Assignment 8

Due March 30th, 2012

1. Read Nishimura Ch. 7
2. Nishimura 7.2
3. Flow Encoding (From midterm II 2010)

   In this question we will consider several situations of flowing spins. We are going to make several approximations: 1) The transverse magnetization decays completely between excitations. 2) The displacement of the spins during a scan is much smaller than the imaging resolution, so it can be neglected for the purpose of determining position. 3) The effects of flow on the excitation are negligible. 4) We assume stroboscopic acquisition, e.g, a spin that moved from position y_0 to $y_0 + \Delta y$ during a TR, in the next TR it will also move from y_0 to $y_0 + \Delta y$. 5) Flow in tubes is NEVER turbulent.

   a) A spin at position $y = y_0$ is moving in the y direction with velocity v_y, so its position is $y(t) = y + v_y t$. Derive an expression for the phase of the spin $\theta_{[y,v]}(t)$ in the presence of a general time varying $G_y(t)$ gradient.

$\theta_{[y,v]}(t) =$

   b) The signal equation of an ensemble of spins at different y positions and y velocities is

   $$s(t) = \int_y \int_v m_{xy}(y, v) e^{-i\theta_{[y,v]}(t)} \, dv \, dy.$$

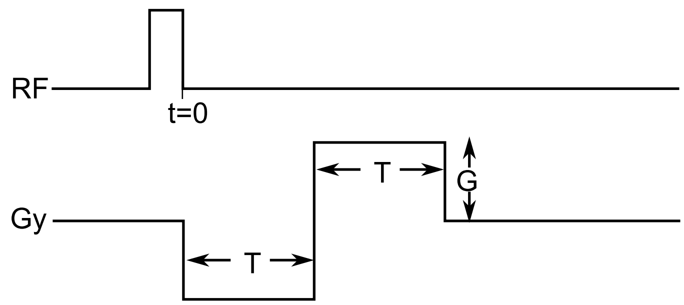
   Show that it can be written in the form of

   $$s(t) = \int_y \int_v m_{xy}(y, v) e^{-i2\pi[k_y(t)y + k_{vy}(t)v_y]} \, dv \, dy,$$

   where $k_y(t) = \frac{\gamma}{2\pi} \int_0^t G_y(\tau) d\tau$ is the usual spatial k-space and $k_{vy}(t)$ is y velocity k-space. Find the expression $k_{vy}(t)$.

$k_{vy}(t) =$

c) Consider the following bipolar gradient pulse:

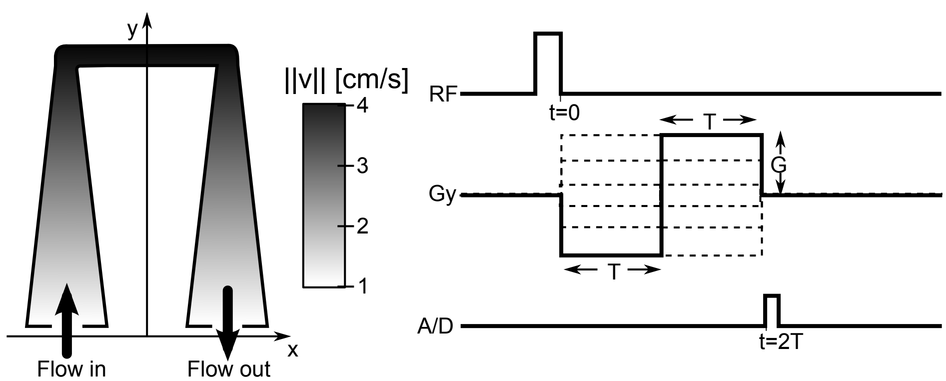


What are k_y and k_{vy} at the end of the pulse ($t = 2T$).

$k_y(2T) =$	$k_{vy}(2T) =$

d) Regardless what you got before, for this question assume that $k_{vy} = \alpha G$, a linear function of G.

You are given the following microfluidic device,



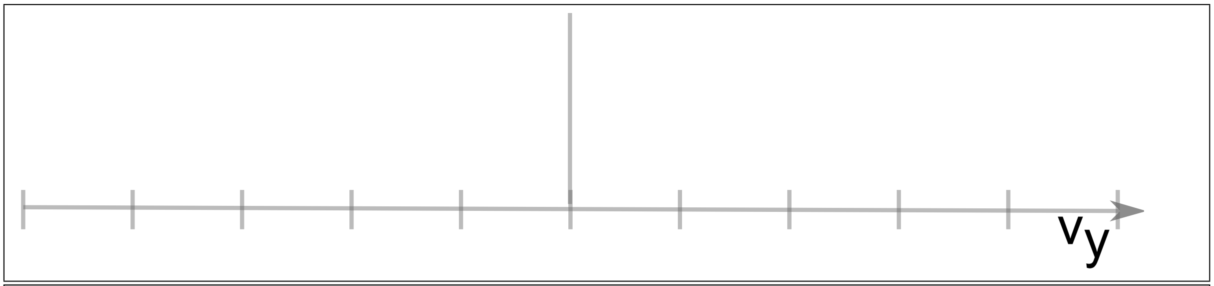
Fluid is injected into the device on the left side and is ejected on the right. The linear narrowing and broadening of the volume of the tubes generates a linear spectrum of velocities which we would like to image using the flow encoding sequence on the right. The flow velocity magnitude (not direction!) is grayscale coded in the image and represents magnitude velocities ranging from 1 cm/s to 4 cm/s.

In a similar way to phase encoding, in each TR the bipolar gradient amplitude is linearly varied from G to $-G$. Every TR we collect a single sample immediately following the bipolar gradient. After N repetitions we get a 1D signal with N samples. We reconstruct a velocity spectrum by computing an inverse 1D Fourier transform.

Qualitatively draw the ideal y-velocity spectrum of the device for this pulse sequence. Annotate, label and explain as much as possible. (You do not need to make calculations in order to do it!)

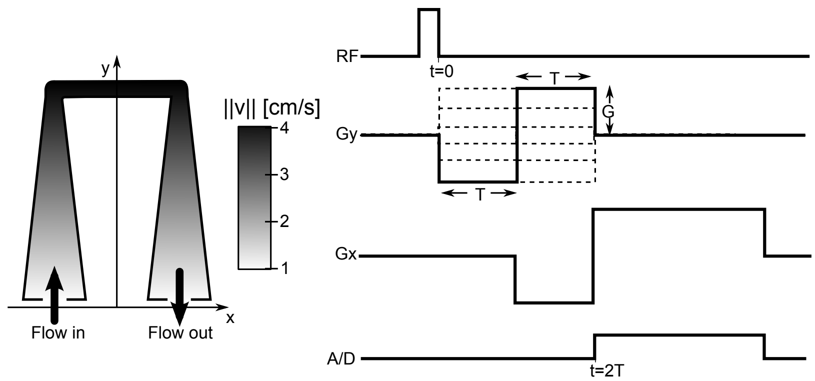
How many velocity encodes N_{kv} are needed to avoid aliasing if $k_{vy_{\max}} = 1$ s/cm?

Ideal Velocity Spectrum:



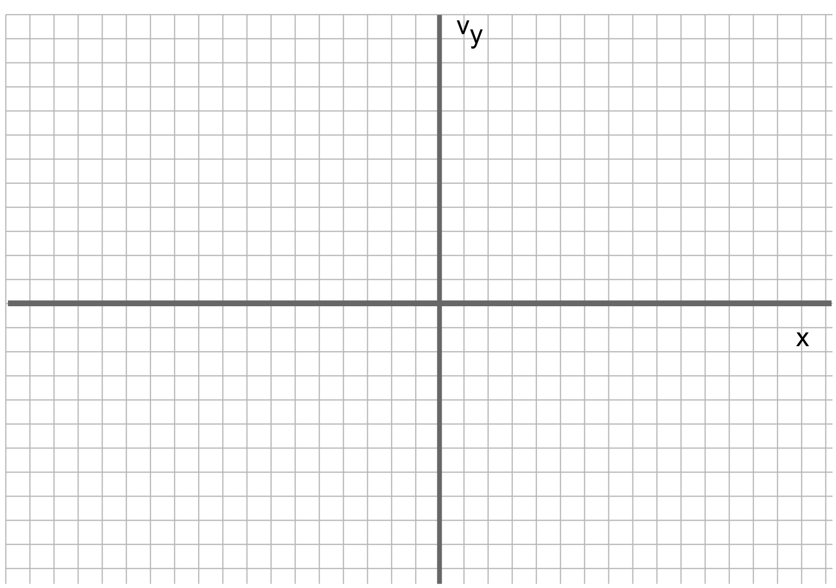
$N_{kv} =$

e) You modify the pulse sequence in d) to include a readout gradient:

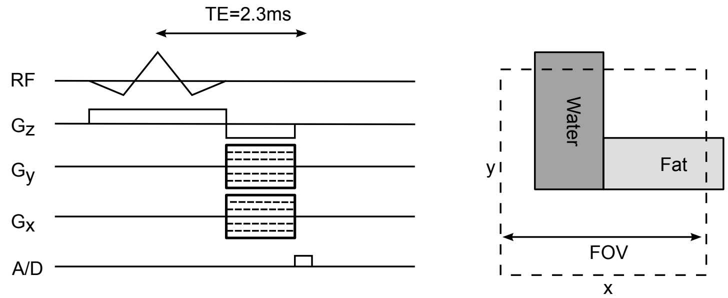


Here we collect M readout samples in each TR, resulting in kv-kx 2D data.

Draw the image that is the result of applying an inverse 2D Fourier transform to the acquired data. Annotate the image to provide intensity information. In the empty space describe in words as clearly as possible your thought process. In your analysis, ignore any flow effects in the x direction due to the readout gradient.



4. Consider the following image and sequence.



This sequence uses phase encoding in two dimensions. This means that for each excitation we read a single point in k-space and there is no gradient on during the readout. The number of phase encodes are enough to support the FOV given in the figure.

a) Re derive the effect of chemical shift (Eqns 7.5-7.10 in the text) for this pulse sequence.

b) Draw the reconstructed image.

c) How would your answer change if you used a regular 2DFT with readout in the x direction. Draw the image qualitatively pointing out the differences. Assume $TE = 11.5$ms.

5. **Matlab Problem: Twinkle Twinkle T_2^***

a.) In this problem we will simulate the effect of T_2^*. In general, T_2^* is an exponential decay approximation signal loss due to intra-voxel dephasing. As you will see in the next questions, this dephasing can be refocused. Again, we will use the Brian Hargreaves Bloch simulator.

First, we will generate a 90° RF pulse:

```
>> dt = 4e-6; % 4 us sampling rate
>> rf_90 = 90/360/(4257*dt);
>> % impulse RF pulse
>> b1 = [rf_90;zeros(100e-3/dt,1)];
>> g = b1*0; % no gradient
```

We will simulate a spin population (512 spins) with a Gaussian distribution of off-resonances:

```
>> % create gaussian distribution of off-resonance
>> % with std 20Hz
>> df = randn(512,1)*20;

>> % All spins at iso-center
>> dp = 0;
>> t1 = 800e-3; % Set T1
>> t2 = 50e-3; % Set T2

>> % start at Mz
>> mx_0 = 0;
>> my_0 = 0;
>> mz_0 = 1;
```

Simulate the the excitation followed by the free induction decay period:

```
>> % Simulate
>> [mx,my,mz] = bloch(b1,g,dt,t1,t2,df,dp,2,mx_0,my_0, mz_0);
```

The signal from an individual spin should exhibit just T_2 decay. Whereas the sum of the signals from all the spins together should exhibit T_2^* decay:

```
>> % for t2 take only 1 spin
>> s_t2 = mx(:,end/2+1)+i*my(:,end/2+1);
>> % for t2* average across off-resonance
>> s_t2star = mean(mx + i*my,2);

>> figure,
>> plot([1:length(s_t2)]*dt,abs(s_t2),[1:length(s_t2star)]*dt,abs(s_t2star));
>> legend('t2','t2^*');
```

Submit the plot that you get.

Low flip angle spin-echoes. Early in the days of NMR (Hann 1950) people observed that when applying two 90° pulses separated by time τ, the FID signal seems to increases at time 2τ.

b.) First, derive an expression (ignoring $T1$ and $T2$) for the transverse magnetization at time $t = 2\tau$. Show that the signal has magnitude $M_0/2$

c.) Validate the result by simulating a sequence with two 90° pulses separated at $\tau = 20ms$. Design the hard 90° RF pulses with maximum $B1_{\max} = 0.16G$. Simulate uniform off resonance ranging $[-600Hz, +600Hz]$. Use at least 512 discrete off-resonance locations! Use $T1 = 1s$, $T2 = 1s$. Plot $s(t)$, the FID of the sequence, for $0 < t < 60ms$. (don't forget to average across the off-resonance!)

d) Repeat the simulation in c), for 45° and 135° refocusing pulses. Do you see a spin-echo? What can you say about the refocusing properties of pulses as a function of flip angle?

e) Simulate a series of ten 15° pulses separated $3ms$ apart. Plot the the FID for $0 < t < 80ms$. Use off-resonance of=linspace(-2600,2600,1024). The echoes you see are called stimulated echoes and are coherence pathways that refocus at different times. Stimulated echoes can often create artifacts in images. But, if used wisely can boost imaging speed, improve signal to noise ratio and more.