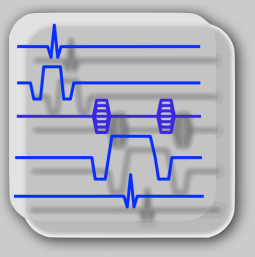


Principles of MRI

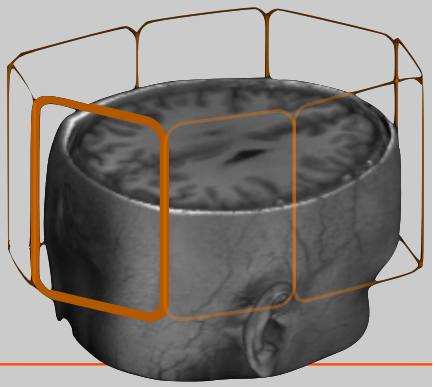
EE225E / BIO265

Lecture 24

Instructor: Miki Lustig
UC Berkeley, EECS

Today

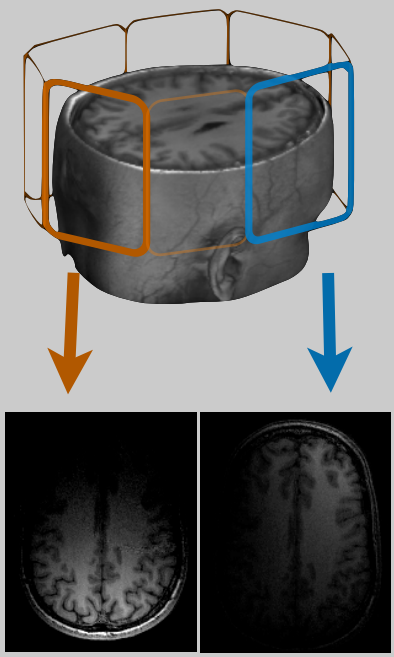
- Imaging with multiple coils
  - Gain SNR
  - Gain Speed (parallel imaging)



Surface Coils

- So far assumed coils sensitivity is uniform
- However....
  - Each coils has Amp+Phase sensitivity

$$S_\gamma(\vec{r}) \quad | \ \gamma = 1, \cdots, N$$



Imaging with Multiple Coils (Roemer 1990)

- Received Data from coil γ

$$s_\gamma(t) = \int_R \underbrace{m(\vec{r})S_\gamma(\vec{r})}_{\text{Weighted image}} \underbrace{e^{-i2\pi\vec{k}(t)\cdot\vec{r}}}_{\text{encoding}} dr$$

$$+ \int_R \underbrace{n'_\gamma(\vec{r})}_{\text{body noise}} \cdot S_\gamma(\vec{r}) dr$$

Noise

- Coils are coupled → Noise correlated

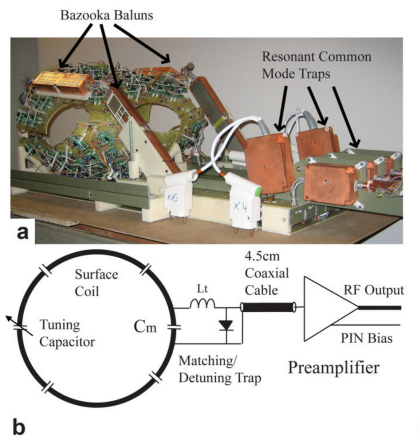
$$n_\gamma = \int_R n'_\gamma(\vec{r}) S_\gamma(\vec{r}) dr \quad \sim \quad \mathcal{N}(0, \sigma^2)$$

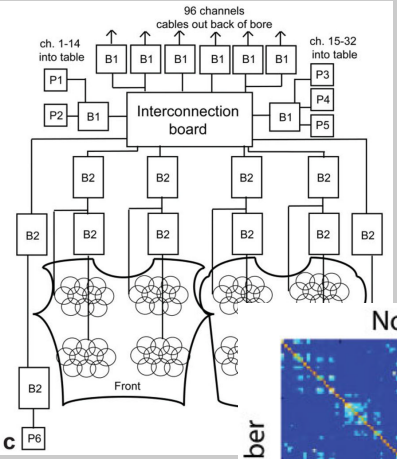
$$E[n_i, n_j] = \Psi^{[N \times N]}$$

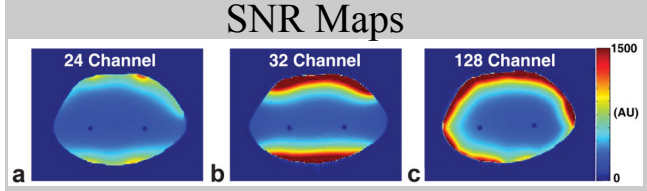
$$n \quad \sim \quad \mathrm{N}(0, \Psi)$$

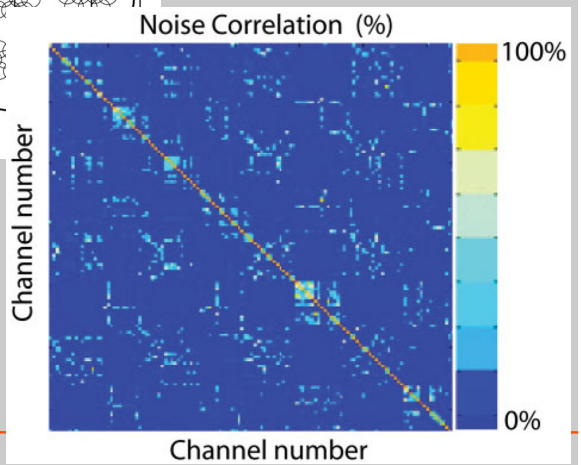
Example

M. Schmitt, et al, MRM 2008;59(6):1431-9



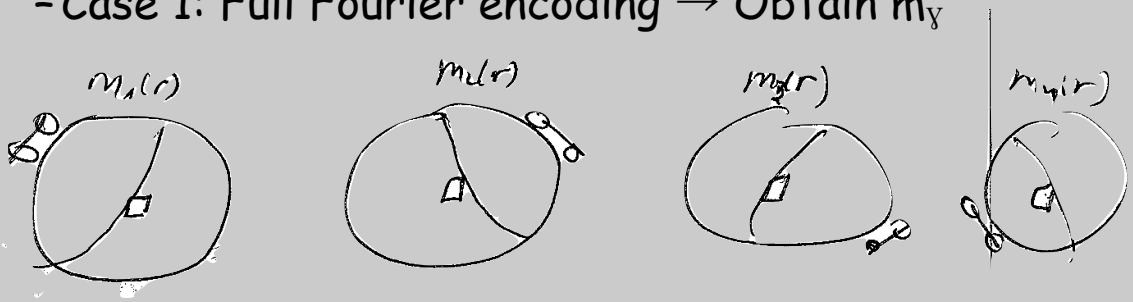




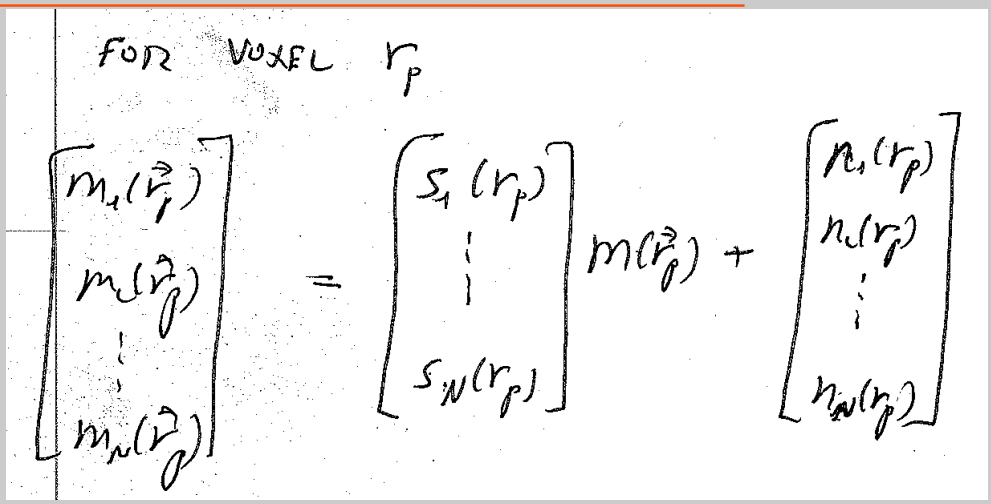


Reconstruction (Roemer 1990)

- How to reconstruct m(r)?
  - Case 1: Full Fourier encoding → Obtain m_y



  - N measurements for each voxel



Or

$$\underbrace{m_s(\vec{r}_p)}_{N\times 1} = \underbrace{S}_{N\times 1} \underbrace{m(\vec{r}_p)}_{1\times 1} + \underbrace{\vec{n}}_{N\times 1}$$

Sensitivity; image pixel; noise

Minimum variance estimate

$$\underbrace{m_s(\vec{r}_p)}_{N\times 1} = \underbrace{S}_{N\times 1} \underbrace{m(\vec{r}_p)}_{1\times 1} + \underbrace{\vec{n}}_{N\times 1}$$

Sensitivity; image pixel; noise

$$\hat{m}(\vec{r}_p) = \underbrace{(S^*\Psi^{-1}S)^{-1}}_{1\times 1}\underbrace{S^*\Psi^{-1}}_{1\times N}\underbrace{m_s(\vec{r}_p)}_{N\times 1}$$

$$\mathrm{var}(\hat{m}(\vec{r}_p)) = (S^*\Psi^{-1}S)^{-1}$$

Special case

$$\Psi = \sigma^2 I$$

$$\hat{m}(\vec{r}_p) = \left(S^*(\sigma^2 I)^{-1}S\right)^{-1} S^*(\sigma^2 I)^{-1} m_s(\vec{r}_p) =$$

$$= (S^*S)^{-1}S^*\vec{m}_s(\vec{r}_p) =$$

$$= \frac{1}{\sum_\gamma |S_\gamma(\vec{r}_p)|^2}\sum_\gamma S_\gamma^*(\vec{r}_p)\, m_\gamma(\vec{r}_p)$$

SOS

- Often sqrt of sum of squares used

$$M_{ss} = \sqrt{\sum M_\gamma^* M_\gamma}$$

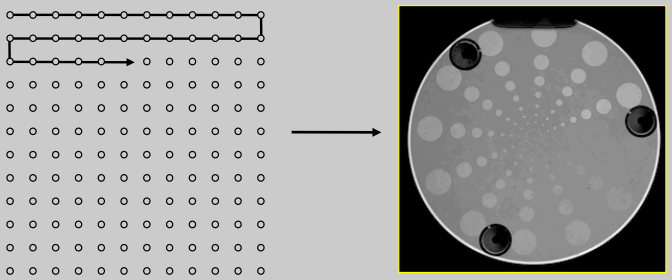
for SNR >20, within 10% of optimum

(Long thought to be adequate)

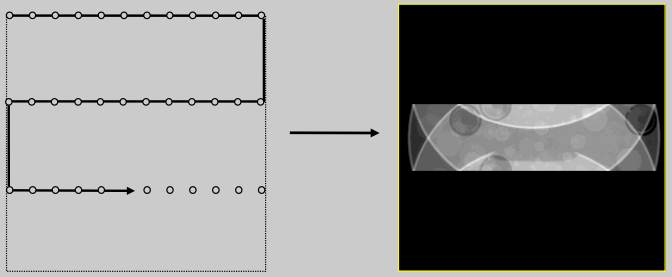
Multi-Coils for Speed

- Multi coils buy SNS → Tradeoff for Speed
- Many methods:
  - Original: SMASH - k-space approach
  - Popular:
    - SENSE - image approach, requires sensitivities
    - GRAPPA - k-space approach, autocalibrating
  - Here we only talk on SENSE in 2DFT
    - assume S_γ are known! (not so good assumption)

K-space Sampling



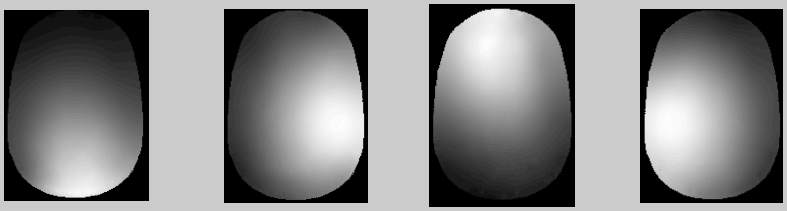
Standard k-space sampling



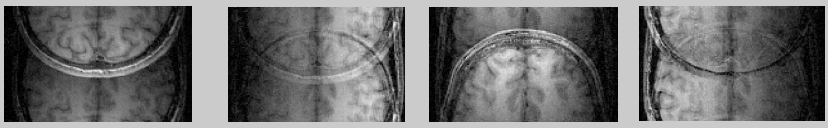
Reduced k-space sampling

Parallel Imaging (Basic Idea)

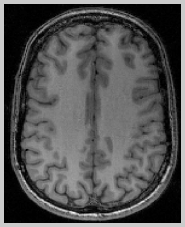
coil 1 coil 2 coil 3 coil 4



coil sensitivities



3x undersampling



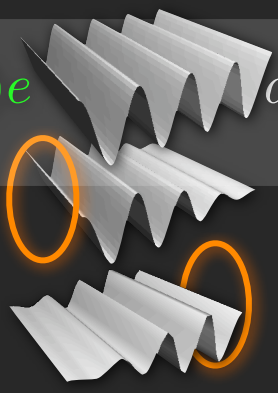
reconstruction

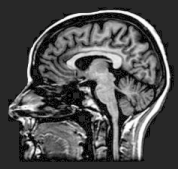
Sensitivity Encoding

$$s(t,k(t)) = \int_R M(\vec{r})S(\vec{r})e^{-i2\pi\vec{k}(t)\cdot\vec{r}}\,dr$$

$$s(t,k(t)) = \int_R M(\vec{r})S(\vec{r})e^{-i2\pi\vec{k}(t)\cdot\vec{r}}\,dr$$

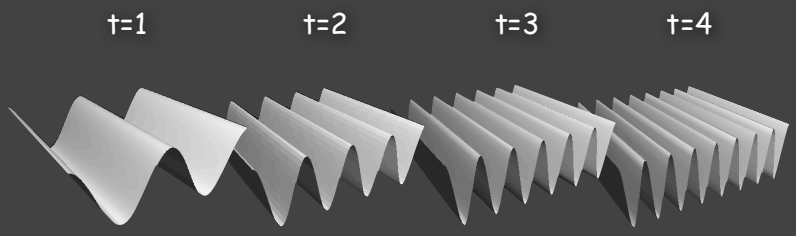
Fourier Encoding

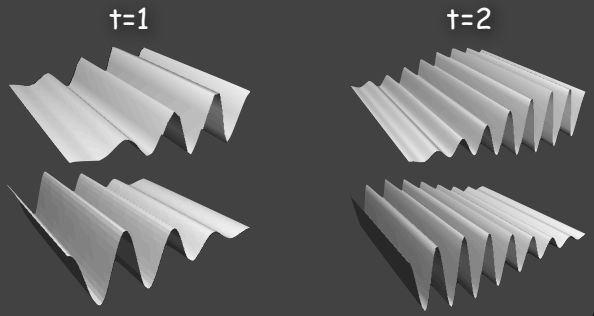




Sensitivity Encoding

Parallel Imaging



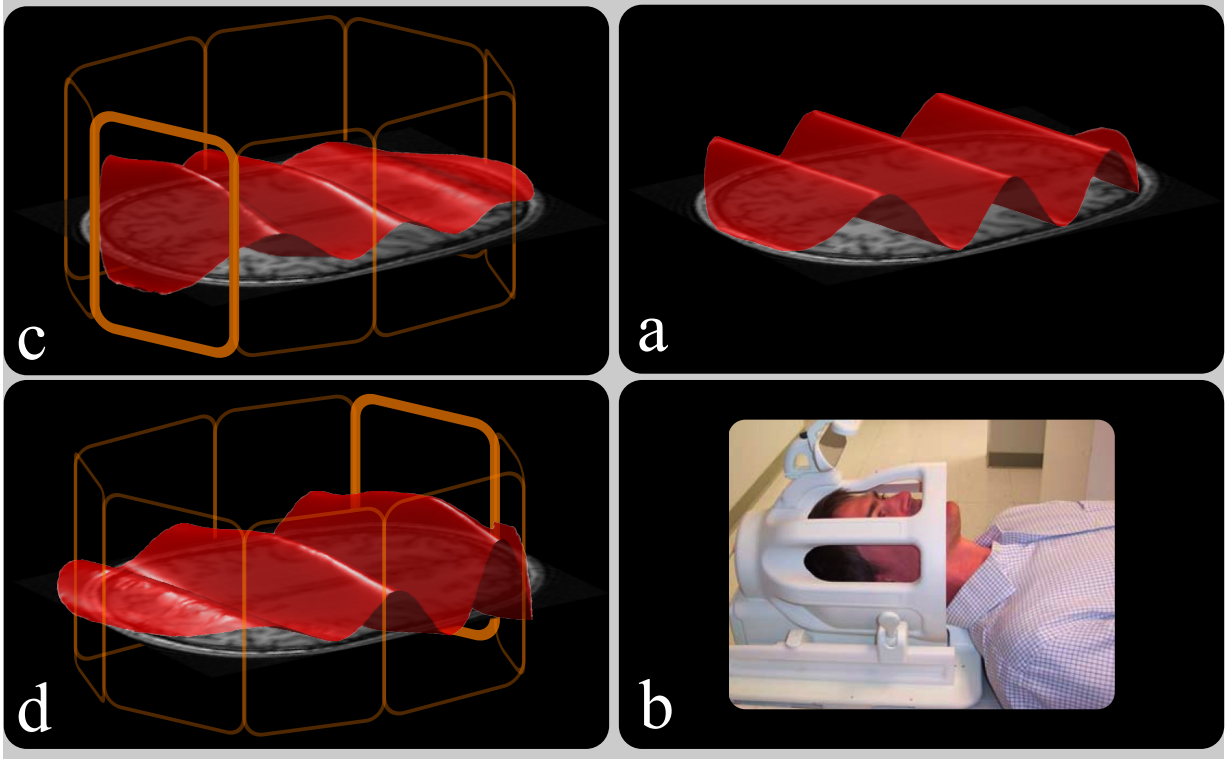


Parallel Imaging

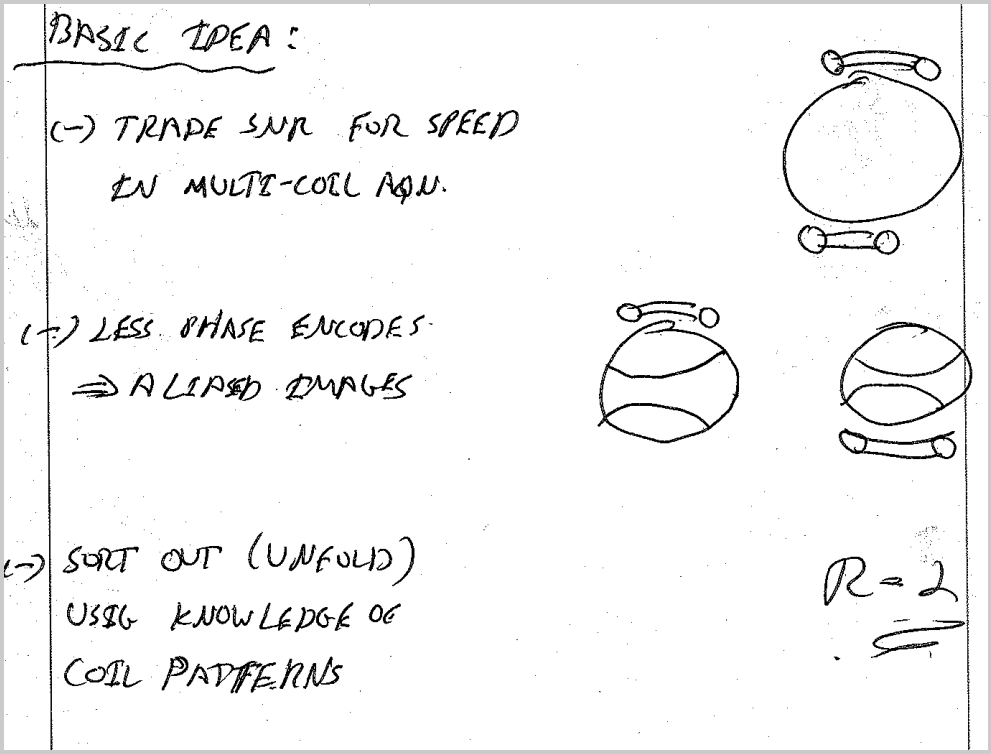
$$s_\gamma(t) = \int_R \underbrace{m(\vec{r})}_{\text{Image}} \underbrace{S_\gamma(\vec{r}) e^{-i2\pi\vec{k}(t)\cdot\vec{r}}}_{\text{Encoding}} dr$$

- Previously: 1 Enc/ time point, N images
- Now: N Enc/time point, 1 image

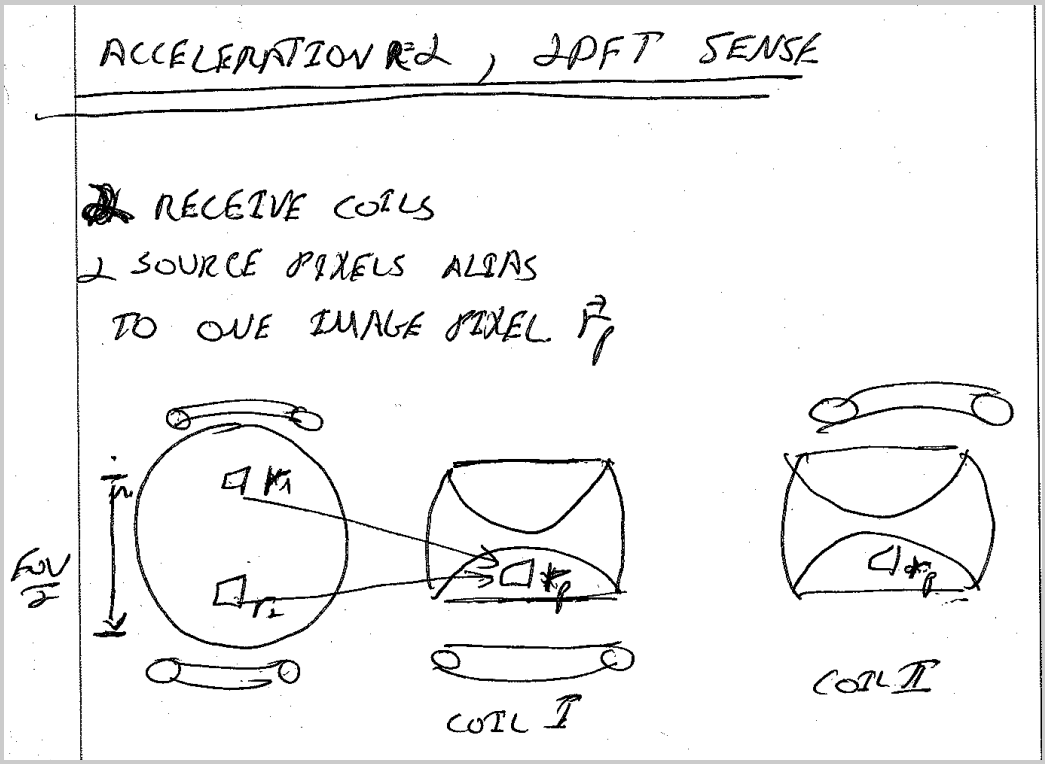
- Faster by skipping phase encodes!



SENSE (Pruessmann 99)



SENSE



SENSE

2 MEASUREMENTS OF $m(\vec{r}_1), m(\vec{r}_2)$

EACH WITH DIFFERENT SENSITIVITIES

$$\begin{bmatrix} m_1(\vec{r}_p) \\ m_2(\vec{r}_p) \end{bmatrix} = \underbrace{\begin{pmatrix} S_1(\vec{r}_1) & S_1(\vec{r}_2) \\ S_2(\vec{r}_1) & S_2(\vec{r}_2) \end{pmatrix}}_{\text{SENSITIVITY AT SOURCE VOXEL}} \begin{pmatrix} m(\vec{r}_1) \\ m(\vec{r}_2) \end{pmatrix} + \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}$$

$$\underset{2\times 1}{m_s(\vec{r}_p)} = \underset{(2\times 2)\ (2\times 1)}{S\,\vec{m}} + \underset{2\times 1}{\vec{n}}$$

SENSE

SAME MINIMUM VARIANCE SOLUTION

$$\underset{2\times 1}{\hat{m}} = \underbrace{\left(S^* \Psi^{-1} S\right)^{-1}}_{2\times 2} \underbrace{S^* \Psi^{-1}}_{2\times 2} \underbrace{\vec{m}_s(\vec{r}_p)}_{2\times 1}$$

2 ALIASED RECONSTRUCTIONS "UNFOLDED" INTO 2 IMAGE PIXELS.

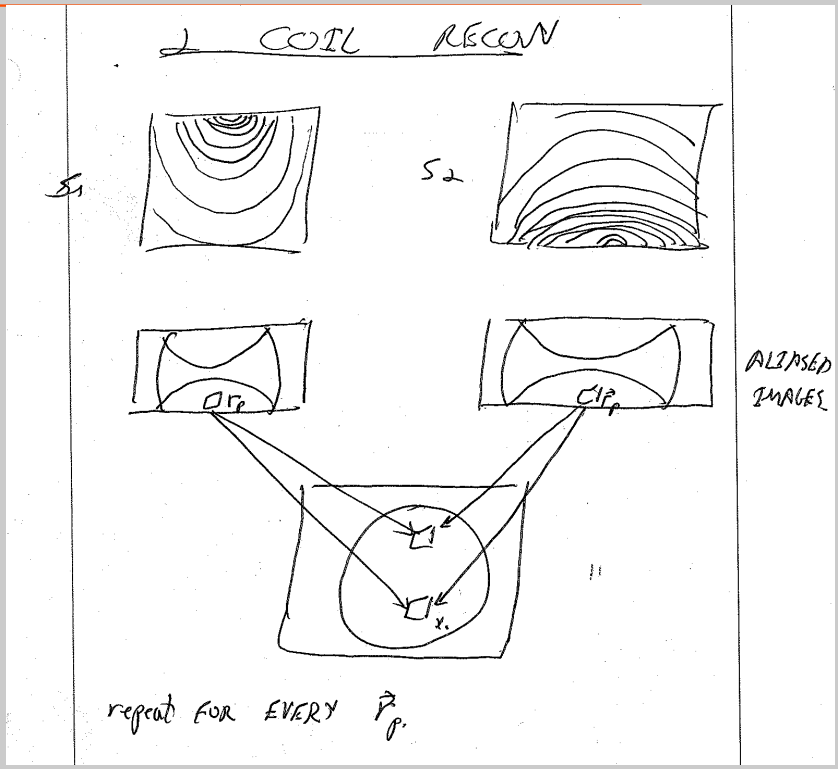
SENSE

(-) GENERAL CASE $R = M$ $\gamma = 1 \ldots N$

$$\underset{N\times 1}{m_s(\vec{r}_\rho)} = \underset{N\times M}{S} \cdot \underset{M\times 1}{\vec{m}} + \underset{N\times 1}{\vec{n}}$$

$$\underset{M\times 1}{\hat{m}} = \underset{M\times M}{(S^H \Psi^{-1} S)^{-1}} \underbrace{S^H \Psi^{-1}}_{M\times N} \underset{N\times 1}{m_s(\vec{r}_\rho)}$$

SENSE

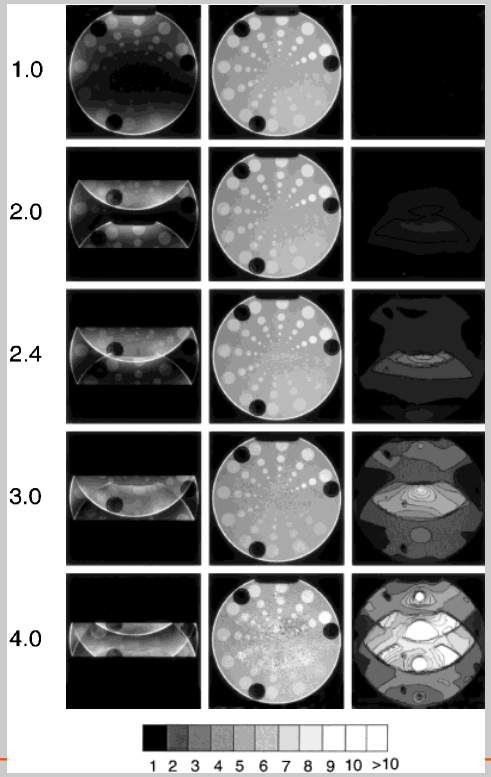


Noise Amplification g-factor

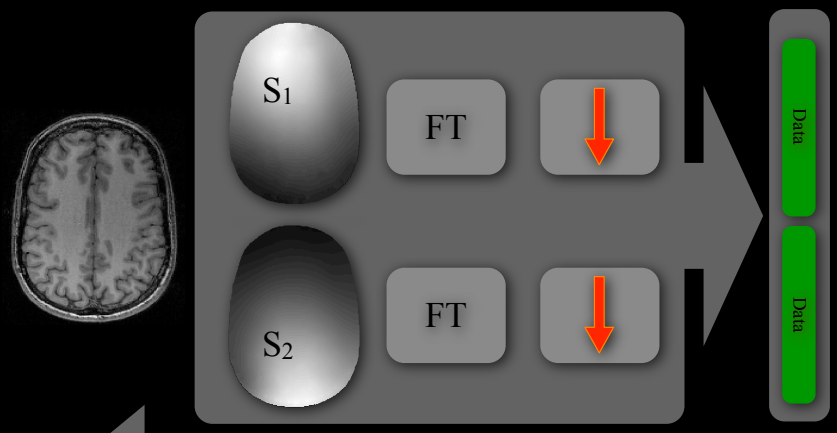
$$SNR_{\rho}^{red} = \frac{SNR_{\rho}^{full}}{g_{\rho}\sqrt{R}}$$

$$R = \frac{n_K^{full}}{n_K^{red}}$$

$$g_{\rho} = \sqrt{[(S^H \Psi^{-1} S)^{-1}]_{\rho,\rho} \, (S^H \Psi^{-1} S)_{\rho,\rho}}$$



SENSE model



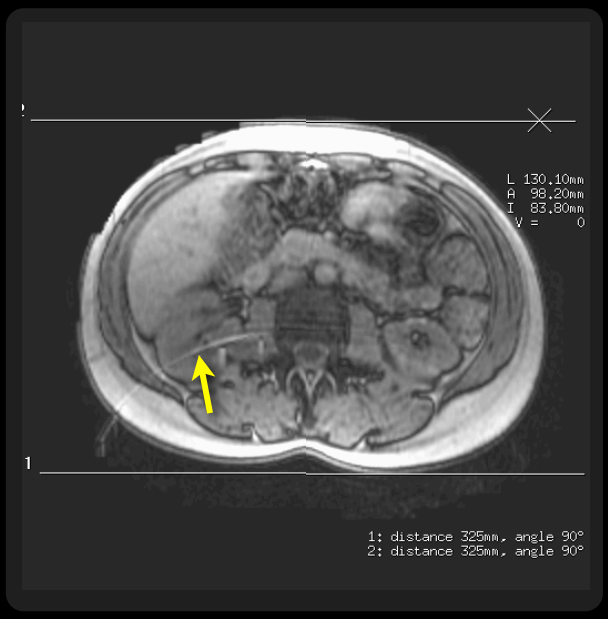
$$Ex = y$$

Pruessmann et. al., 1999

SENSE

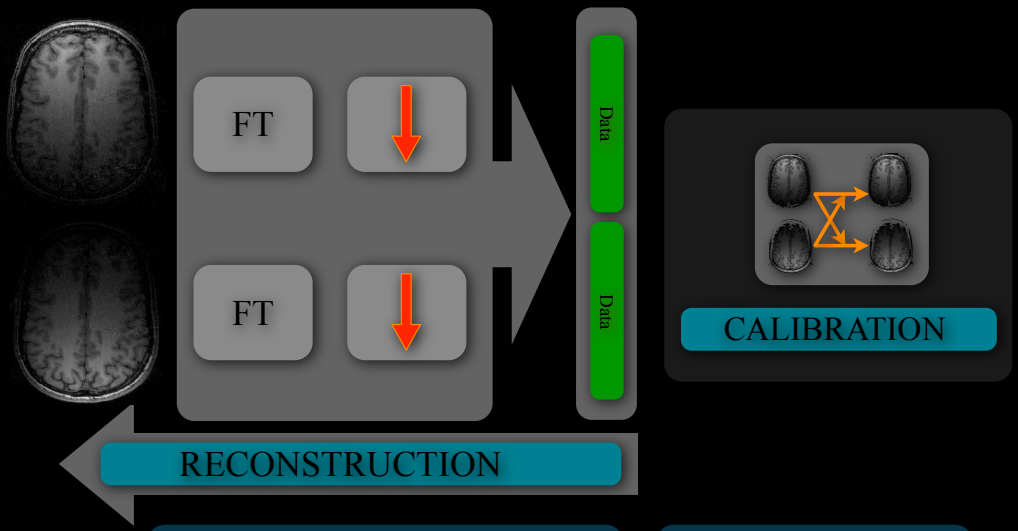
- Full inverse model
- Noise optimal
- One combined image

- Sensitive to map errors
  - Too low resolution
  - Misregistration
  - Folded maps



*image, courtesy of Kevin King

Autocalibrating Model (GRAPPA)

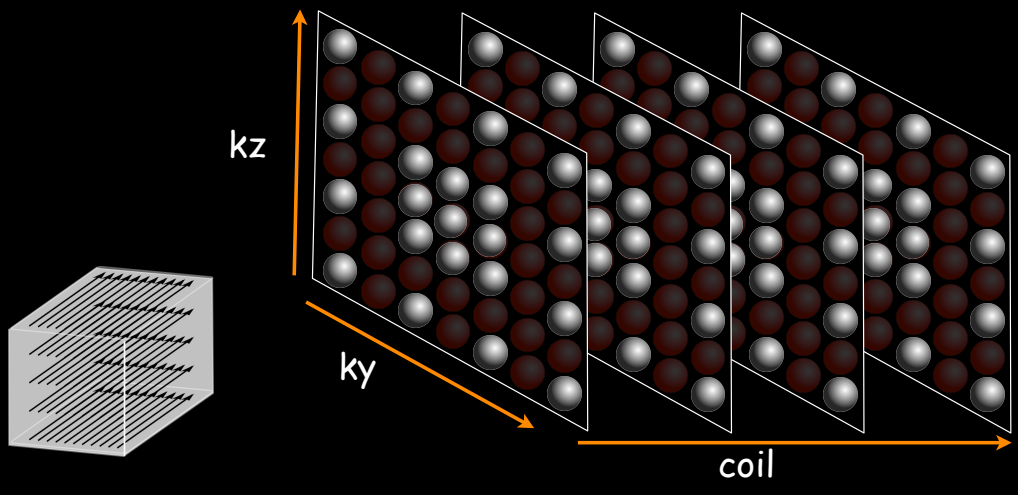


$$F_u x = y \qquad |x \in \mathbb{G}$$

Griswold et. al., 2002

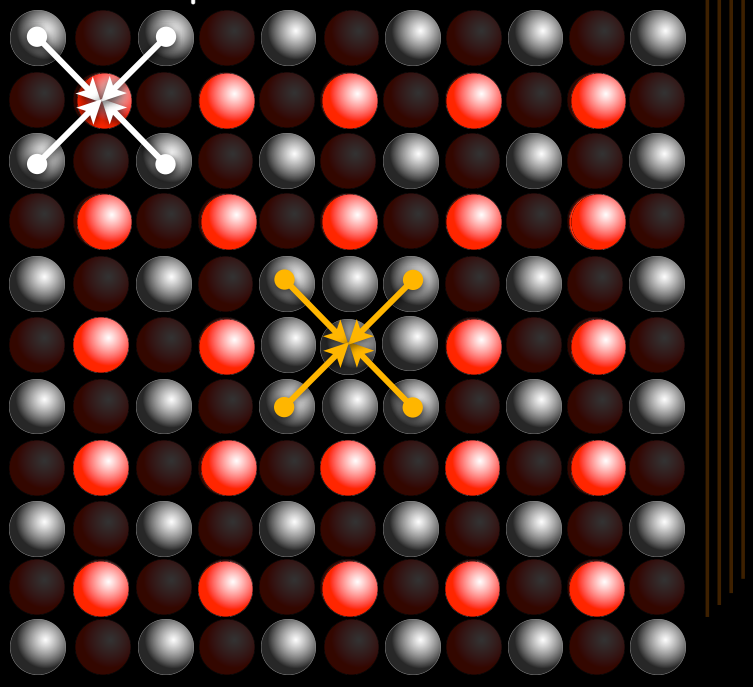
GRAPPA

- Reconstruction through interpolation
- Based on generalized sampling theory
- k-space vs. coil sampling domain



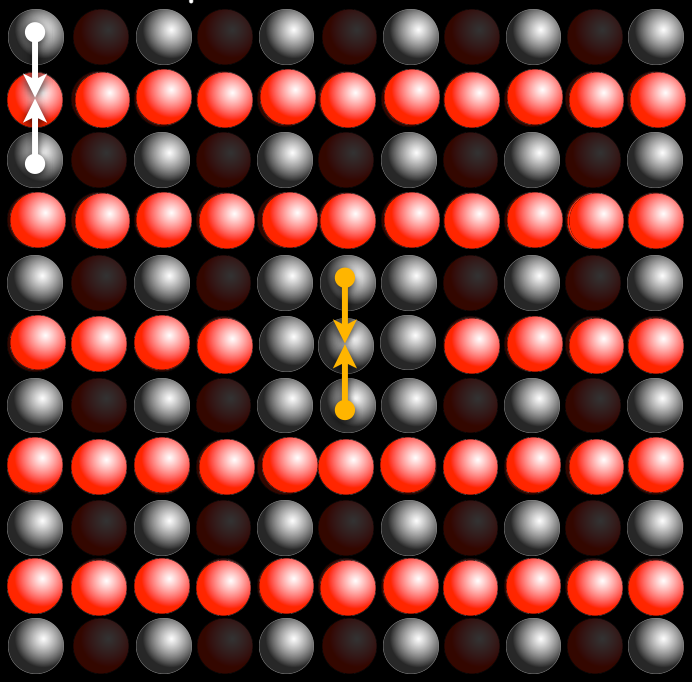
GRAPPA/ARC

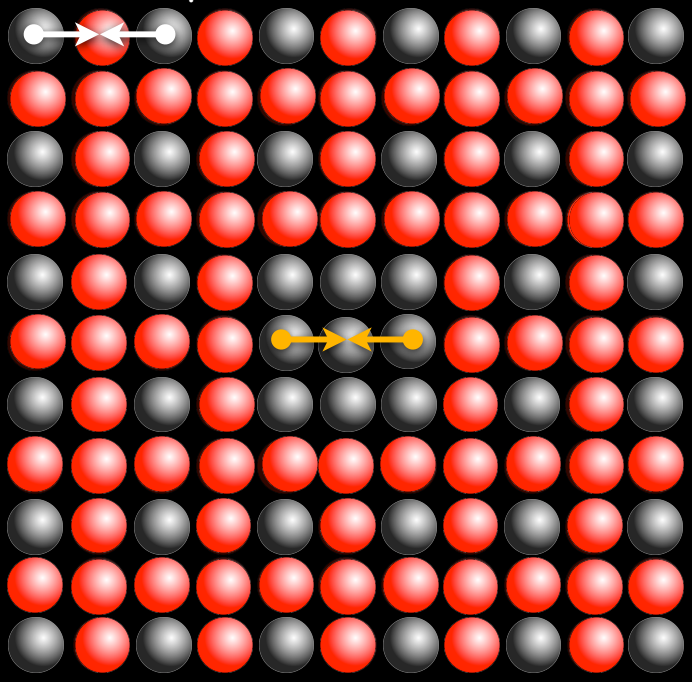
k-Space variant interpolation



GRAPPA/ARC

k-Space variant interpolation



GRAPPA/ARC

k-Space variant interpolation