Image Restoration

Frequency domain techniques

Periodic noise.

What if interference pattern is not "clean"?

Sources of periodic interference patterns:
Coupling + amplification of low level signals in electro optical scanners's electronic circuitry.

Approach ① First isolate principal contributions (spikes). of the interference pattern.

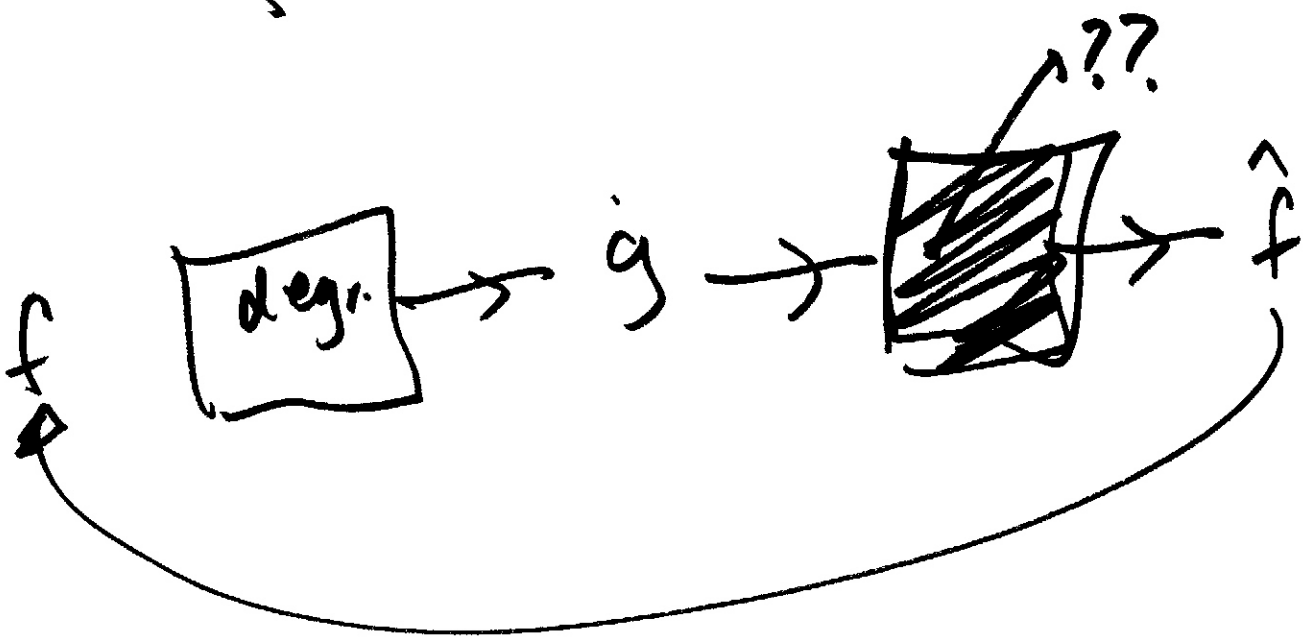
② subtract a variable weighted portion of the pattern from the corrupt image.

objective: i.e. e.g. minimize local variance of a processed image.

$g(x,y) \longleftrightarrow G(\omega_1, \omega_2)$ observed degraded signal.

$f(x,y) \longleftrightarrow F(\omega_1, \omega_2)$ → clean signal. original undegraded.

$\hat{f}(x,y) \longleftrightarrow \hat{F}(\omega_1, \omega_2)$ → processed version of g approximate f.



Step ① put a notch filter $H(\omega_1, \omega_2)$ at location of each spike.

Find filter

$$N(\omega_1, \omega_2) = H(\omega_1, \omega_2)\, G(\omega_1, \omega_2)$$

$$\eta(x, y) = \mathcal{F}^{-1}\{ H(\omega_1, \omega_2)\, G(\omega_1, \omega_2) \}$$

N: noise.

η: noise in space domain.

Step (2)

$$\hat{f}(x,y) = g(x,y) - w(x,y)\,\eta(x,y)$$

weight fn.

Optimization: choose local weight $w(x,y)$ to minimize local variance of $\hat{f}$ at (x,y)

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neighborhood $(2a+1) \times (2b+1)$

local variance over this neighborhood:

$$\sigma^2_{xy} = \frac{1}{(2a+1)(2b+1)} \sum_{s=-a}^{+a} \sum_{t=-b}^{+b} \left[\hat{f}(x+s, y+t) - \bar{\hat{f}}(x,y) \right]^2$$

$\bar{\hat{f}}$ = local mean = $\bar{\hat{f}}(x,y)$

$$= \frac{1}{(2a+1)(2b+1)} \sum_{s=-a}^{+a} \sum_{t=-b}^{+b} \hat{f}(x+s, y+t)$$

plug in $\hat{f} = g - w\eta$ into

assume $W(x,y)$ is constant over $[2a+1] \times [2b+1]$ region.

$W(x+s, y+t) \simeq W(x,y)$

$s,t \in [-a, +a] \times [-b, +b]$

$$\sigma^2_{xy} = \frac{1}{(2a+1)(2b+1)} \sum_{-a}^{+a} \sum_{-b}^{+b}$$

$$\Big\{ \Big[g(x+s, y+t) \Big] - w(x,y)\, \eta(x+s, y+t) \Big]$$

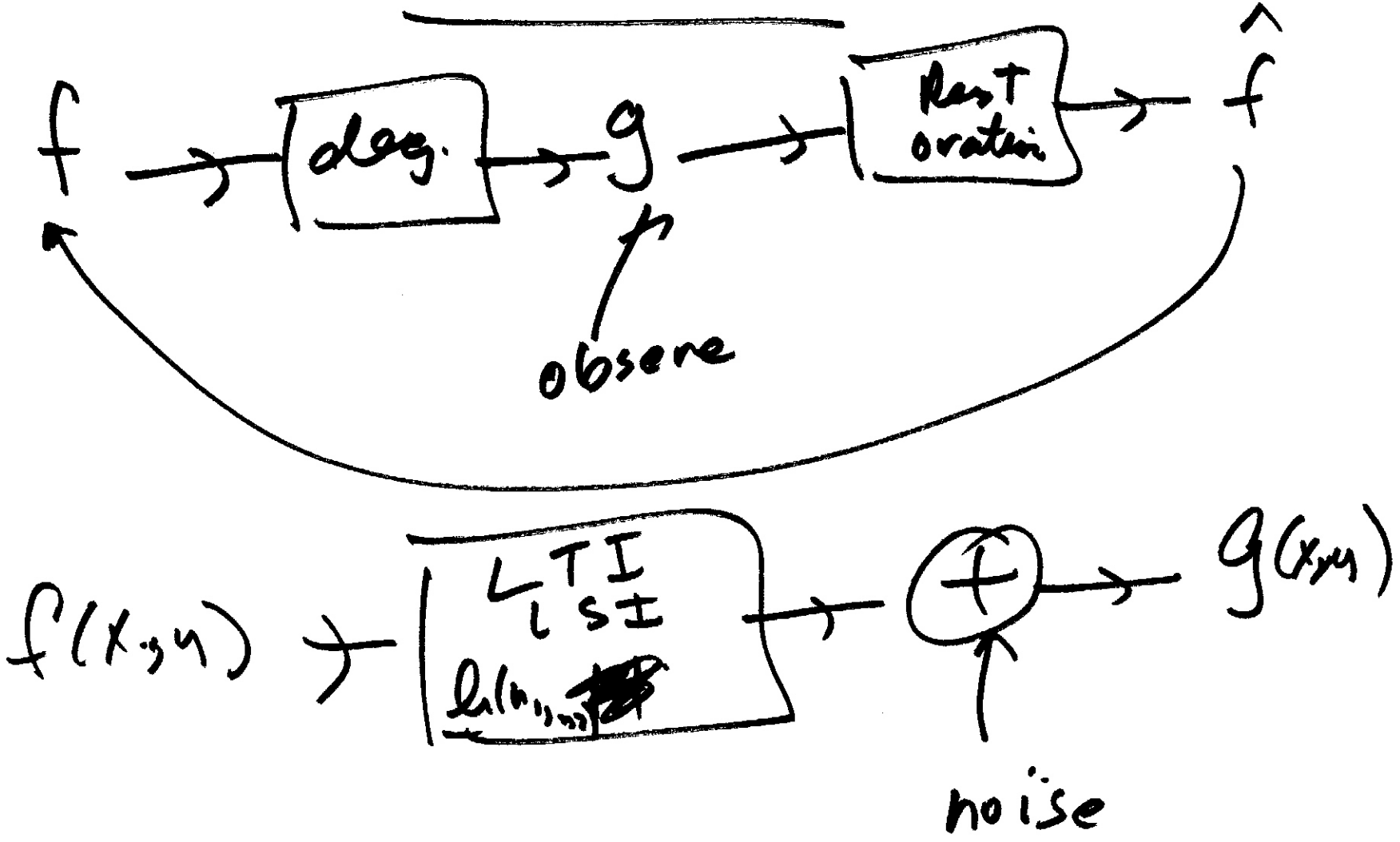
$$- \Big[\bar{g}(x,y) - w(x,y)\, \bar{\eta}(x,y) \Big] \Big\}^2$$

optimal weight

$$\frac{\partial \sigma^2_{xy}}{\partial w(x,y)} = 0 \Rightarrow$$

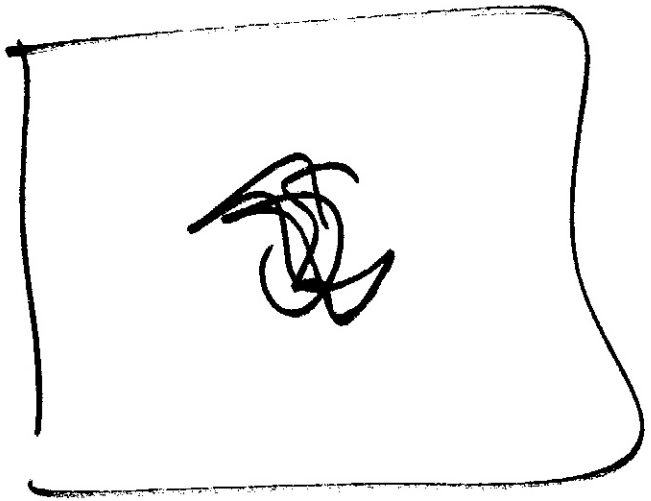
$$w(x,y) = \frac{\overline{g(x,y)\,\eta(x,y)} - \bar{g}(x,y)\,\bar{\eta}(x,y)}{\overline{\eta^2}(x,y) - \bar{\eta}^2(x,y)}$$

Restoration



Estimating the degradation fn

① Observation



go to parts of g that you have apriori knowledge.

see how it looks.

compare to what it should have looked like.

⟹ make some intelligent guess about h and noise

② Experimentation: put a known signal into your system to calibrate

③ Modeling

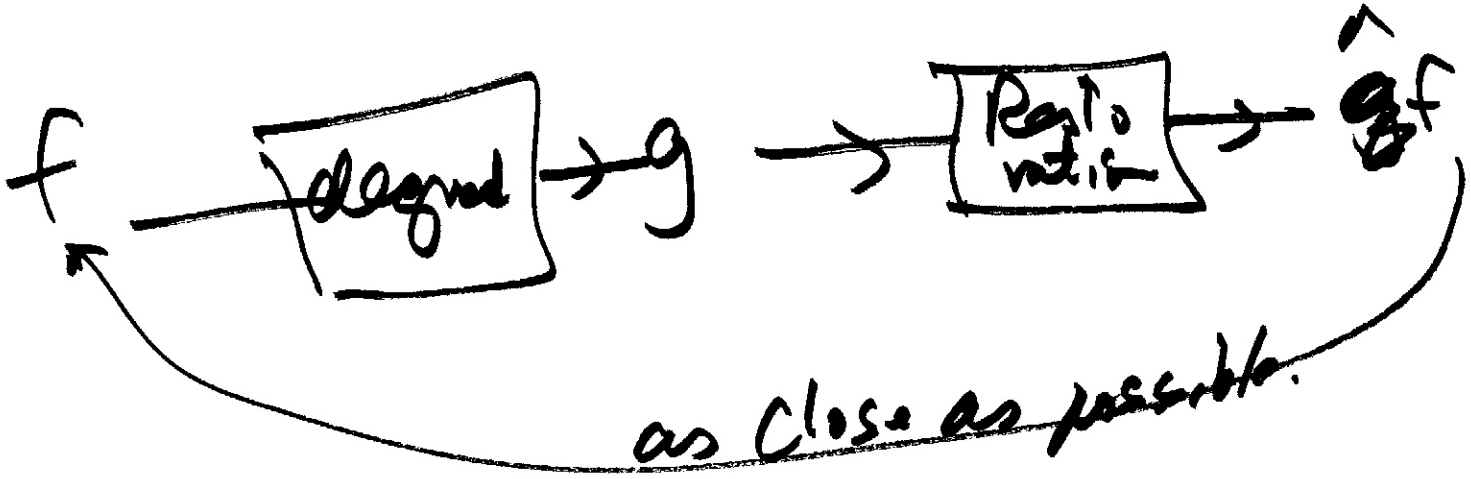
Stanley + Hufnagel.

Atmospheric turbulence $-k(\omega_1^2 + \omega_2^2)^{5/6}$

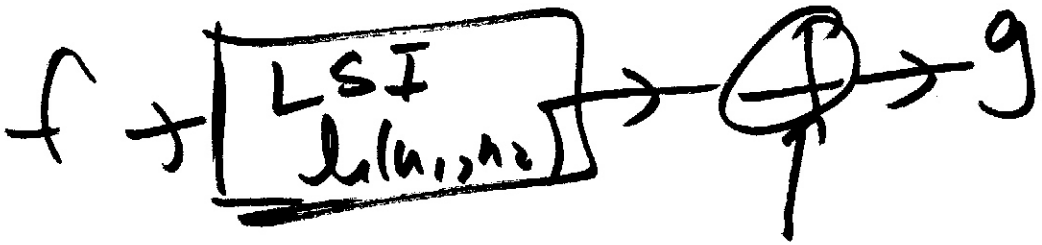
$H(\omega_1, \omega_2) = e$

Motion Blur ←

Restoration



Degradation can be modelled as.



Use domain specific knowledge to model degradation.

Motion Blur modeling

image $f(x,y)$

$x_0(t)$ Time varying component of motion along x.

$y_0(t)$ " " " " " y.

T = duration of exposure.

$g(x,y)$ = observed or captured signal.

$$g(x,y) = \int_0^T f(x - x_0(t),\ y - y_0(t))\ dt$$

$$\text{F.T.}\{g(x,y)\} = G(\omega_x, \omega_y) = \int_{-\infty}^{+\infty}\int g(x,y)\, e^{-j2\pi(\omega_x x + \omega_y y)}\ dx\,dy.$$

$$G(\omega_x,\omega_y)=\int_0^T \left(\int_{-\infty}^{+\infty}\int f(x-x_0(t),y-y_0(t))\, e^{-j2\pi(\omega_x x+\omega_y y)}\, dx\, dy \right) dt$$

$$= \int_0^T F(\omega_x,\omega_y)\, e^{-j2\pi(\omega_x x_0(t)+\omega_y y_0(t))}\, dt$$

$$G(\omega_x,\omega_y) = F(\omega_x,\omega_y) \underbrace{\int_0^T e^{-j2\pi(\omega_x x_0(t)+\omega_y y_0(t))}\, dt}_{H(\omega_x,\omega_y)}$$

$g(x,y)=T\, f(x,y)$

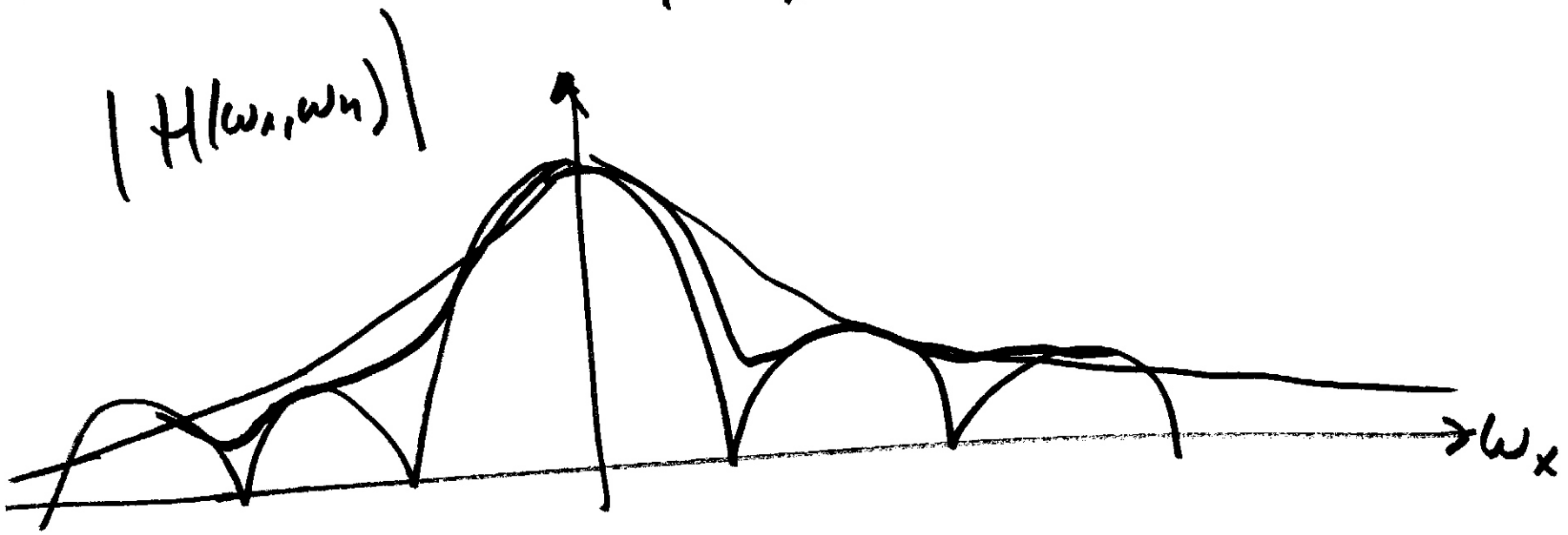
- $x_0(t)=0 \quad y_0(t)=0 \Longrightarrow$
- $x_0(t)=\frac{at}{T}$ = constant speed along x direction $y_0(t)=0$

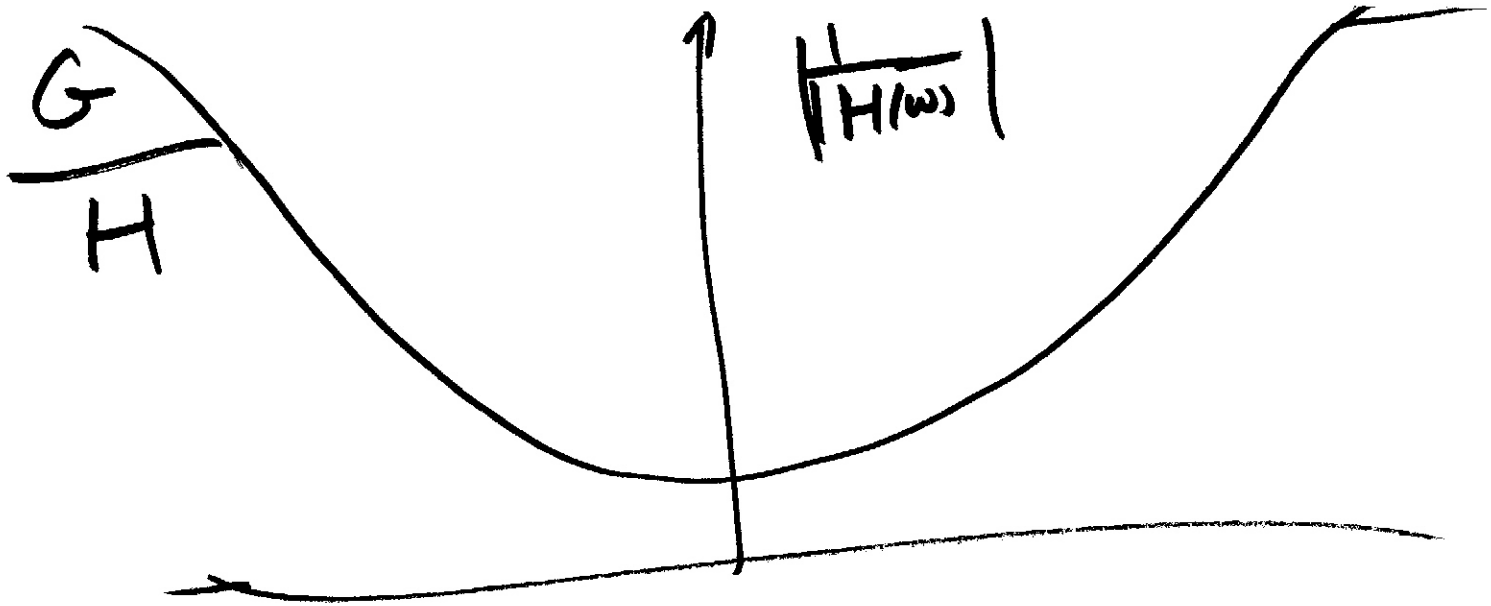
$$\boxed{G(\omega_x,\omega_y) = F(\omega_x,\omega_y)\, H(\omega_x,\omega_y)}$$

$$H(\omega_x,\omega_y) = \int_0^T e^{-j2\pi(\omega_x x_1(t))}\,dt$$

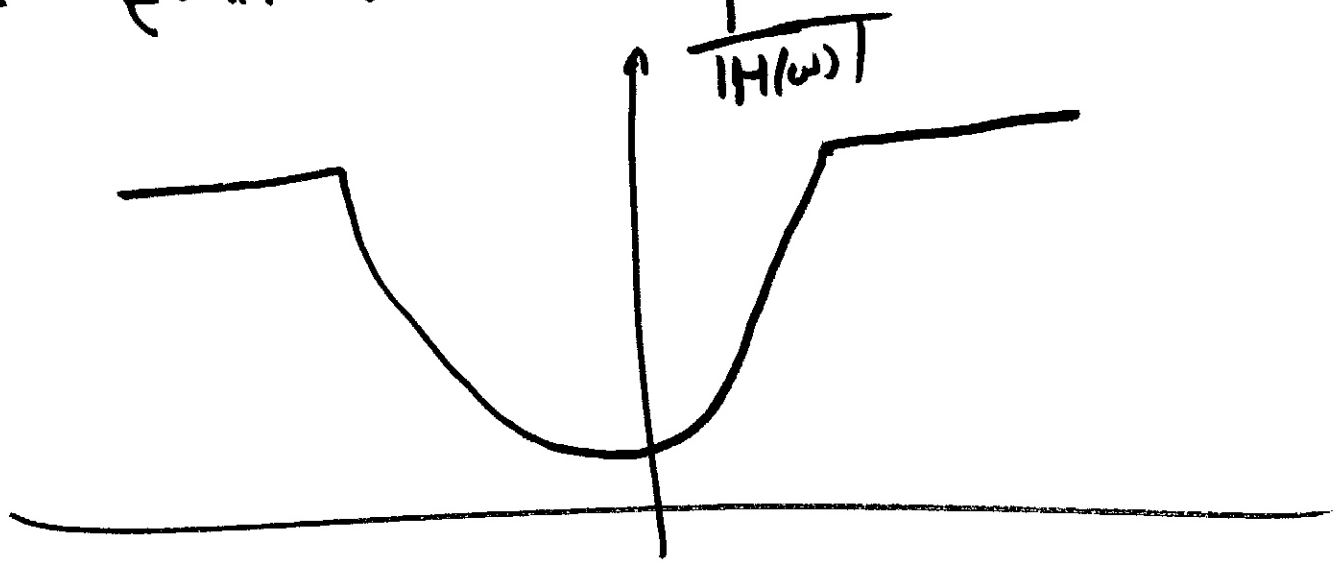
$$= \int_0^T e^{-j2\pi \frac{\omega_x a t}{T}}\,dt$$

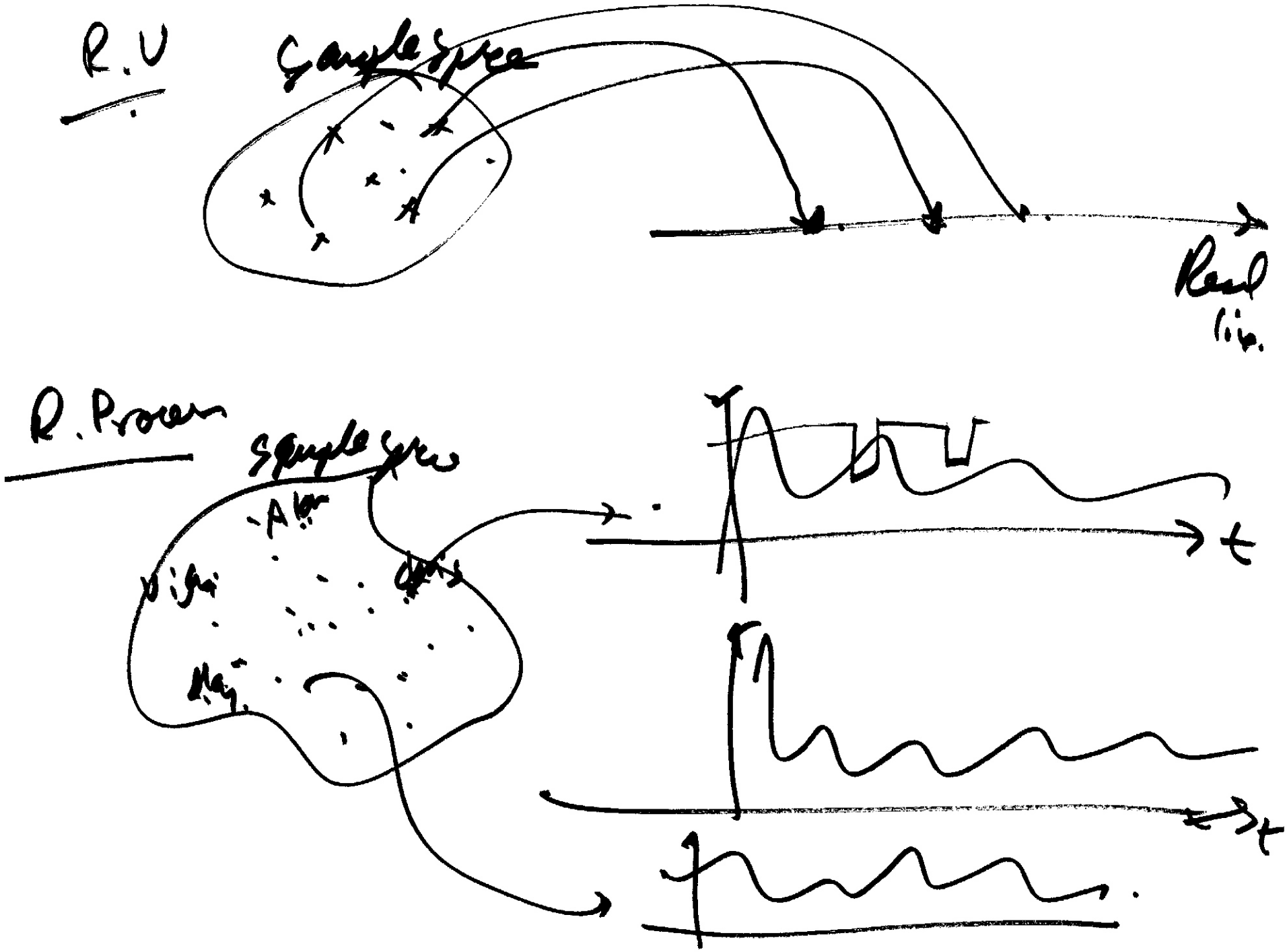
$$H(\omega_x,\omega_y) = \frac{T}{\pi\omega_x a}\,\mathrm{Sin}(\pi\omega_x a)\,e^{-j\pi\omega_x a}$$





$$G \quad \xi(\omega_x, \omega_y) = F(\omega_x, \omega_y)\, H(\omega_x, \omega_y) + \underline{\underline{\text{Noise}}}$$




R.V
Sample space
Real line
R. Process
Sample space
t
t

Stationarity

$$P_{x(t_1)\,x(t_2)\,\ldots\,x(t_N)}\left(X_1, X_2, X_3, \ldots, X_N\right)$$

$$=$$

$$P_{x(0),\,x(t_2-t_1),\,\ldots\,x(t_N-t_1)}\left(X_1, X_2, \ldots, X_N\right)$$