1. Enhancement → image "look" better
   subjective improvement.

2. Restoration
   Image has been degraded by smoothing out noise
   - noise
   - blur
   - atmospheric turbulence.
   objective model → Error → MSE
\[ f(x,y) \rightarrow \text{Degraded, } h(h_{11}, h_{22}) \rightarrow g(x,y) \]

\[ g(x,y) = h \ast f + \eta \]

\[ g(x,y) = h(x,y) \ast f(x,y) + \eta(x,y) \]

\[ g(x,y) \rightarrow \text{Restoration Box} \rightarrow \hat{f}(x,y) \]
minimize \[ E \left[ (f(x,y) - \hat{f}(x,y))^2 \right] \]

Today.

\( \mu \) is identity.

Only corruption \( \rightarrow \) noise.

\[ g(x,y) = f(x,y) + \eta(x,y) \]

Assume noise is independent of spatial coordinates, uncorrelated w.r.t. itself.
Gaussian

\[ p(x) = \frac{1}{\sqrt{2\pi} \sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \]

\[ \text{mean} = \mu \]

\[ \text{var} = \sigma^2 \]

To 99.7% of values are within one sigma [\( \mu - \sigma, \mu + \sigma \)]

\[ 95.5% \text{ within 2 sigma} \]

sensor noise due to poor illumination or high temperature
\( p(x) = \begin{cases} \frac{z}{b} & \text{if } 2 < a, \\ \frac{2}{(2-a) - 3} & \text{if } 2 \leq a. \end{cases} \)

\( \mu = \sqrt{\frac{8}{a^2 (a-2)^2}} \)

\( \rho = \frac{1 + \sqrt{1 + \frac{4}{a^2}}} {2a} \)

noise in range imaginary
3) Erlang (Gamma) noise:

\[ p(z) = \begin{cases} \frac{z^{b-1} \cdot e^{-az}}{(b-1)!} & z \geq 0 \\ 0 & \text{else} \end{cases} \]

\[ \mu = \frac{b}{a} \]

\[ \sigma^2 = \frac{b}{a^2} \]

4) Exponential special case of Erlang.

\[ p(z) = \begin{cases} az & z \geq 0 \\ 0 & \text{else} \end{cases} \]
\( \text{uniform.} \quad p(x) = \begin{cases} \frac{1}{b-a} & a \leq x \leq b \\ 0 & \text{otherwise.} \end{cases} \)

\[ \mu = \frac{a+b}{2} \]

\[ \text{random # generator.} \]

\[ \text{Impulse, salt/pepper noise.} \]

\[ p(z) = \sum p_a \delta_{a} p_b \]

if \( b > a \), \( b \) show up as light dots

\( r \) dark dot.
1. Arithmetic Mean:

\[ f(x,y) = \frac{1}{mn} \sum_{(s,t) \in S_{xy}} g(s,t) \]

2. Geometric Mean:

Similar to arithmetic but loses detail:

\[ f(x,y) = \left[ \prod_{(s,t) \in S_{xy}} g(s,t) \right]^{1/mn} \]
3. Harmonic mean filter.

$$\hat{F}(x,y) = \frac{mn}{\sum_{(s,t) \in S_{xy}} \frac{1}{g(s,t)}}$$

Works for salt noise, fails for pepper.

4. Contour Harmonic.

$$\hat{f}(x,y) = \frac{\sum_{(s,t) \in S_{xy}} g(s,t)}{Q}$$

$Q > 0 \rightarrow$ remove pepper

$Q < 0 \rightarrow$ remove salt noise
Order Statistics 6.14

1. Median
   - Reduce Sxy. Find median replace \( (x,y) \) with the median value.

2. Max
   - Also removes some data points.

3. Min
   - Also removes some data points.

4. Midpoint
   \[
   f(x,y) = \max \{ g(x,t) \}, \quad \forall y,
   \]
   \[
   f(x,y) = \min \{ g(x,t) \}, \quad \forall y,
   \]
   \[
   f(x,y) = \frac{1}{2} \{ \max + \min \}.
   \]
\[ f(x,y) = \frac{1}{mn - d} \sum_{(s,t) \in S_{xy}} g(\Theta(x,y)) \]

S\_xy \_m\_n.

g(r(s,t)) \rightarrow g(s,t) \text{ excluding } d/2 \text{ bright and gray level, and } d/2 \text{ darkest.}
Adaptive local noise reduction

\[ \hat{f}(x, y) = g(x, y) - \frac{6\hat{z}}{6L^2} \left( g(x, y) - m_L \right) \]

\[ m_L = \text{local mean} = \frac{1}{mn} \sum_{(x,t) \in S_{xy}} g(x, t) \]

\[ \sigma_L^2 = \text{local variance} \]

\[ \sigma_N^2 = \text{noise variance} \]

- if \( \sigma_N^2 \ll \sigma_L^2 \rightarrow \hat{f} = g \quad \text{good} \]
- if \( \sigma_N^2 \gg \sigma_L^2 \) then approximates \( \frac{\sigma_N^2}{\sigma_L^2} \approx 1 \)

\[ \rightarrow \hat{f} \approx m_L \]
- $Z_{max} = \max \{ y | (x,y) \}$
- $Z_{min} = \min \{ y | (x,y) \}$
- Pixel $(x,y)$
- Looking at neighborhood $S(x,y)$ around

- Adaptive Median Filter

- Median Filter

- Sort/Order

- Start/Begin
- $Z_{\text{min}} = \text{min value in } S_{xy}$
- $Z_{\text{med}} = \text{median value in } S_{xy}$

Outline

- Keep increasing window size until $Z_{\text{mod}}$ is not an impulse.

- When this happens, check $Z_{xy}$.
  - If $Z_{xy}$ is not an impulse $\rightarrow$ output $Z_{xy}$
  - If $Z_{xy}$ is an impulse $\rightarrow$ output $Z_{\text{mod}}$

Since $Z_{\text{mod}}$ is not an impulse.
**Psuedo Code**

**Part A**

if $Z_{min} < Z_{med} < Z_{max}$

Then go to part B. $\Rightarrow Z_{med}$ is not an impulse.

else if $\text{window size} < S_{max}$

window $\leftarrow$ window + 1, go to part A

else output $Z_{xy}$.

**Part B**

If $Z_{min} < Z_{xy} < Z_{max}$ $\Rightarrow Z_{xy}$ is not an impulse

output $Z_{xy}$

else output $Z_{med}$