Image Enhancement

March 9, 2007

- unsharp masking + high boost filtering

- unsharp Masking.

$$f_s(x,y) = f(x,y) - \bar{f}(x,y)$$

$f_s(x,y)$: processed; $f(x,y)$: original; $\bar{f}(x,y)$: LPF = blurred.

- General version unsharp masking. : high boost filtering

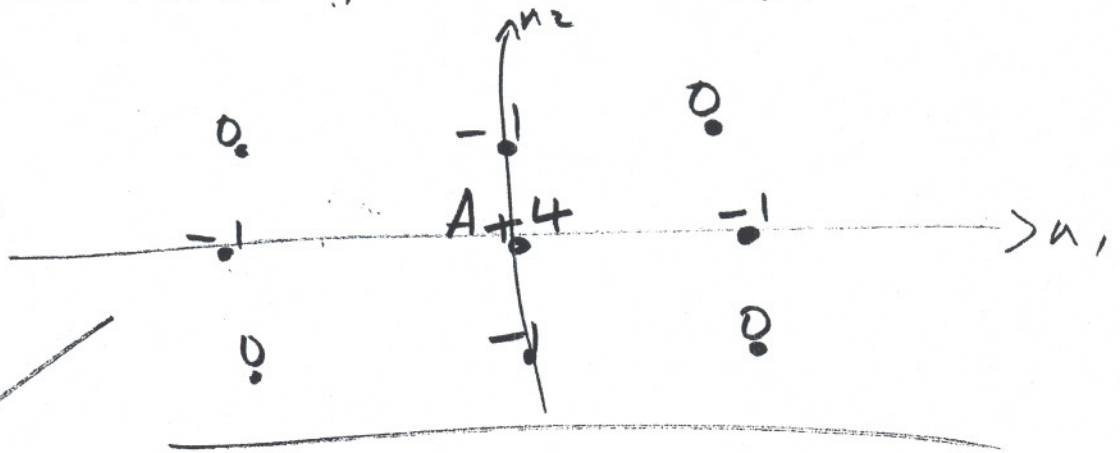
$$f_{hb}(x,y) = A\, f(x,y) - \bar{f}(x,y)$$

A: scaling factor.

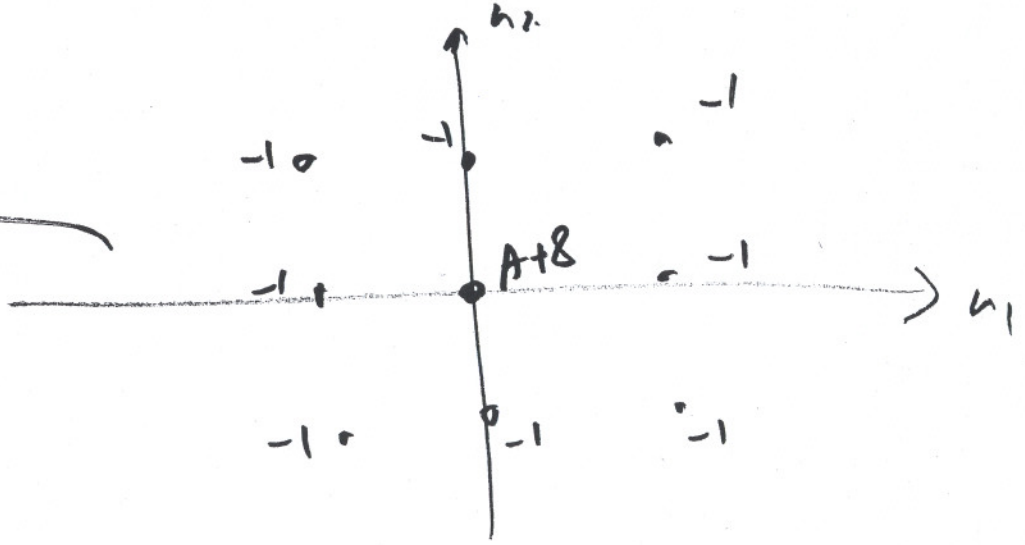
$A \geq 1$

$$f_{hb}(x,y) = (A-1) f(x,y) + f_s(x,y).$$

hb = linear operator



possible impulse response HB filtering.



Homomorphic Processing

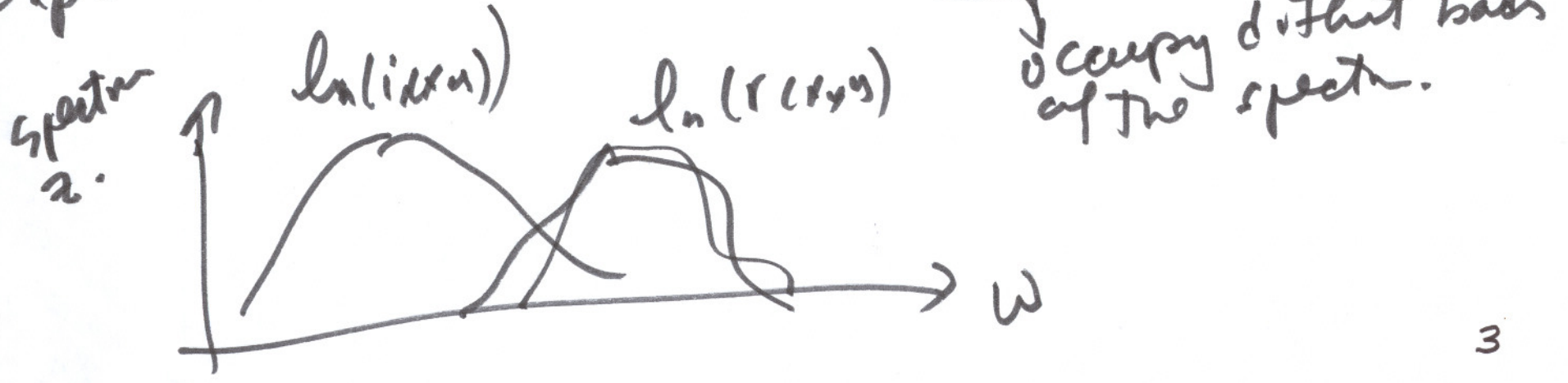
$$f(x,y) = i(x,y)\, r(x,y)$$

illumination — reflectance

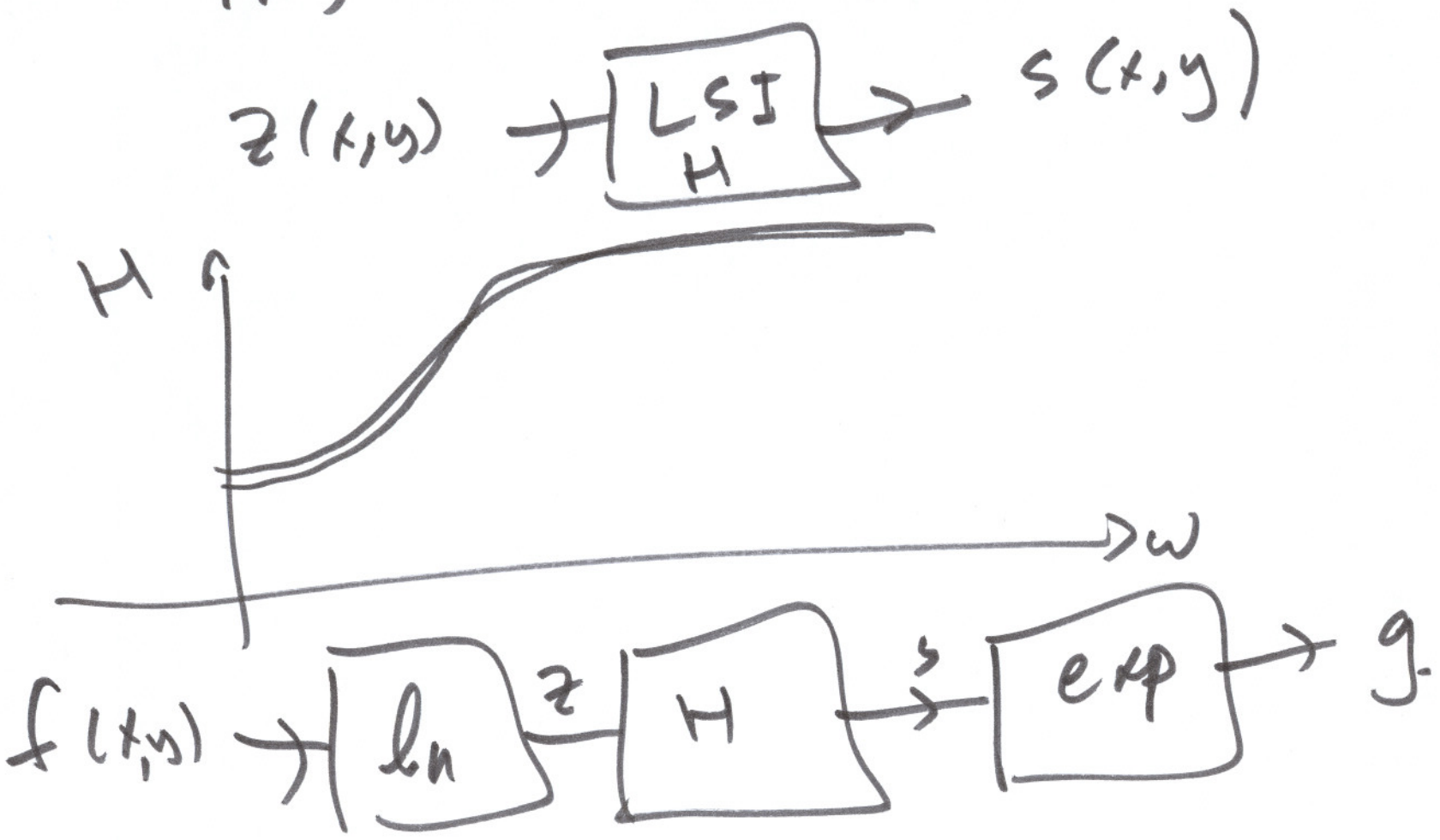
Define $z(x,y) = \ln\left(f(x,y)\right)$

$$z(x,y) = \ln i(x,y) + \ln r(x,y)$$

Exploit the fact that they occupy different bands of the spectrum.



Apply linear filtering.



Using Gradients for Evaluation

First derivative:

$$\vec{\nabla f} = \begin{bmatrix} \frac{\partial f(x,y)}{\partial x} \\ \frac{\partial f(x,y)}{\partial y} \end{bmatrix} = \begin{bmatrix} G_x \\ G_y \end{bmatrix}$$

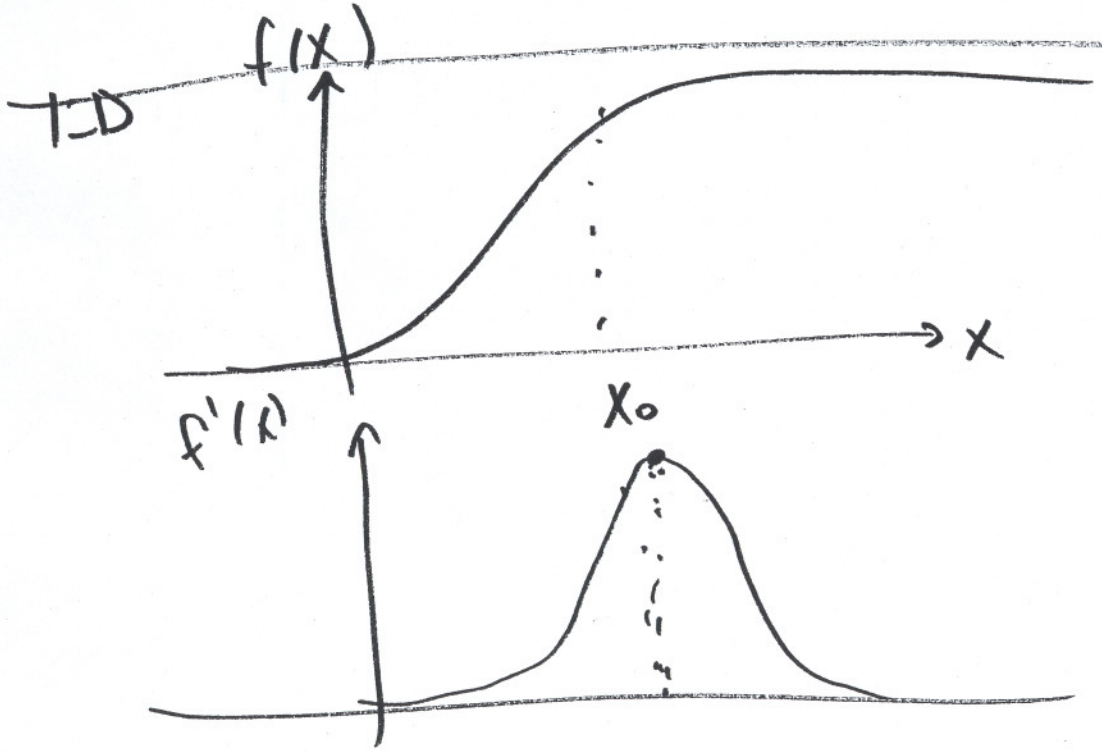
$$|\vec{\nabla f}| = \sqrt{G_x^2 + G_y^2} \longrightarrow \text{Edge Detection.}$$

↓ To simplify computationally

$$|\vec{\nabla f}| \approx |G_x| + |G_y|$$

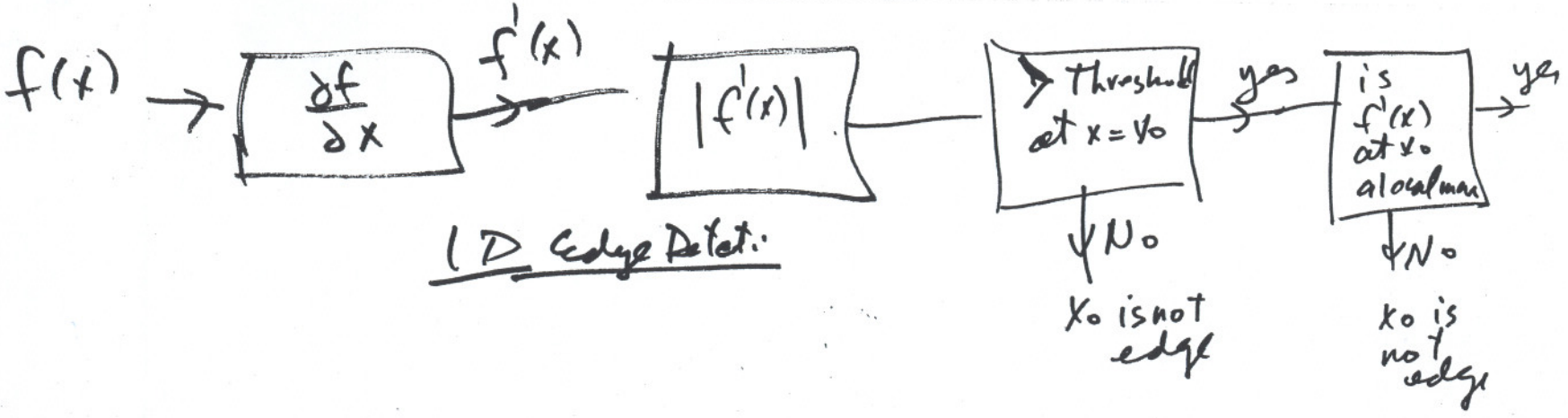
Edge Detection

- Gradient Method.
- Laplacian $\nabla^2 f$
- Lof G = Laplacian of Gaussian: Hildreth/Marr.



Derivative

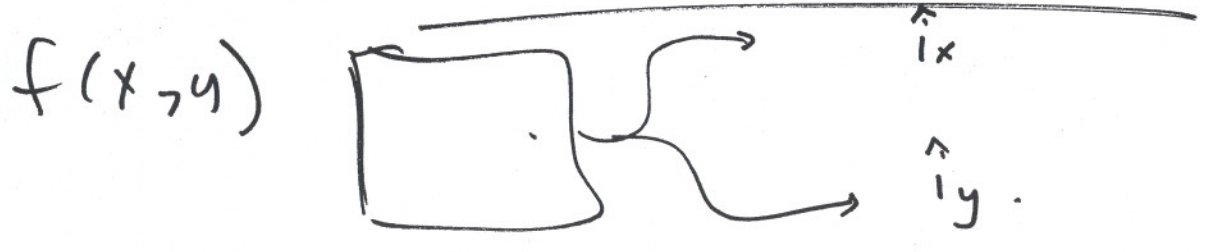
if $\frac{\partial f}{\partial x}$ large $\longrightarrow$ likely an edg.



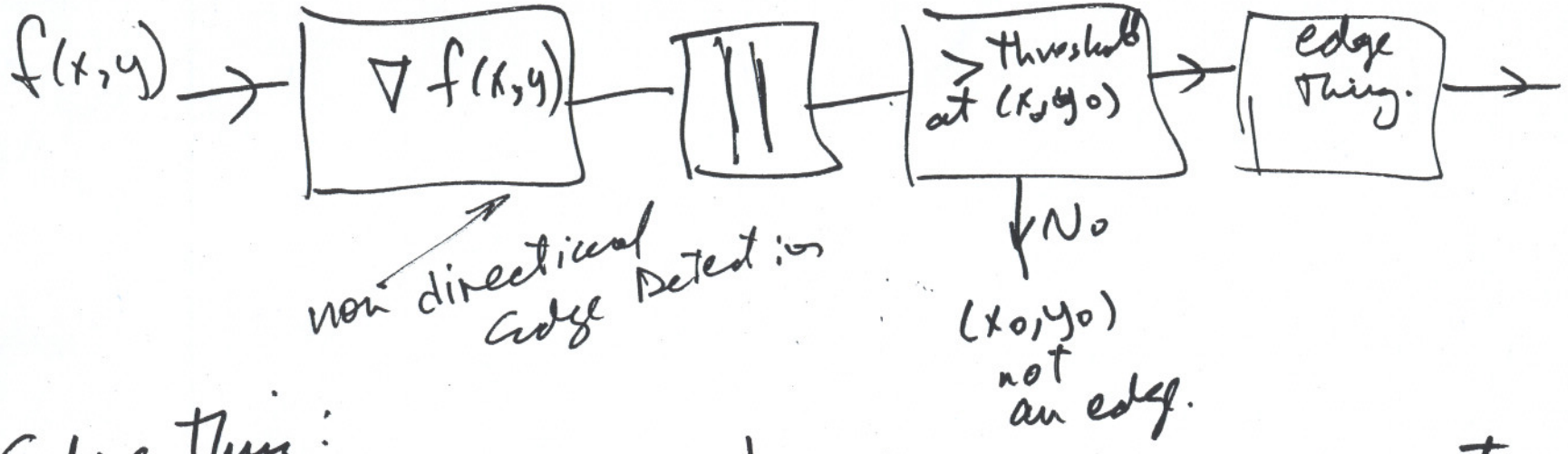
1D Edge Detn.

Show 10.7 of b/w

Extension To 2D



$$\nabla f(x,y) = \frac{\partial f(x,y)}{\partial x}\,\hat{i}_x + \frac{\partial f(x,y)}{\partial y}\,\hat{i}_y$$



Edge thing:

(a) If $|\nabla f(x,y)|$ has a local max at (x_0,y_0) in horizontal direction, but not vertical, we declare (x_0,y_0) an edge when

$$\left|\frac{\partial f(x,y)}{\partial x}\right|_{(x_0,y_0)} > K\left|\frac{\partial f(x,y)}{\partial y}\right|_{(x_0,y_0)}$$

$K \geq 2$

(b) If $|\nabla f(x,y)|$ has a local max at (x_0, y_0) in vertical direction but not horizontal, declare (x_0, y_0) an edge. when

$$\left|\frac{\partial f}{\partial y}(x,y)\right|_{(x_0,y_0)} > K \left|\frac{\partial f(x,y)}{\partial x}\right|_{(x_0,y_0)}$$

Directional Gradient Edge Detectors

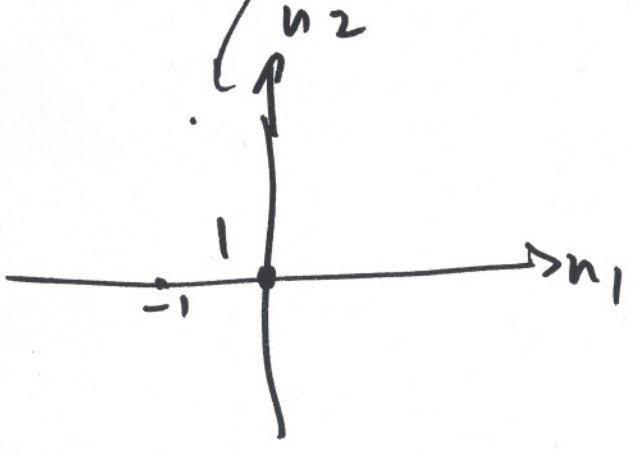
- Bias Towards a particular direction.

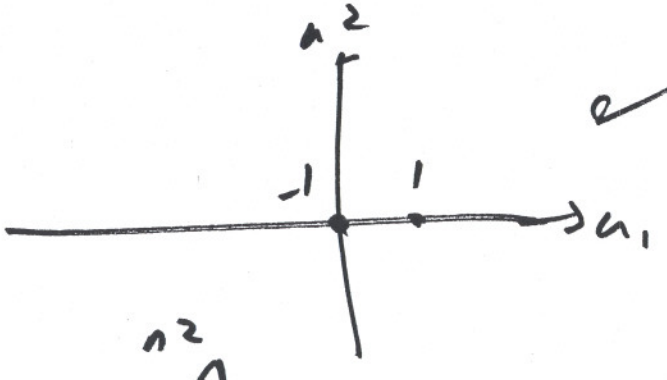
compute vertical edg: $\frac{\partial f(x,y)}{\partial x}$

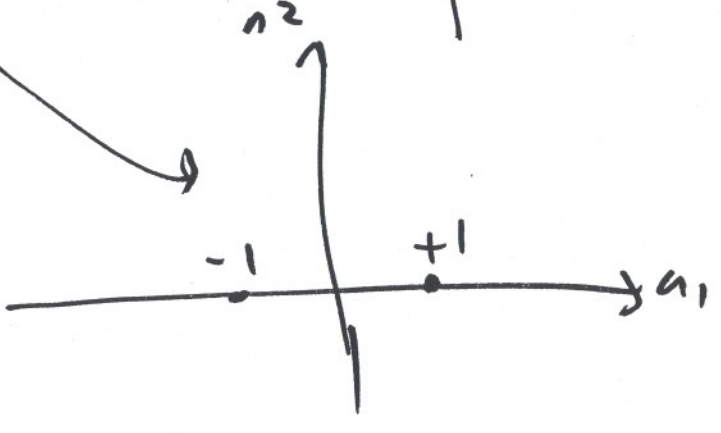
horizontal edg $\frac{\partial f(x,y)}{\partial y}$

$$\frac{\partial f(x,y)}{\partial x} = \begin{cases} \dfrac{f(x,y) - f(x-1,y)}{T} \\ \dfrac{f(x+1,y) - f(x,y)}{T} \\ \dfrac{f(x+1,y) - f(x-1,y)}{2T} \end{cases}$$

$x \overset{T}{\longleftrightarrow} x$

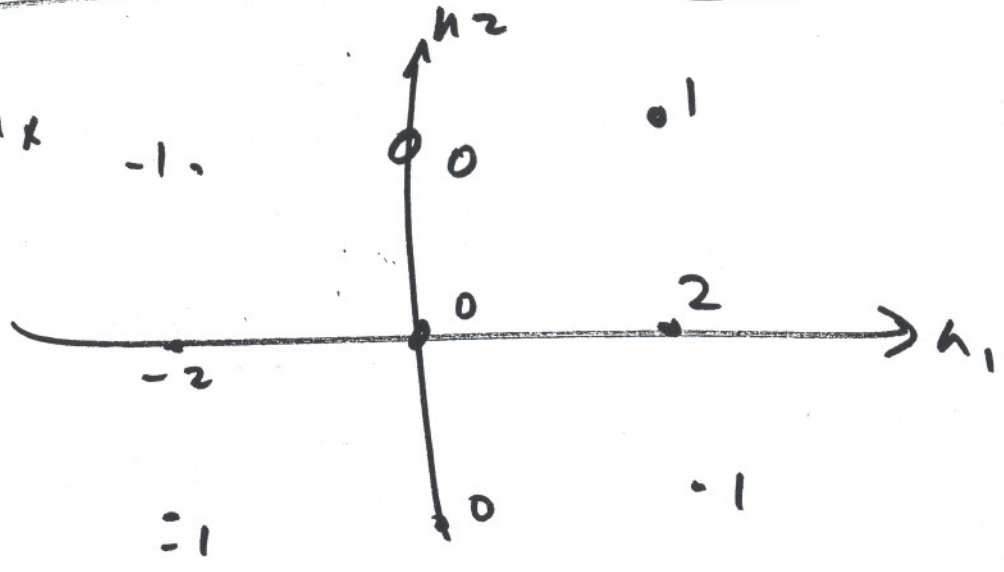




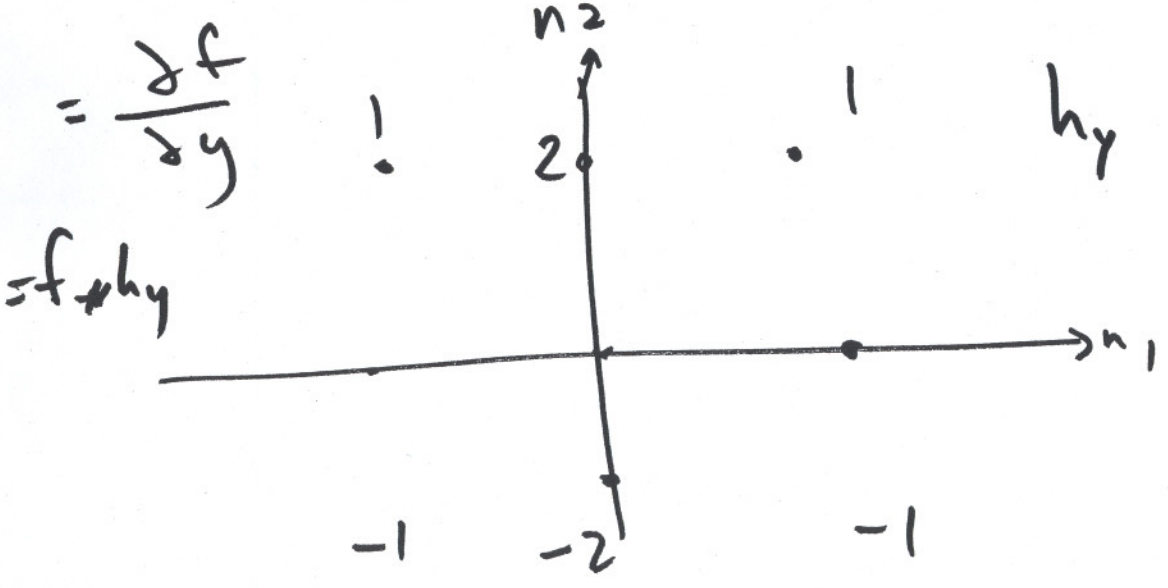


Farras Directional Edge Detection — Sobel

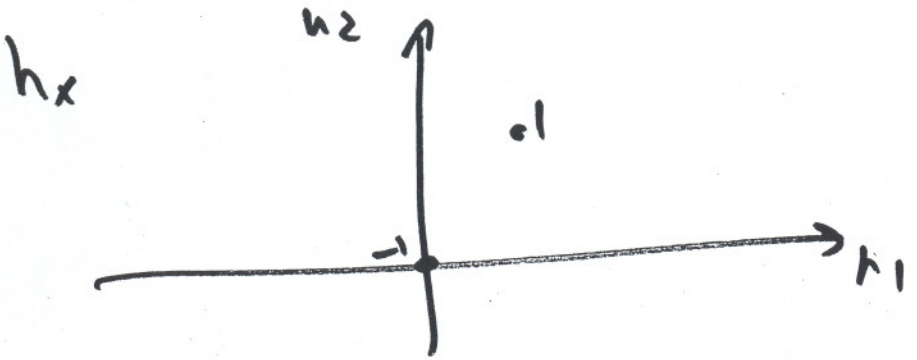
$\frac{\partial f}{\partial x}$ $f * h_x$

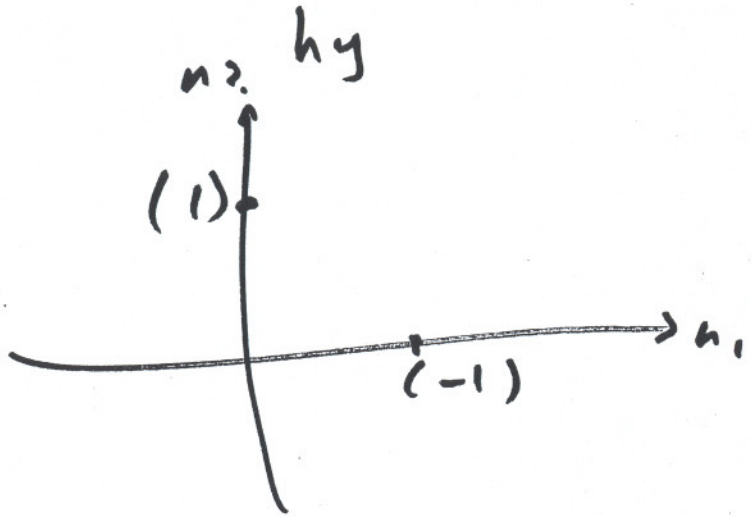


$= \frac{\partial f}{\partial y}$ $f * h_y$



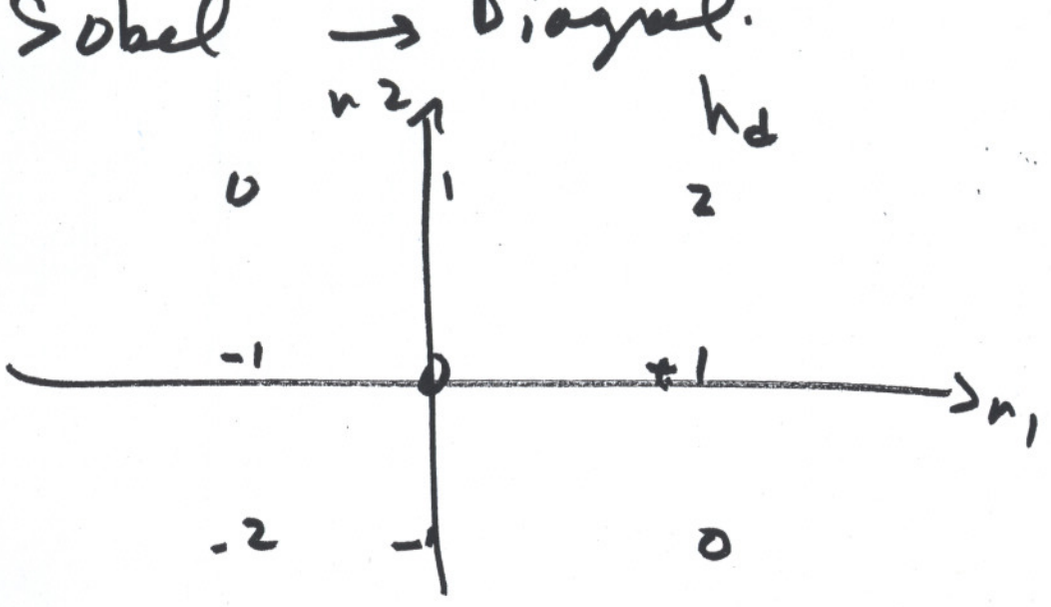
Roberts cross detector

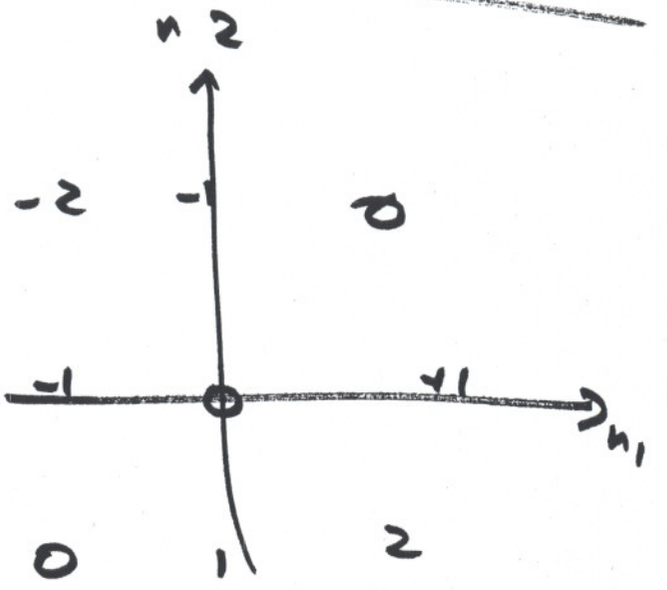




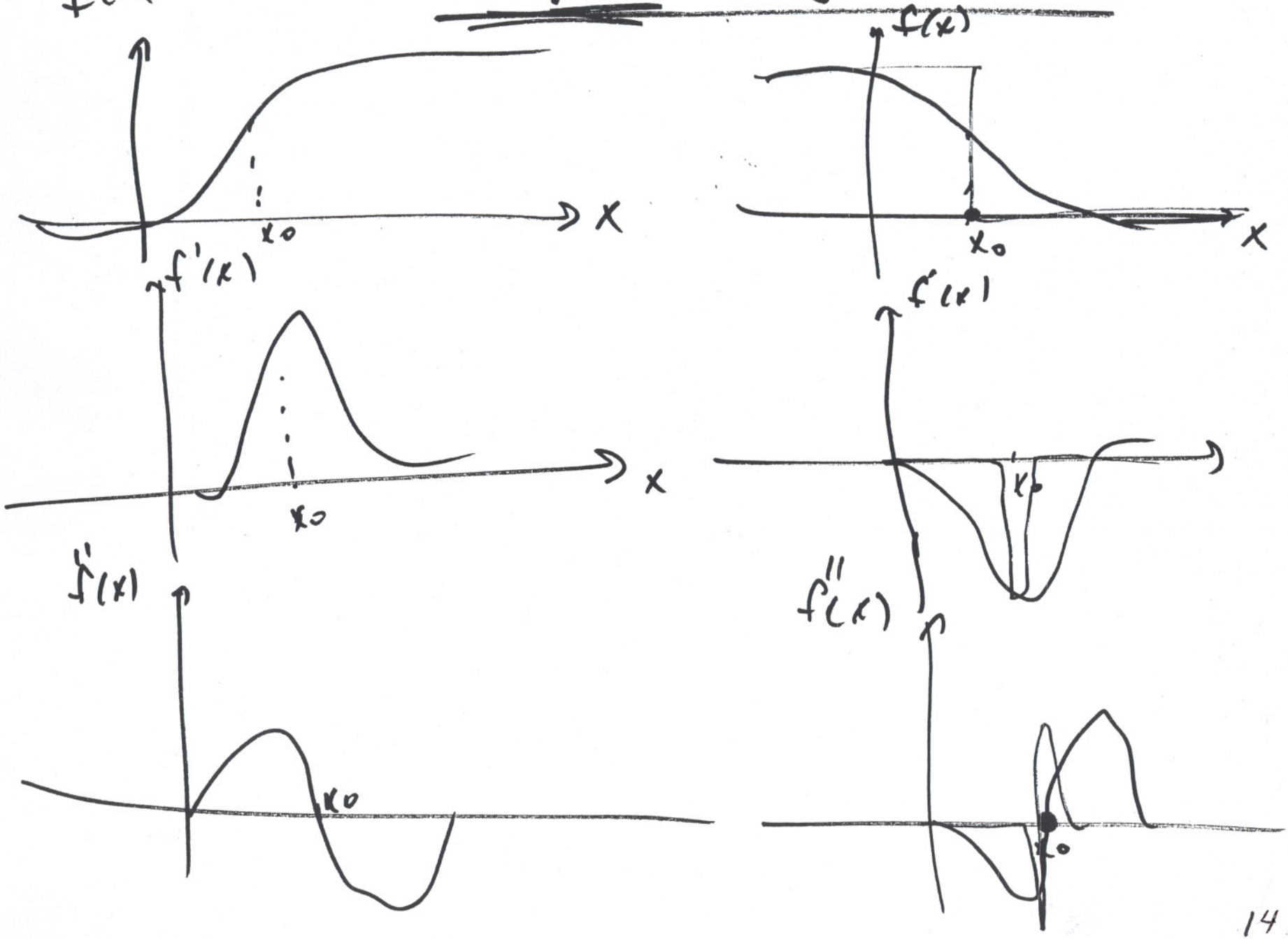
Diagonal Directional Gradient Filters

Sobel → Diagonal.





Laplacian Edge Detection



- zero crossing of $f''(x) \longrightarrow$ edge.

Extension To 2D

$$\nabla^2 f(x,y) = \nabla\left(\nabla f(x,y)\right)$$

$$= \frac{\partial^2 f(x,y)}{\partial x^2} + \frac{\partial^2 f(x,y)}{\partial y^2}$$

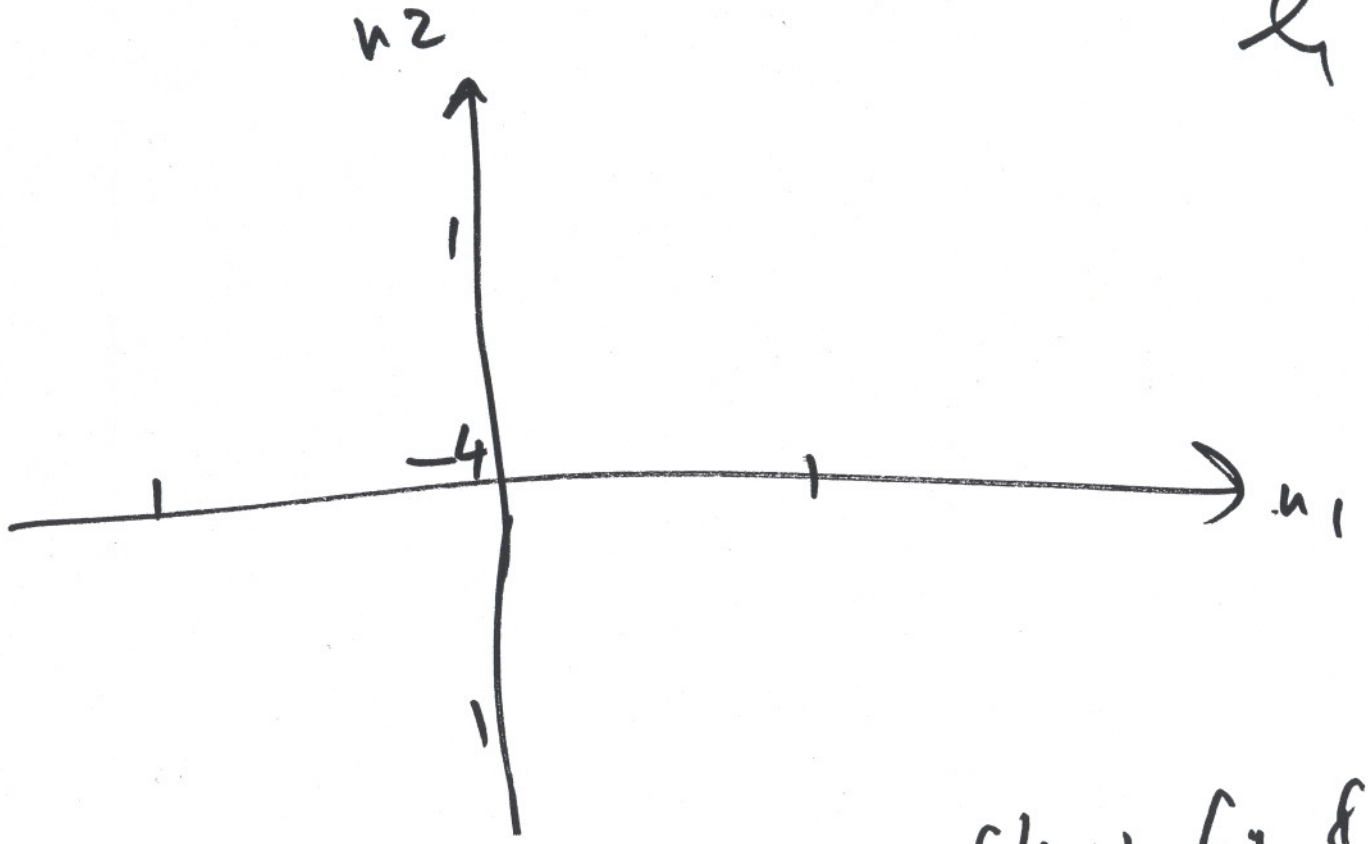
$$\left.\begin{aligned} \frac{\partial f}{\partial x} &\longrightarrow f(n_1+1, n_2) - f(n_1, n_2) \\ \frac{\partial^2 f}{\partial x^2} &\longrightarrow f_x(n_1, n_2) - f_x(n_1-1, n_2) \end{aligned}\right\} \Rightarrow$$

$$\frac{\partial^2 f}{\partial x^2} \Longrightarrow f(n_1+1, n_2) - 2f(n_1, n_2) + f(n_1-1, n_2)$$

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} \longrightarrow f(n_1+1, n_2) + f(n_1-1, n_2) + f(n_1, n_2+1) + f(n_1, n_2-1) - 4f(n_1, n_2)$$

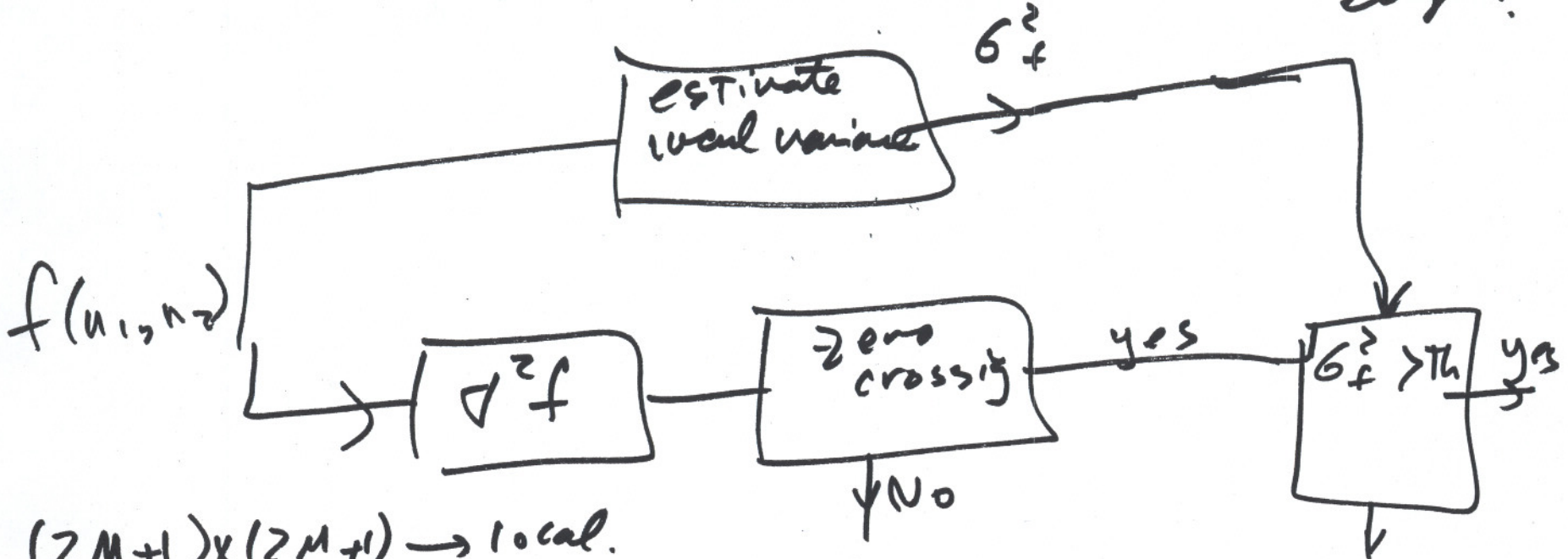
$$= f * h.$$

h



show fig 8.33
J Lim

How To use $\nabla^2 f$ operation to detect edge?



local mean

$$m_f(n_1,n_2) = \frac{1}{(2M+1)^2} \sum_{k_1=n_1-M}^{n_1+M} \sum_{k_2=n_2-M}^{n_2+M} f(k_1,k_2)$$

local variance

$$\sigma_f^2(n_1,n_2) = \frac{1}{(2M+1)^2} \sum_{k_1=n_1-M}^{n_1+M} \sum_{k_2=n_2-M}^{n_2+M} \left(f(k_1,k_2) - m_f(k_1,k_2) \right)^2$$

Window $(2M+1) \times (2M+1)$ centered about (n_1,n_2)

Edge Detection Laplacian of Gaussian

Gaussian $h(x,y) = e^{-\frac{(x^2+y^2)}{2\pi\sigma^2}}$

$$H(\Omega_x, \Omega_y) = e^{-2\pi\sigma^2 \frac{(\Omega_x^2 + \Omega_y^2)}{2}}$$

$$2\pi^2\sigma^2$$

"Laplacian of Gaussian" = LoG.

$$\nabla^2 \left[f(x,y) * h(x,y) \right] =$$

(Gaussian)

$$= f(x,y) * \left[\nabla^2 h(x,y) \right]$$

$$= f(x,y) * \left[\frac{\partial^2 h(x,y)}{\partial x^2} + \frac{\partial^2 h(x,y)}{\partial y^2} \right]$$

$$\nabla^2 h(x,y) = \frac{e^{-(x^2+y^2)/2\pi\sigma^2}}{(2\pi\sigma^2)^2}\left(x^2+y^2-2\pi\sigma^2\right)$$

$$F\left\{\downarrow\right\} = -2\pi^2\sigma^2 \; e^{-\pi\sigma^2(\Omega_x^2+\Omega_y^2)/2}\left(\Omega_x^2+\Omega_y^2\right)$$

Fig. 8.36 J. Lim