

LSI \longrightarrow Impulse response.

$$h(n_1, n_2)$$

- enables to entirely characterize

an LSI system.

- tells you output given input.

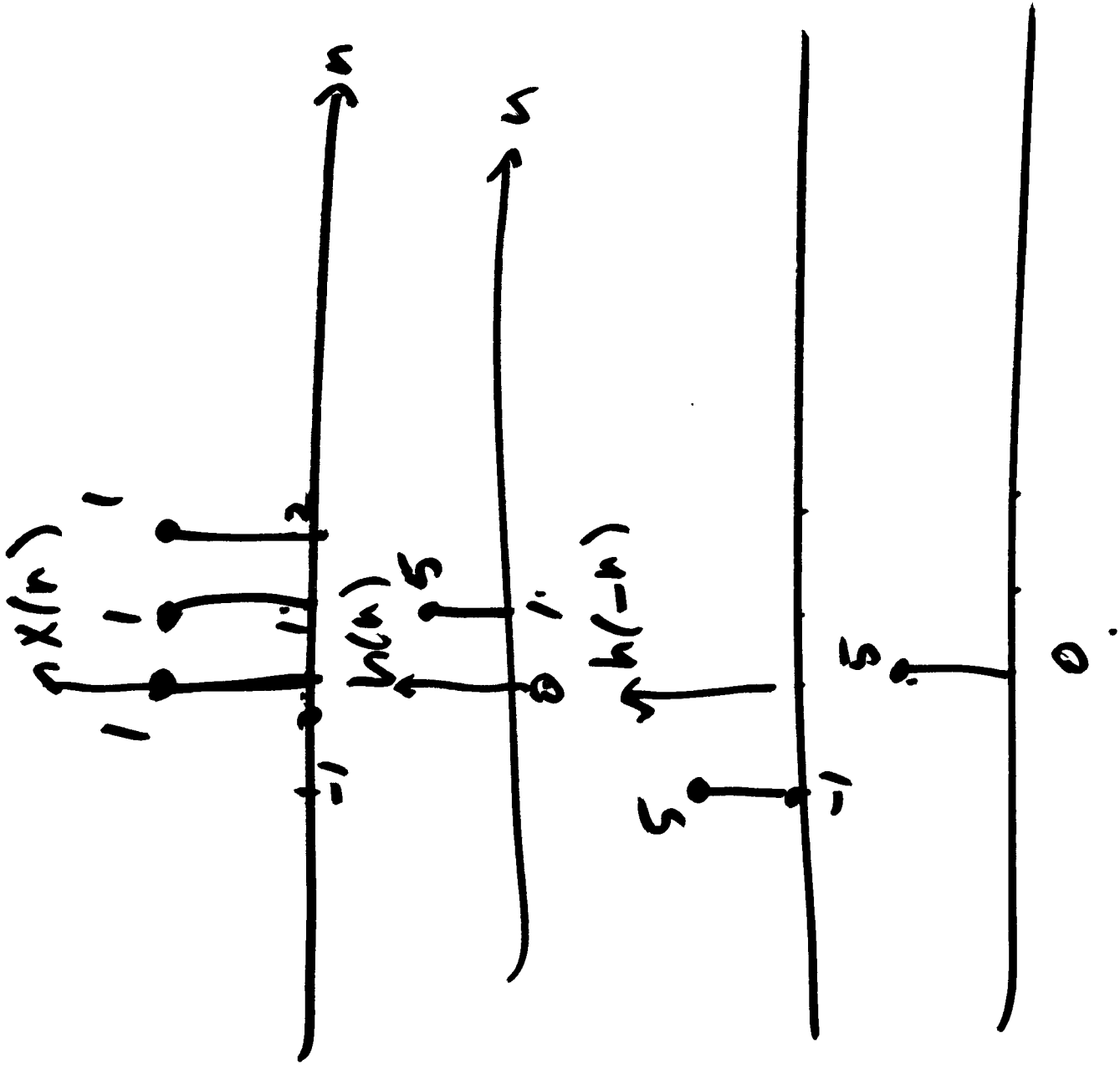
$$Y(n_1, n_2) = X * h \longrightarrow \text{convolution}$$

$$y(n_1, n_2) = \sum_{k_1} \sum_{k_2} x(k_1, k_2) h(n_1 - k_1, n_2 - k_2)$$

$$Y(u) = \sum_k X(k) h(u-k) \quad 1D$$

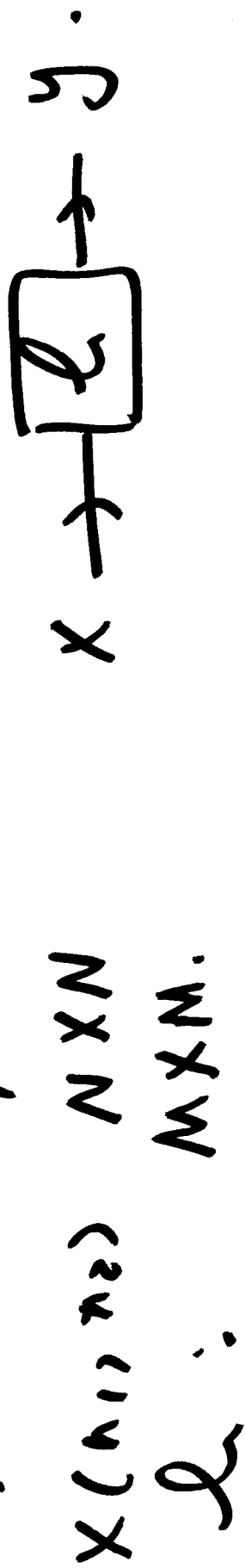
$$y(1) = 1$$

$$y(0) = 0$$



Separable LSI systems

$$h(n_1, n_2) = h_1(n_1) h_2(n_2)$$



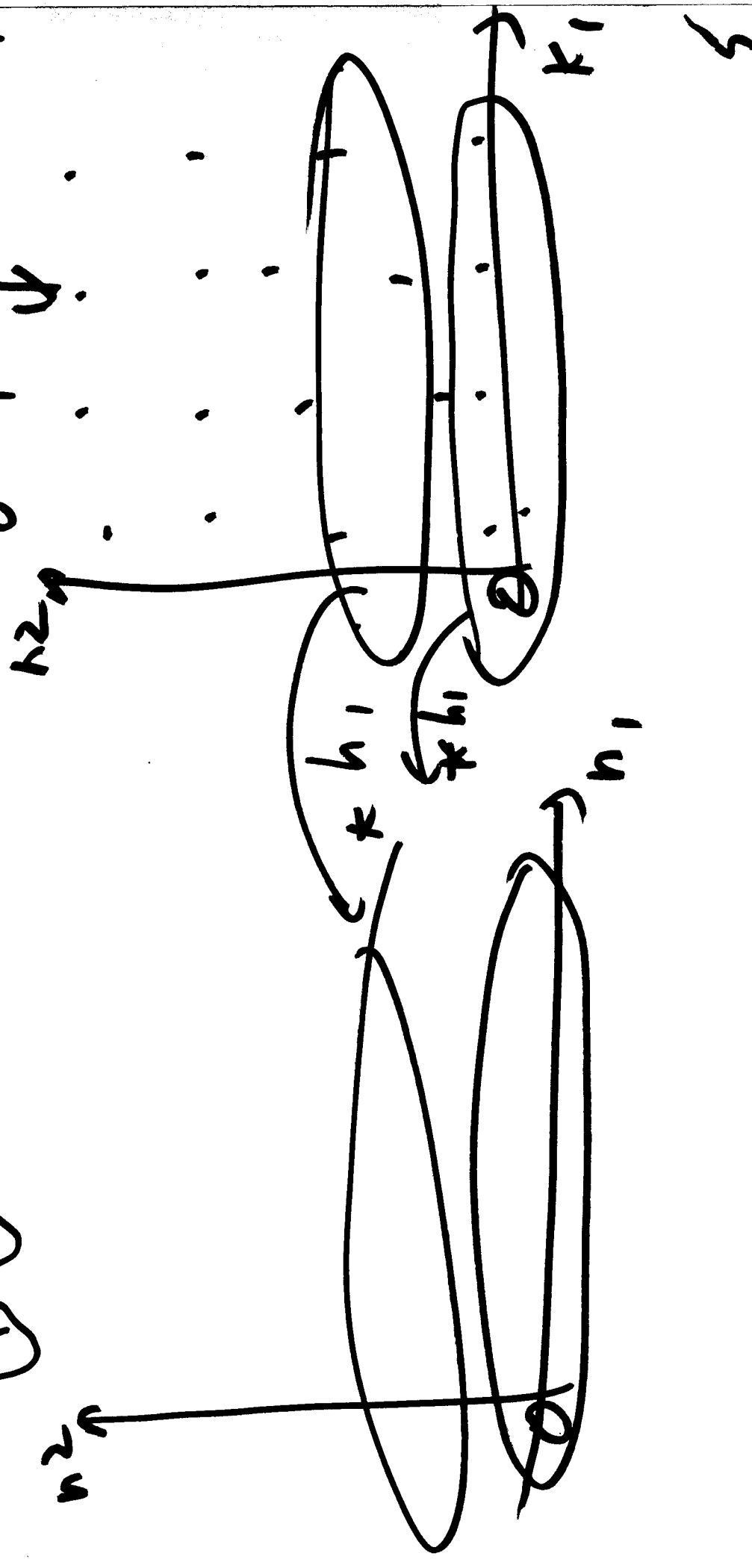
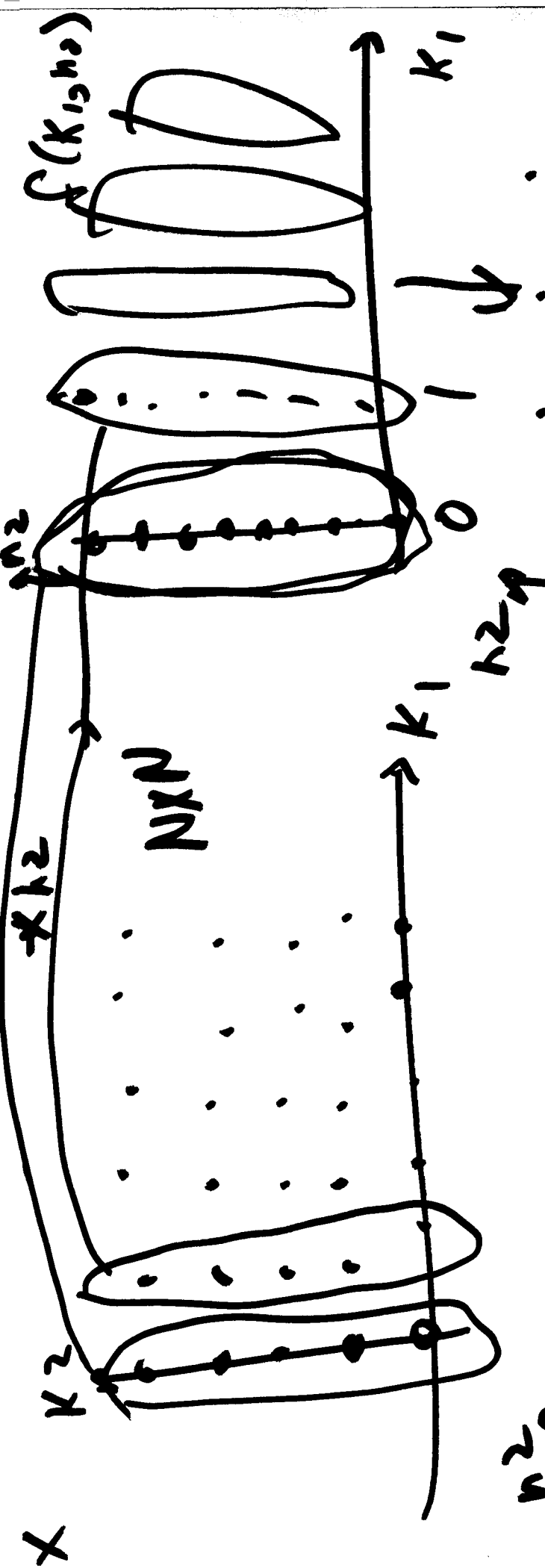
general:

$$y(n_1, n_2) = \sum_{k_1} \sum_{k_2} x(k_1, k_2) h_1(n_1 - k_1) h_2(n_2 - k_2)$$

sep. sys.

$$= \sum_{k_1} \sum_{k_2} x(k_1, k_2) h_1(n_1 - k_1) h_2(n_2 - k_2)$$

$$= \underbrace{\sum_{k_2} x(k_1, k_2) h_2(n_2 - k_2)}_{f(k_1, n_2)} = x(k_1) * h_2$$



Summary

1. Series of 1D convolutor To get.

$f(k, n_2)$:

N columns \rightarrow $\left\{ \begin{array}{l} N \text{ convolutor} \\ NM \text{ oper / convol} \end{array} \right.$

$N^2 M$ op To build

② Series of 1-D conv To get $g(n, n_1)$

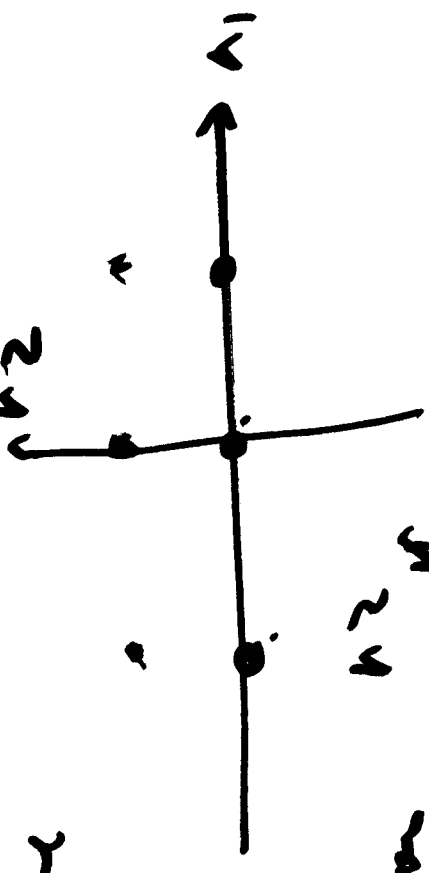
N rows \rightarrow $\left\{ \begin{array}{l} N \text{ convolutor} \\ NM \text{ op / convol} \end{array} \right.$

$N^2 M$

Total $2 \cdot N^2 M$.

Finite impulse response

(FIR) \Rightarrow Always stable.



Infinite impulse response

(IIR) some stable, some unstable.



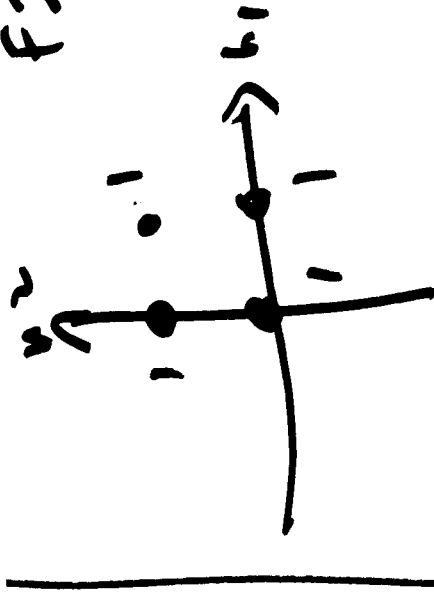
$\sum_{n=-\infty}^{\infty} \frac{1}{n}$ blows up $\sum_{n=1}^{\infty} \frac{1}{n^2}$ does not blow up.

$$\sum_{n_1, n_2} h(n_1, n_2) \xrightarrow{Z.T.} \sum_{n_1, n_2} h(n_1, n_2) z_1^{-n_1} z_2^{-n_2}$$

$$= H(z_1, z_2) = \sum_{n_1, n_2} h(n_1, n_2) z_1^{-n_1} z_2^{-n_2}$$

(FIR)

FIR



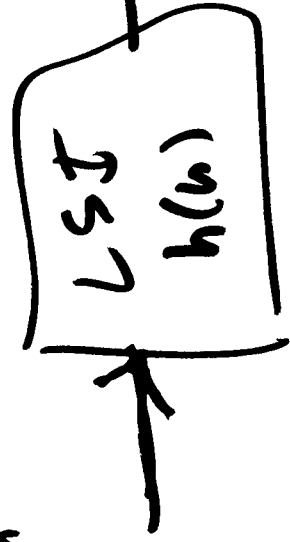
$$H(z_1, z_2) = 1 + z_1^{-1} + z_2^{-1} + z_1^{-1} z_2^{-1}$$

2D polynomial in z_1^{-1} and in z_2^{-1}

2.D.F.I.T.

$$H(\omega_1, \omega_2) = \sum_{n_1} \sum_{n_2} h(n_1, n_2) e^{-j\omega_1 n_1 - j\omega_2 n_2}$$

$e^{j\omega n}$



$$H(\omega_0) e^{j\omega_0 n}$$

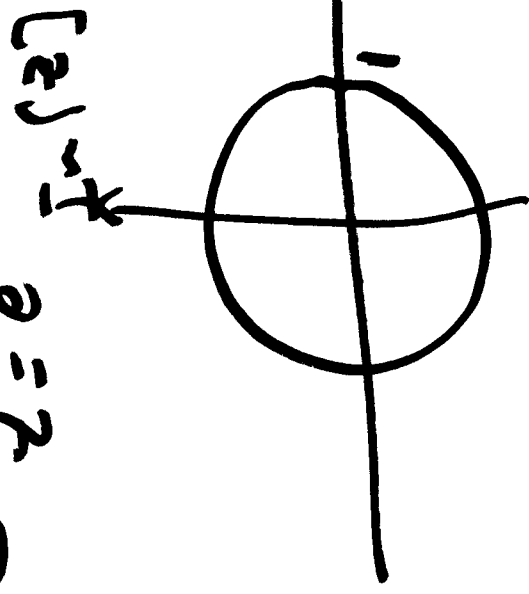
NOT a fr of $n \cdot j\omega n$

$$H(\omega_0) = [H(\omega)]_{\omega=\omega_0} = \sum_{n=-\infty}^{+\infty} h(n) e^{-j\omega n}$$

$$H(\omega) = \sum_{n=-\infty}^{+\infty} h(n) e^{-j\omega n} \quad \Rightarrow$$

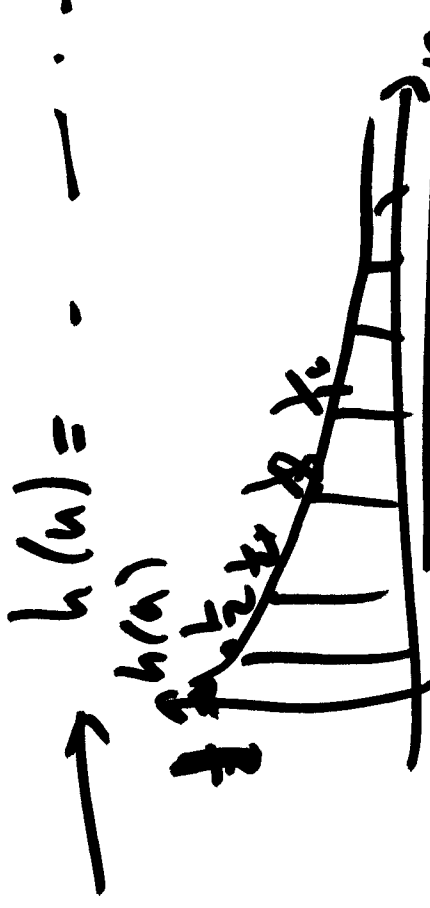
$$H(z) = \sum_{n=-\infty}^{+\infty} h(n) z^{-n}$$

$$H(j\omega) = [H(z)]_{z=e^{j\omega}}$$



$$H(z) = \frac{1}{1 - \frac{1}{2}z^{-1}}$$

ROC: outside circle



$$\sum_{n=-\infty}^{\infty} |h(n)| < \infty$$

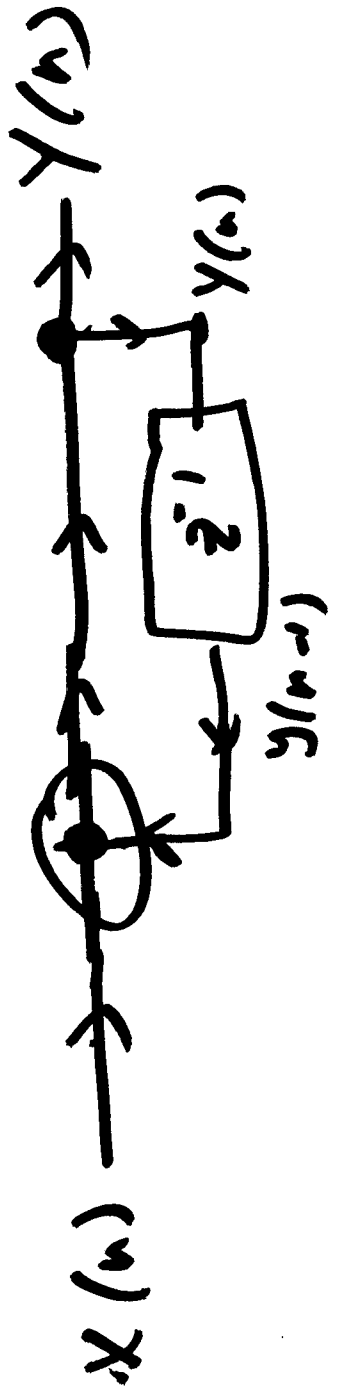
BIBO

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1 - \frac{1}{2}z^{-1}} \cdot X(z)$$

$$Y(z) \left[1 - \frac{1}{2}z^{-1} \right] = X(z)$$

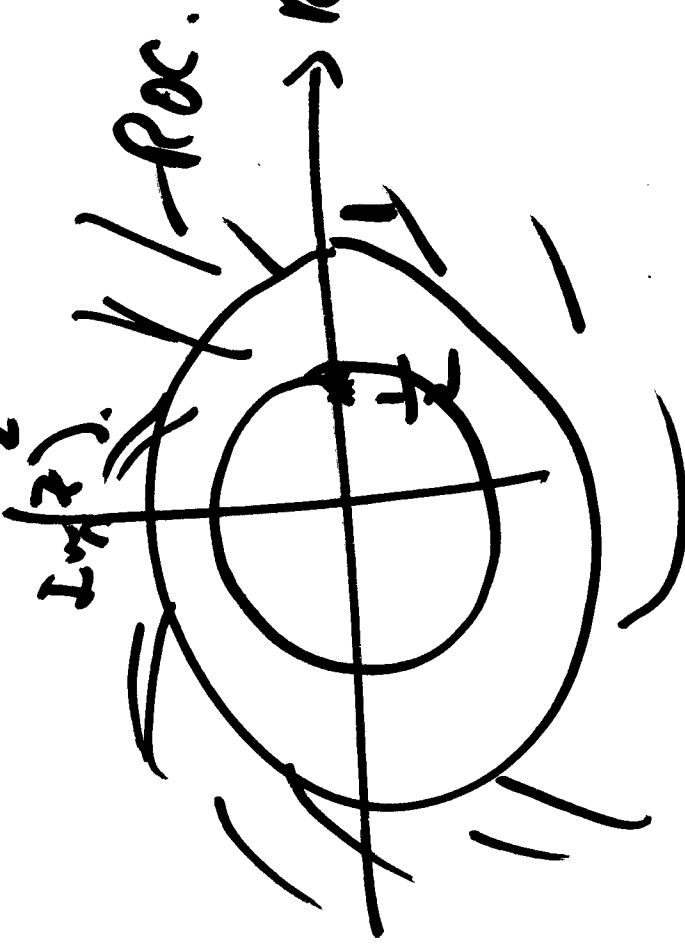
~~$$Y(n) - \frac{1}{2}Y(n-1) = X(n)$$~~

$$Y(n) = X(n) + \frac{1}{2}Y(n-1)$$



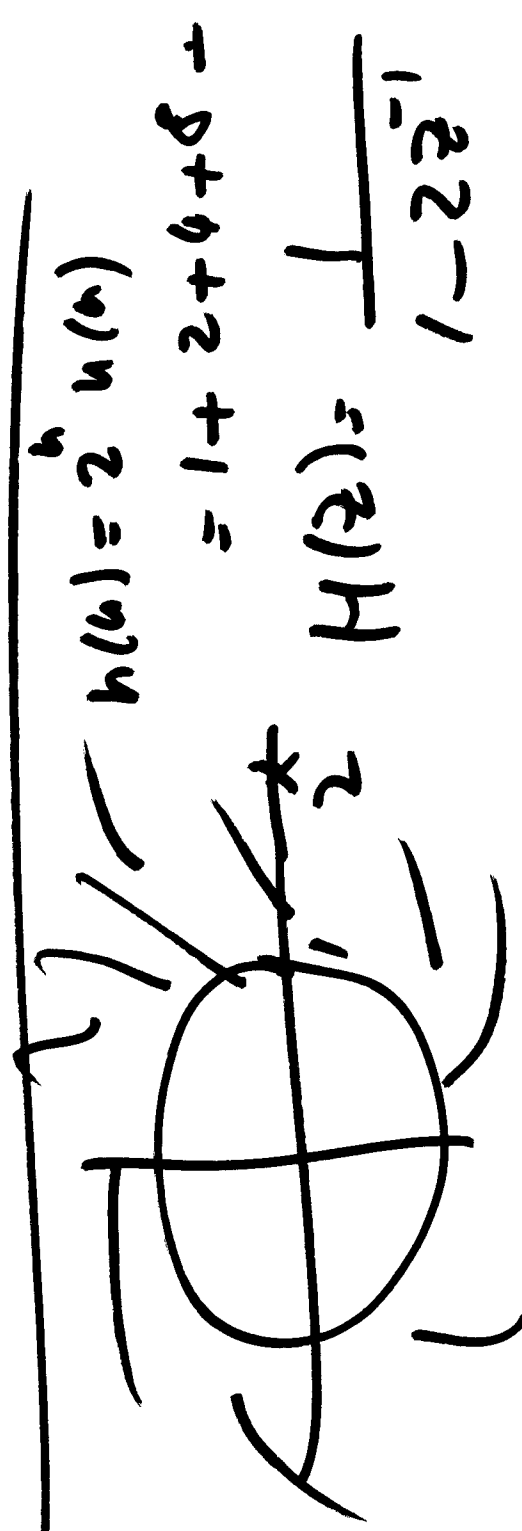
pole $1 - \frac{1}{2}z^{-1} = 0 \Rightarrow z = \frac{1}{2}$

$z = \frac{1}{2} \Rightarrow z = \frac{1}{2}$
ROC.



$|z| > \frac{1}{2}$
 $h(n) = \frac{1}{2}^n u(n)$

Unstable



$h(n) = 2^n u(n)$

$= 1 + 2 + 4 + 8 + \dots$

$H(z) = \frac{1}{1 - 2z^{-1}}$

$h(z) :=$

$$1 + 5z + 3z^2 + 9z^3 + 12z^4 + 18z^5$$

2-D polynomial can always

BS factored.

→ Fundamental Thm of Algebra.

Any 1-D polynomial ~~can~~ of deg. n
can be factored into n
polynomials of deg. 1.

$$\prod_{i=1}^n (1 - \alpha_i z^{-1})$$

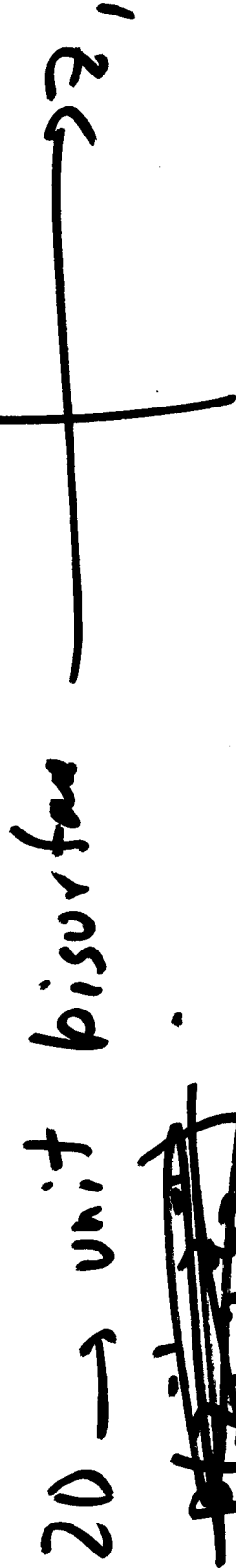
Story in 2D

$$\text{IIR: } H(z_1, z_2) = \frac{1}{P(z_1, z_2)}$$

$$P(z_1, z_2) = (1 - z_1^{-1})(1 - z_2^{-1})$$

Thm: Hayn 1980's: \leftarrow

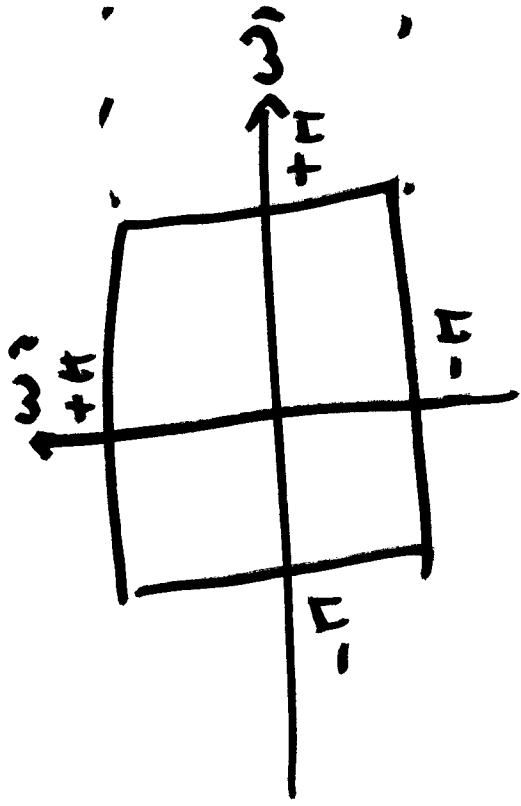
Set of reducible 2D polynomials is of measure 0 in the set of all 2D polynomials.



~~$P(z_1, z_2)$~~

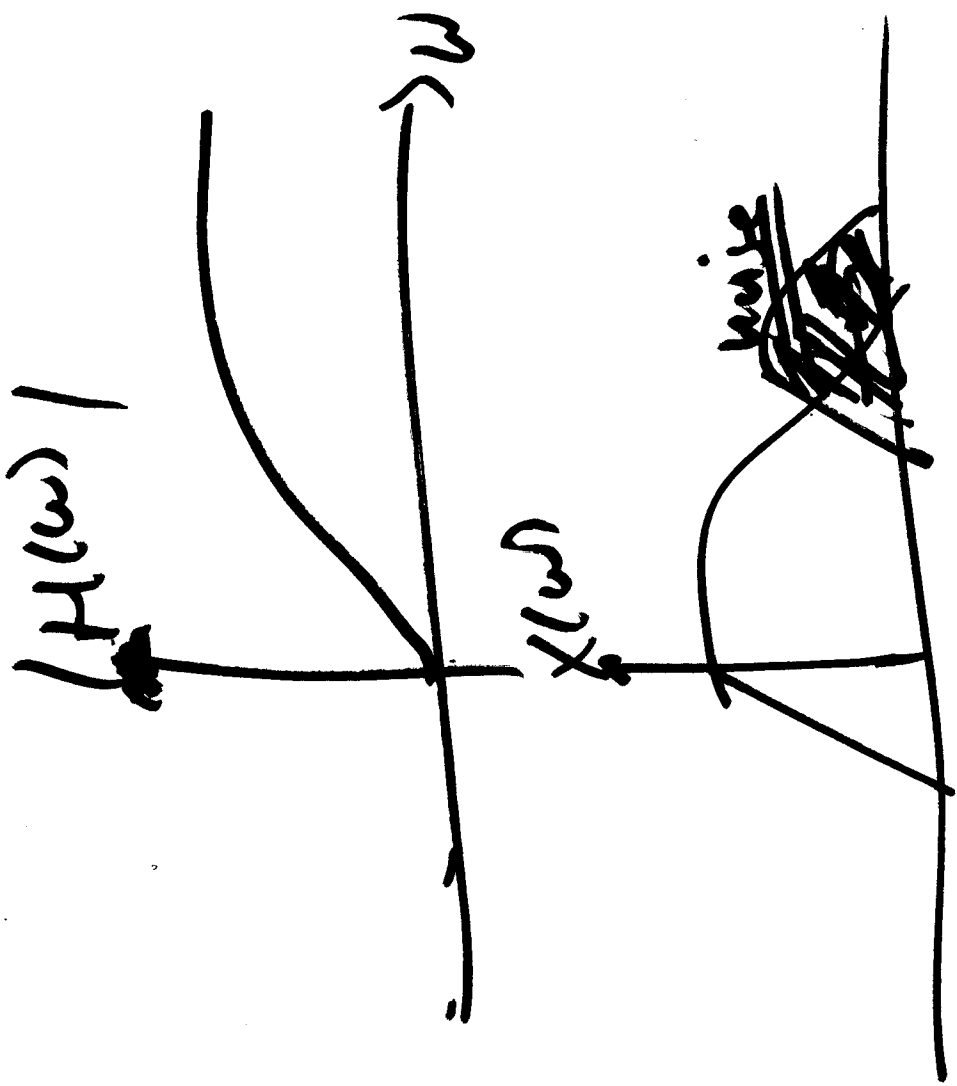
Do the pde surface cross the
unit Bi-surface? \Rightarrow Some things
covered later.

Proving stability in 2D is hard



$x(n_1, n_2)$ → discrete integers

$X(\omega_1, \omega_2)$ → Real variables.



Signal.

