Detour into Numerical Analysis

Problem

- well conditioned
- ill conditioned

Well Condi

- ill conditioned

Algorithm applied to problem

Condition $\# = \text{measure of how \ "well conditioned\" an \ algorithm \ is \ on \ a \ problem}$
Solving Linear Eqns. \[ Ax = b \]

Weather

Initial Conditions

Forecast satisfied with

\[ ax + bx^2 + cx + d \]
Algorithm:

$A \times x = b$

List of possible algorithms:

1. Gaussian Elimination
2. Inverse Tomble
3. Cholesky
4. QR or SVD
5. Decomposition
6. Iterative alg. large prob.
Condition # = \[ \frac{\text{Perturbation in output}}{\text{Perturbation in input}} \]

\[ A, b, A\tilde{x} = \tilde{b} \]

\[ \text{Input} \rightarrow \text{Problem} \rightarrow \text{Output} \]

Condition # = \[ \frac{\text{Relative change in output}}{\text{Relative change in input}} \]

If Condition # is \[ 10^6, 10^9, 10^{12} \] \[ \rightarrow \text{Problem is ill-conditioned} \]

\[ \text{If } \] \[ \text{Condition } \leq 1 \] \[ \rightarrow \text{Problem is well-conditioned} \]
\[ A \mathbf{y} = b \]

\[ A \rightarrow \text{Condition #} = \frac{\text{target singular value}}{\text{smallest singular value}} \]

\[ A = \begin{bmatrix} a & 5 \\ c & d \end{bmatrix} \]

Condition # of problem depends on the "Intrinsic" aspect of the problem itself.

Well cond. \rightarrow \begin{align*} & \text{well Alg.} \\ & \text{good answer} \end{align*}

Ill cond. \rightarrow \begin{align*} & \text{ill Alg.} \\ & \text{wrong answer} \end{align*}

Problem

\[ \text{Ill cond.} \rightarrow \text{out of luck.} \]
Condition # of Alg

\[ A \times x = b \]

\[ \text{relative change in output} \]
\[ \text{relative change in input} \]

A: Very Ray Condition
Even though theoretically Hayk/McElhollan showed uniqueness in practice $\Rightarrow$ ill conditioned.

- 2D polynomials can almost closely be approximated by fearful close form $\rightarrow$ Izraelevitz + Lin

- Iterative

Oppenheim + Mersereau $\rightarrow$ 1972 Proceedings of IEEE P.S.T.
Recon from F.T. Phase

$X(k_1,k_2) \xrightarrow{D.T.F.T} X(\omega_1,\omega_2) = \sum_{n_1} \sum_{n_2} x(n_1,n_2) e^{-j\omega_1 n_1} e^{-j\omega_2 n_2}$

Patrick Van Hove \( \approx \) 1982

Need to have couple of \( \phi \) at more than \( N^2 \)
Two Alg. iterative.

direct.

Then 1982 even quantizing phase to one bit & yet remains signal successful.

of F.I.