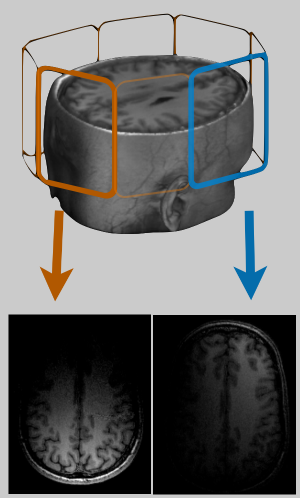
- Imaging with multiple coils
  - Gain SNR
  - Gain Speed (parallel imaging)



Surface Coils

- So far assumed coils sensitivity is uniform
- However....
  - Each coils has Amp+Phase sensitivity

$$S_\gamma(\vec{r}) \quad | \; \gamma = 1, \cdots, N$$



Imaging with Multiple Coils (Roemer 1990)

- Received Data from coil γ

$$s_\gamma(t) = \int_R \underbrace{m(\vec{r})S_\gamma(\vec{r})}_{\text{Weighted image}} \underbrace{e^{-i2\pi\vec{k}(t)\cdot\vec{r}}}_{\text{encoding}} dr$$

$$+ \int_R \underbrace{n'_\gamma(\vec{r})}_{\text{body noise}} \cdot S_\gamma(\vec{r}) dr$$

Noise

- Coils are coupled → Noise correlated

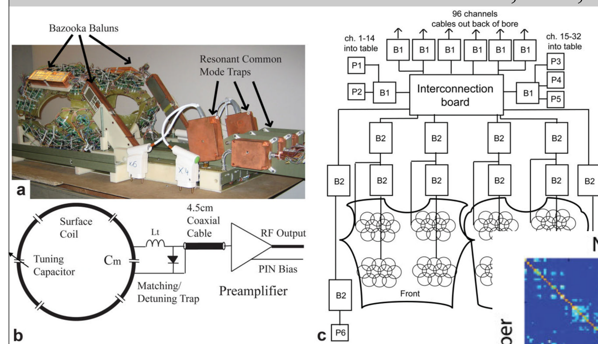
$$n_\gamma = \int_R n'_\gamma(\vec{r}) S_\gamma(\vec{r}) dr \quad \sim \quad \mathcal{N}(0, \sigma^2)$$

$$E[n_i, n_j] = \Psi^{[N \times N]}$$

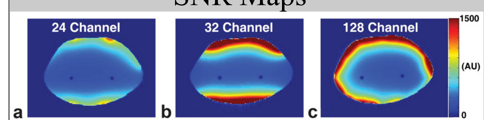
$$n \quad \sim \quad \mathrm{N}(0, \Psi)$$

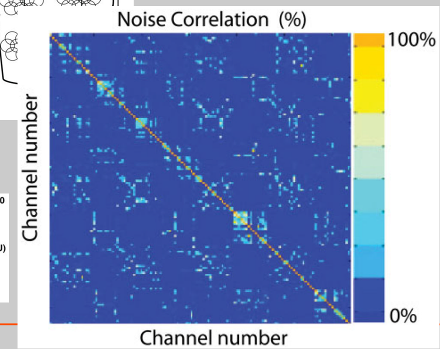
Example

M. Schmitt, et al, MRM 2008;59(6):1431-9



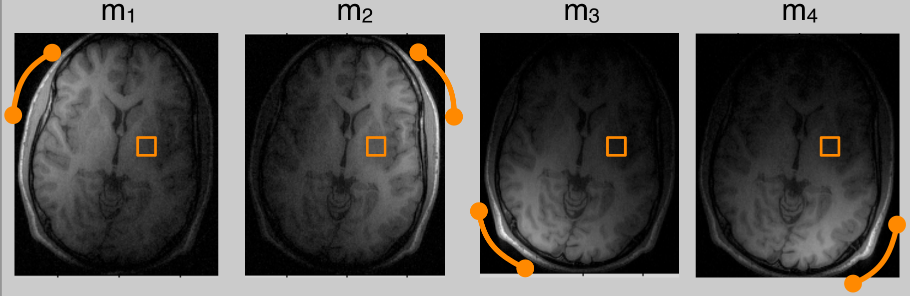
SNR Maps





Reconstruction (Roemer 1990)

- How to reconstruct m(r)?
  - Case 1: Full Fourier encoding → Obtain m_γ



  - N measurements for each voxel

FOR VOXEL r_p

$$\begin{bmatrix} m_1(\vec{r}_p) \\ m_2(\vec{r}_p) \\ \vdots \\ m_N(\vec{r}_p) \end{bmatrix} = \begin{bmatrix} S_1(r_p) \\ \vdots \\ S_N(r_p) \end{bmatrix} m(\vec{r}_p) + \begin{bmatrix} n_1(r_p) \\ n_2(r_p) \\ \vdots \\ n_N(r_p) \end{bmatrix}$$

Or

$$\underbrace{m_s(\vec{r}_p)}_{N \times 1} = \underbrace{S}_{\substack{N \times 1 \\ \text{Sensitivity}}} \underbrace{m(\vec{r}_p)}_{\substack{1 \times 1 \\ \text{image pixel}}} + \underbrace{\vec{n}}_{\substack{N \times 1 \\ \text{noise}}}$$

Minimum variance estimate

$$\underbrace{m_s(\vec{r}_p)}_{N\times 1} = \underbrace{S}_{\substack{N\times 1\\ \text{Sensitivity}}} \underbrace{m(\vec{r}_p)}_{\substack{1\times 1\\ \text{image}\\ \text{pixel}}} + \underbrace{\vec{n}}_{\substack{N\times 1\\ \text{noise}}}$$

$$\hat{m}(\vec{r}_p) = \underbrace{(S^*\Psi^{-1}S)^{-1}}_{1\times 1} \underbrace{S^*\Psi^{-1}}_{1\times N} \underbrace{m_s(\vec{r}_p)}_{N\times 1}$$

$$\mathrm{var}(\hat{m}(\vec{r}_p)) = (S^*\Psi^{-1}S)^{-1}$$

Special case

$$\Psi = \sigma^2 I$$

$$\hat{m}(\vec{r}_p) = (S^*(\sigma^2 I)^{-1} S)^{-1} S^*(\sigma^2 I)^{-1} m_s(\vec{r}_p) =$$

$$= (S^*S)^{-1} S^* \vec{m}_s(\vec{r}_p) =$$

$$= \frac{1}{\sum_\gamma |S_\gamma(\vec{r}_p)|^2} \sum_\gamma S_\gamma^*(\vec{r}_p) m_\gamma(\vec{r}_p)$$

SOS

- Often sqrt of sum of squares used

$$M_{ss} = \sqrt{\sum M_\gamma^* M_\gamma}$$

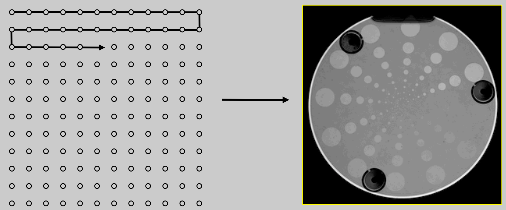
for SNR >20, within 10% of optimum

(Long thought to be adequate)

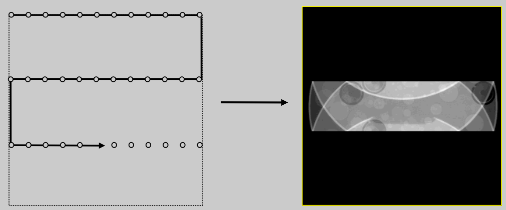
Multi-Coils for Speed

- Multi coils buy SNS → Tradeoff for Speed
- Many methods:
  - Original: SMASH - k-space approach
  - Popular:
    - SENSE - image approach, requires sensitivities
    - GRAPPA - k-space approach, autocalibrating
  - Here we only talk on SENSE in 2DFT
    - assume S_γ are known! (not so good assumption)

K-space Sampling

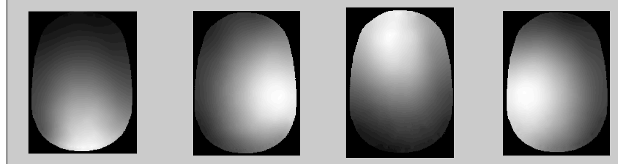


Standard k-space sampling

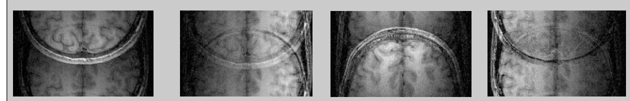


Reduced k-space sampling

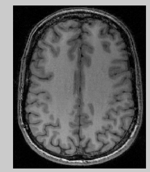
Parallel Imaging (Basic Idea)



coil sensitivities



3x undersampling

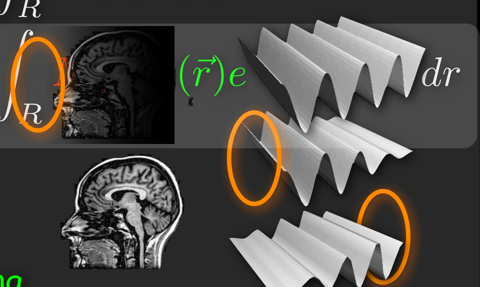


reconstruction

Sensitivity Encoding

$$s(t,k(t)) = \int_R M(\vec{r})S(\vec{r})e^{-i2\pi\vec{k}(t)\cdot\vec{r}}\,dr$$

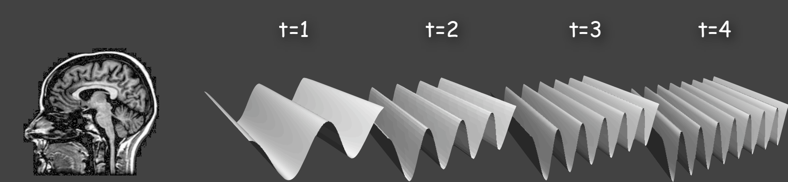
$s(t,k(t)) = \int_R$ $(\vec{r})e$ dr

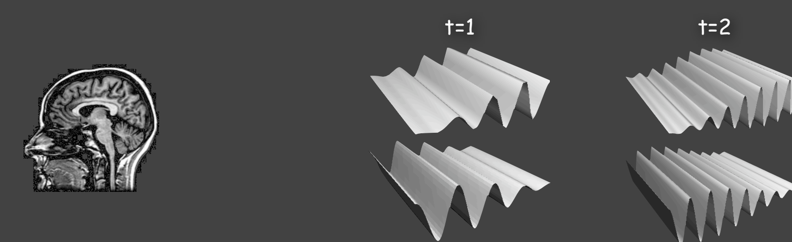


Fourier Encoding

Sensitivity Encoding

Parallel Imaging

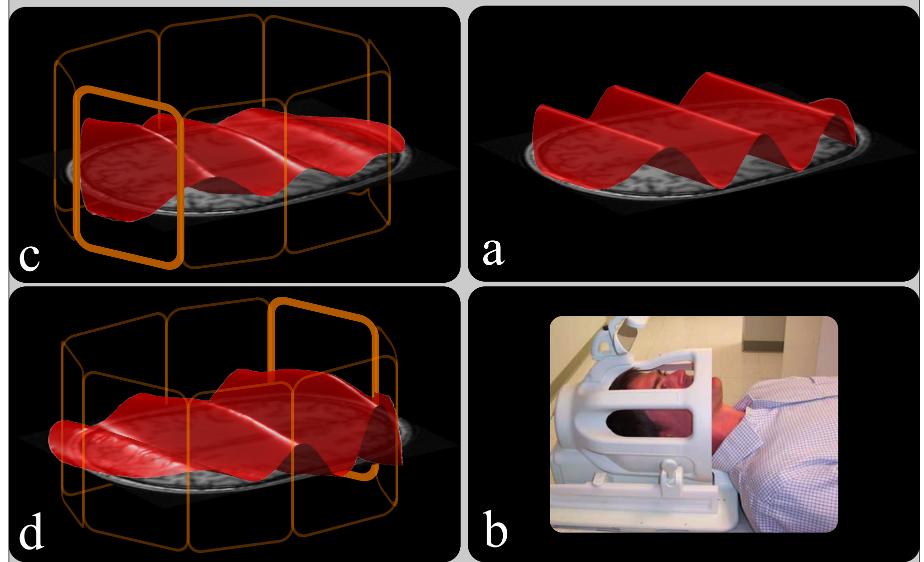




Parallel Imaging

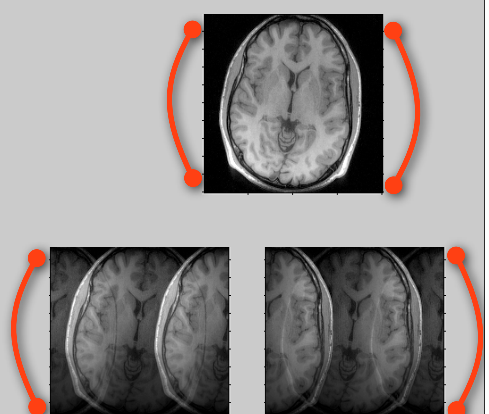
$$s_\gamma(t) = \int_R \underbrace{m(\vec{r})}_{\text{Image}} \underbrace{S_\gamma(\vec{r}) e^{-i2\pi \vec{k}(t)\cdot\vec{r}}}_{\text{Encoding}} dr$$

- Previously: 1 Enc/ time point, N images
- Now: N Enc/time point, 1 image
- Faster by skipping phase encodes!



SENSE (Pruessmann 99)

- Basic IDEA:
  - Trade SNR for speed in multi-coil acq.
  - Less Phase encodes ⇒ aliased images
  - Sort out (unfold) using knowledge of coil patterns



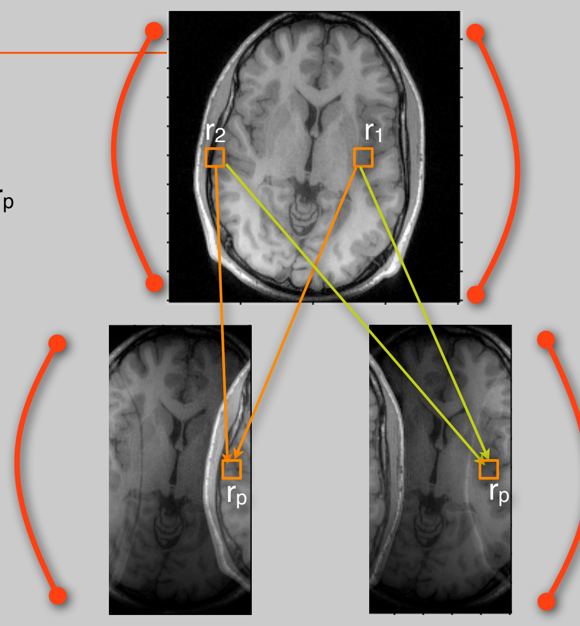
coil 1 image

coil 2 image

R = x2

SENSE

2 receive coils
2 source pixels alias
to one image pixel @r_p



SENSE

2 measurements of m(r1), m(r2)
each with different weighting

$$\begin{bmatrix} m_1(r_p) \\ m_2(r_p) \end{bmatrix} = \underbrace{\begin{bmatrix} S_1(r_1) & S_1(r_2) \\ S_2(r_1) & S_2(r_2) \end{bmatrix}}_{\text{sensitivity at source voxel}} \begin{bmatrix} m(r_1) \\ m(r_2) \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \end{bmatrix}$$

$$m_s(r_p) = Sm + n$$

SENSE

SAME MINIMUM VARIANCE SOLUTION

$$\underset{2\times 1}{\hat{m}} = (\underbrace{S^* \Psi^{-1} S}_{2\times 2})^{-1} \underbrace{S^* \Psi^{-1}}_{2\times 2} \underbrace{\vec{m}_s(\vec{r}_p)}_{2\times 1}$$

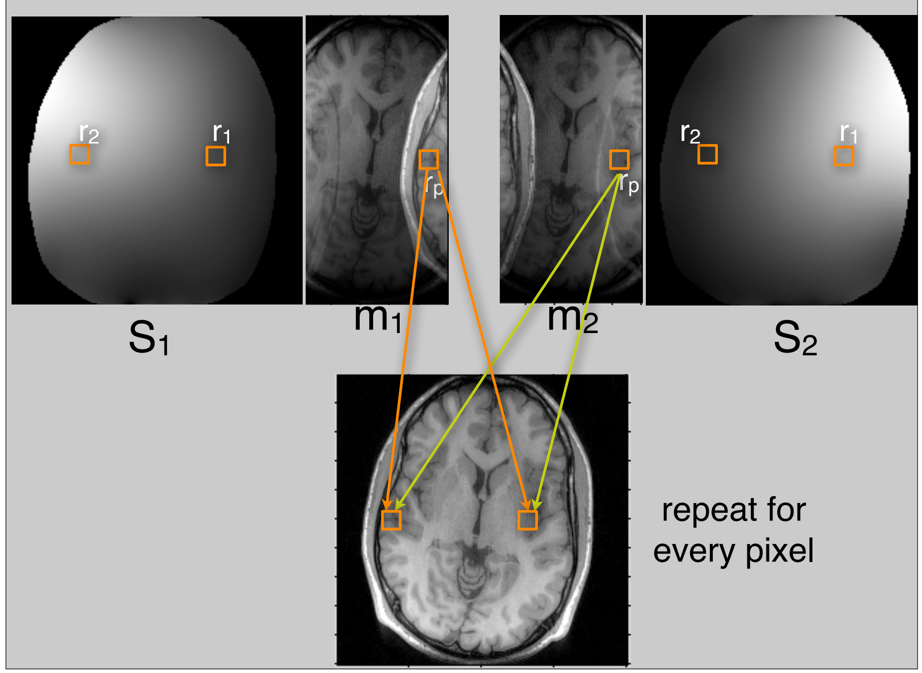
2 ALIASED RECONSTRUCTIONS "UNFOLDED" INTO 2 IMAGE PIXELS.

SENSE

(-) GENERAL CASE $R = M$ $\gamma = 1 \ldots N$

$$\underset{N\times 1}{m_s(\vec{r}_p)} = \underset{N\times M}{S} \cdot \underset{M\times 1}{\vec{m}} + \underset{N\times 1}{\vec{n}}$$

$$\underset{M\times 1}{\hat{m}} = \underset{M\times M}{(S^* \Psi^{-1} S)^{-1}} \underbrace{S^* \Psi^{-1}}_{M\times N} \underset{N\times 1}{m_s(\vec{r}_p)}$$

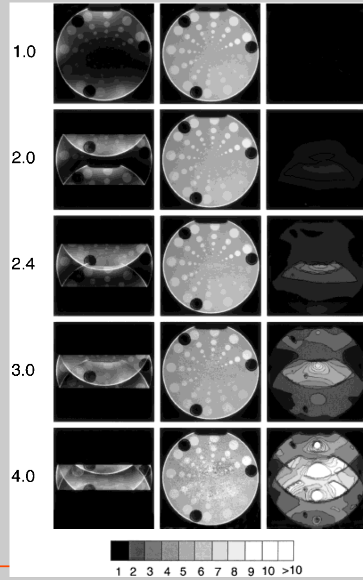


Noise Amplification g-factor

$$SNR_\rho^{red} = \frac{SNR_\rho^{full}}{g_\rho \sqrt{R}}$$

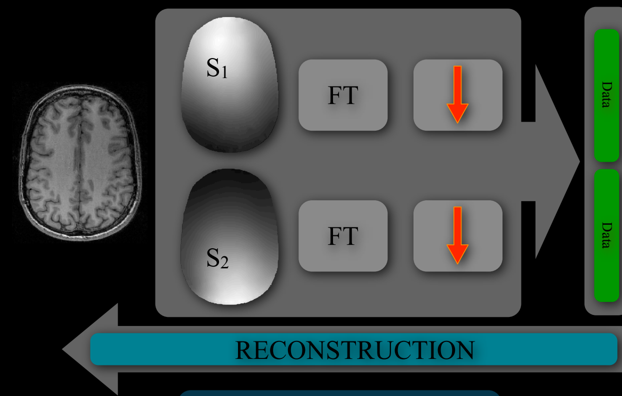
$$R = \frac{n_K^{full}}{n_K^{red}}$$

$$g_\rho = \sqrt{[(S^H \Psi^{-1} S)^{-1}]_{\rho,\rho} \, (S^H \Psi^{-1} S)_{\rho,\rho}}$$



M. Lustig, EECS UC Berkeley

SENSE model



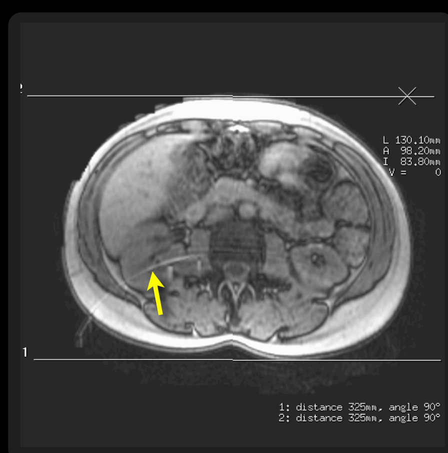
$$Ex = y$$

Pruessmann et. al., 1999

SENSE

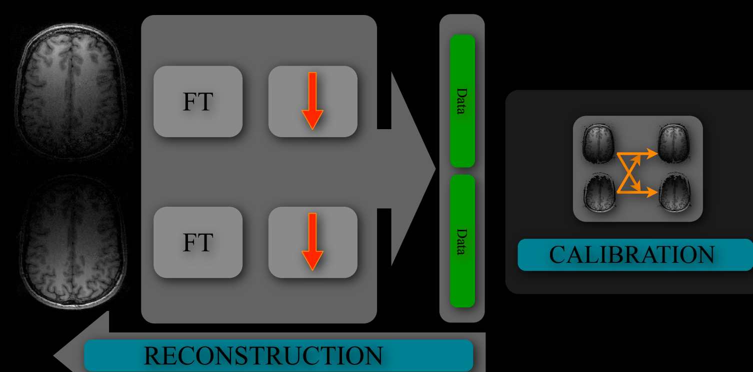
- Full inverse model
- Noise optimal
- One combined image

- Sensitive to map errors
  - Too low resolution
  - Misregistration
  - Folded maps



*image, courtesy of Kevin King

Autocalibrating Model (GRAPPA)

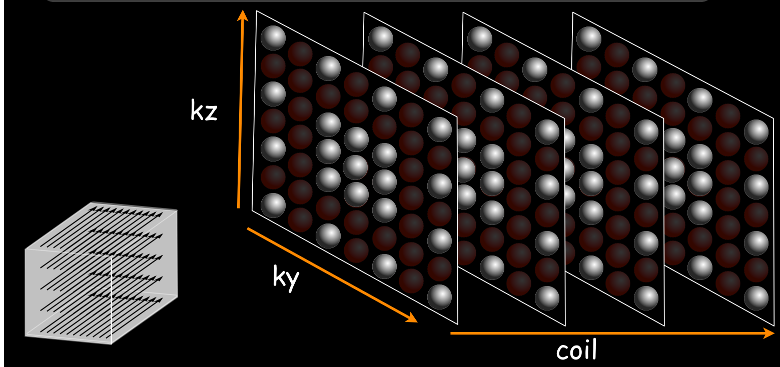


$$F_u x = y \qquad | x \in \mathbb{G}$$

Griswold et. al., 2002

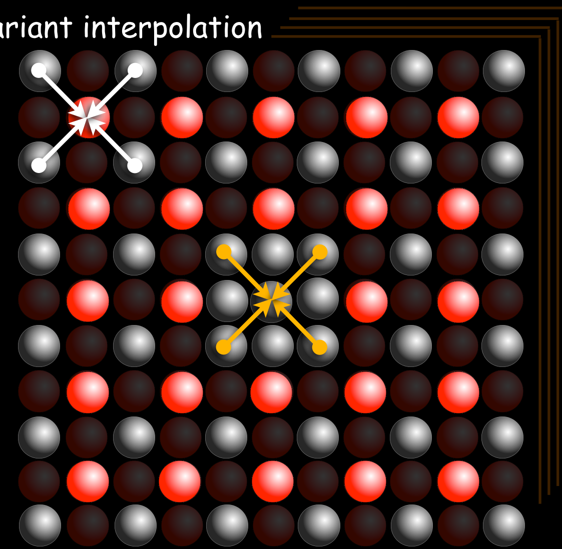
GRAPPA

- Reconstruction through interpolation
- Based on generalized sampling theory
- k-space vs. coil sampling domain



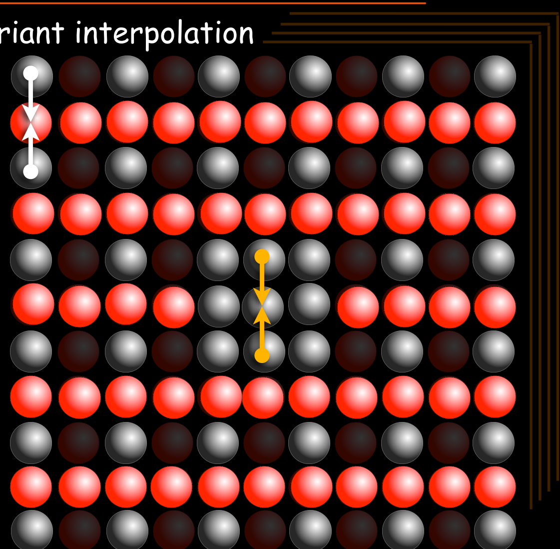
GRAPPA/ARC

k-Space variant interpolation



GRAPPA/ARC

k-Space variant interpolation



GRAPPA/ARC

k-Space variant interpolation