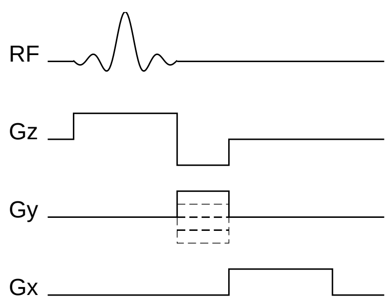
EE225E/BIOE265 Spring 2013 Miki Lustig
Principles of MRI

Assignment 4

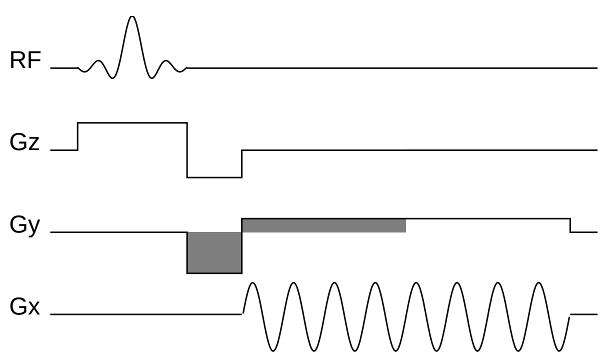
Due Feb 27, 2013

1. Finish reading Nishimura Ch.4 and Ch. 5.
2. The following pulse sequences are defective in that they don't cover a symmetric region in k-space. For each, (i) sketch the region of k-space that each trajectory covers, and (ii) propose a modification that provides symmetric coverage.

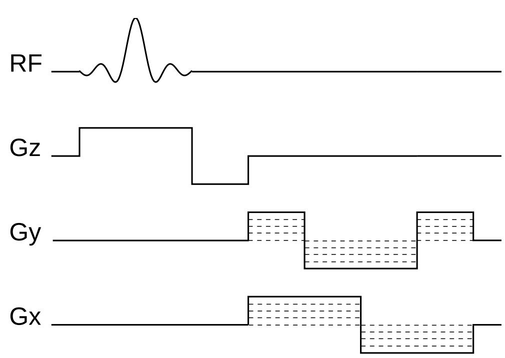
   (a)



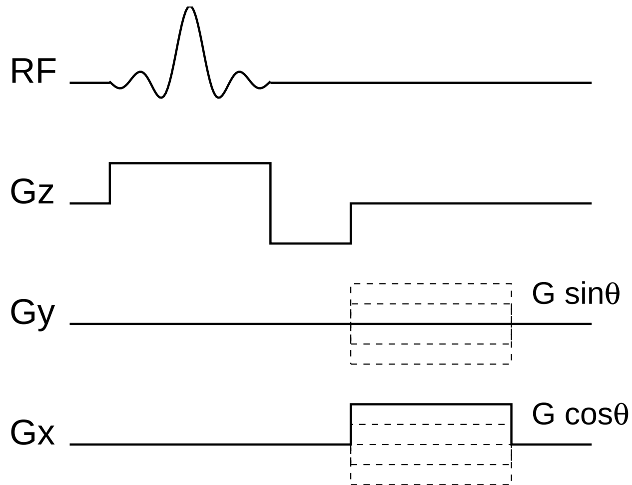
   (b) The shaded areas are equal in this pulse sequence.



   (c) The amplitude of the readout gradients are incremented together, so that for the i^{th} acquisition the gradient amplititudes are $G_x = G_y = G_{max}\left(\frac{i}{N}\right)$.



3. The following pulse sequence is performed repeatedly, with varying values of θ.



We would like the pulse sequence to cover a symmetric region of k-space.

(a) What range should the range of the angle θ be? Find the minimum and maximum values of θ, and don't worry about how finely θ is sampled. Sketch the resulting k-space trajectory, and k-space coverage.

(b) Compute the readout gradient duration if the maximum gradient is 4 G/cm, and we want 0.5 mm resolution.

4. A common problem in MRI is determining what scan parameters to use to maximize the contrast between two materials with different T_1 and T_2 times. Assume the two materials have the same M_0, and that they both have been excited by a 90° pulse. Define the difference in magnetization at a time t as

$$\begin{aligned} \Delta M_{xy}(t) &= M_{xy,A}(t) - M_{xy,B}(t) \\ \Delta M_z(t) &= M_{z,A}(t) - M_{z,B}(t) \end{aligned}$$

where material A has relaxation parameters $T_{1,A}$ and $T_{2,A}$, and material B has $T_{1,B}$ and $T_{2,B}$.

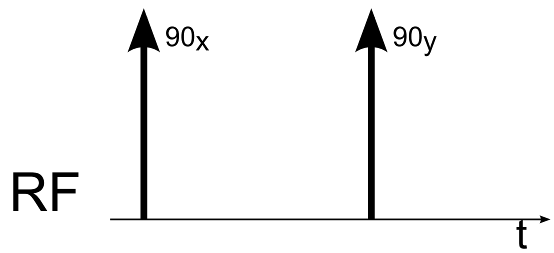
(a) Find an expression for the time that maximizes $|\Delta M_{xy}(t)|$.

(b) Find an expression for the time that maximizes $|\Delta M_z(t)|$.

(c) Assume that we are imaging the brain, and want to maximize the contrast between gray matter and white matter. The relaxation times for gray matter are $T_1 = 920$ ms, and $T_2 = 100$ ms, and for white matter are $T_1 = 790$ ms and $T_2 = 92$ ms. What are the optimum times for T_1 and T_2 contrast?

5. RF Excitation (Midterm I 2012)

Consider the following sequence of two RF pulses:



The notation 90_x, 90_y means that B_1 is applied respectively in the x and y directions (in the rotating frame). Assume that the RF pulse is instantaneous. At time $t = 0$ the magnetization is: $[0, 0, M_0]^T$. You can neglect T_2 decay and T_1 recovery.

a) What is the M_z component of the magnetization immediately after each RF pulse? explain.

$M_{z1} \ =$	$M_{z2} =$

b) Unfortunately, your RF excitation coil produces a B_1 inhomogeneous field with $\pm10\%$ variation in the flip angle. What is the resulting percentage variation in M_z due to the inhomogeneity (numbers!) ? Comment on the advantages or/and disadvantages of the 2-pulse scheme compared to a single RF when the target is to achieve maximum magnetization in the transverse plane.

$M'_{z1} \ = \ M_{z1} \ \pm$ comment :	$M'_{z2} \ = \ M_{z2} \ \pm$

c) The above analysis was done for spins at the Larmor frequency. However, due to field inhomogeneity some spins in the volume will precess at slightly different frequencies. What is the resulting M_z component of the magnetization after the second RF for spins with $\gamma\Delta B_0 = 250$Hz? Assume that the time between pulses is 1ms, so the magnetization precesses over 1/4th of a cycle in that time period. Comment on the sensitivity of this sequence to B_0 inhomogeneity.

6. This question is take from midterm Sp '10
TRUE or FALSE

For each of the following statements, identify weather it is True or False. To get a score you need also to provide a **brief explanation** for either case.

a) For sequences with short time-repetitions (TR), doubling the main field (B_0) always results in double the SNR.

TRUE/FALSE

b) The FID from n Conollyum nuclei ($\frac{\gamma}{2\pi} = 4$ kHz/G) at 1.5T is identical to the FID from n Pinesume nuclei ($\frac{\gamma}{2\pi} = 1$ kHz/G) at 6T.

TRUE/FALSE

c) (short T_2, long T_1) species are easier to image *in vivo* than (long T_2, short T_1) species.

TRUE/FALSE

7. **Matlab Excercise: Bloch simulation and spin visualization** In this exercise you will simulate and visualize the behavior of a spin in the presence of magnetic fields. For this purpose we will use a Matlab implementation of a Bloch simulator that was written by Prof. Brian Hargreaves of Stanford Radiology.

   - **The Bloch simulator**. Download the Bloch simulator files : bloch.m and bloch.c from the class website. The file bloch.c is a mex file implementation that is used to accelerate the computation of the solution for the Bloch equation. You will first need to compile the executable for your particular computer architecture. In your Matlab command window type:

     ```
     >> mex bloch.c
     ```

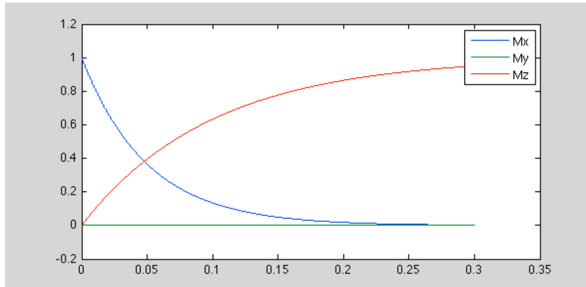
     Ignore any warnings from the compilation. If you encounter problems compiling the the mex file you can try downloading the executable for your platform from the class website
     Read the help for the function bloch by typing

     ```
     >> help bloch
     ```

     Here's a simple example showing T_2 decay and T_1 recovery:

     ```
     >> dt = 4e-6; % 4 us sampling rate
     >> rf_90 = 90/360/(4257*dt);

     >> % impulse RF pulse
     >> b1 = [rf_90;zeros(300e-3/dt,1)];
     >> g = b1*0; % no gradient

     % the spin is on-resonance
     >> df = 0;

     >> % the spin at iso-center
     >> dp = 0;

     >> t1 = 100e-3;
     >> t2 = 50e-3;

     >> % start at Mz
     >> mx_0 = 0;
     >> my_0 = 0;
     >> mz_0 = 1;

     >> % Simulate
     >> [mx,my,mz] = bloch(b1,g,dt,t1,t2,df,dp,2,mx_0,my_0, mz_0);

     >> %plot
     >> time = [1:length(mx)]*dt;
     >> figure, plot(time,mx,time,my,time,mz); legend('Mx','My','Mz');
     ```

     This is the plot you should get:

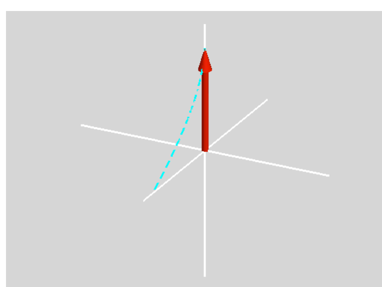


- Design an RF waveform, sampled at $4\mu s$ with maximum $|B_1| < 0.16$G that produces a 90° rotation. What is the length of the pulse and what is the amplitude?
- Design a gradient waveform with $|G| < 4$G/cm and $|\frac{dG}{dt}| < 15000$G/cm/s that produces a rotation of 2 cycles for spins located at 0.2cm off iso-center. You can use the function you wrote in the previous homework! What is the length of the pulse?
- Perform a simulation of a period of $t = 100$ms, in which first the RF is applied, followed by the gradient and then no-filed is applied. Simulate a spin at 0.2cm from iso-center with T_1 =30e-3 and T_2 =15e-3. Plot M_x,M_y, and M_z as a function of time.

- **Visualization:** Download the files: visualizeMagn.m, arrow3D.m, rotatePoints.m from the class website. The function visualizeMagn renders an animation of the spin and the fields. The first argument is a vector of the applied B_1 field in Gauss. The second argument is the vector of the effective B_0 field. In this case it will be the gradient field times the position of the spin. The third argument is an array of the magnetization vector vs time. It should be [N x 3] representing Mx My and Mz at each time point. The last argument is the acceleration factor of the rendering (you should use at least 10 if you are impatient).

  This is the plot you should get for the previous example by running:

  ```
  >> visualizeMagn(zeros(size(mx)), zeros(size(mx)), [mx,my,mz],100);
  ```

  Render the simulation and submit the plot of the final result.
- Now, simulate and visualize the sequence 90RF, Gradient, -90RF for a spin at: 0cm, 0.025cm, 0.05cm, 0.075cm and 0.1cm. Use the same gradient you used previously and $T1 = 1000$ and $T2 = 1000$.

  What can you say about the distribution of Mz in space after this sequence?