Assume that $x(n_1, n_2)$ is a finite-extent sequence. Suppose the above difference equation corresponds to an LSI system with an ROC that satisfies the conditions in the following constraint map.

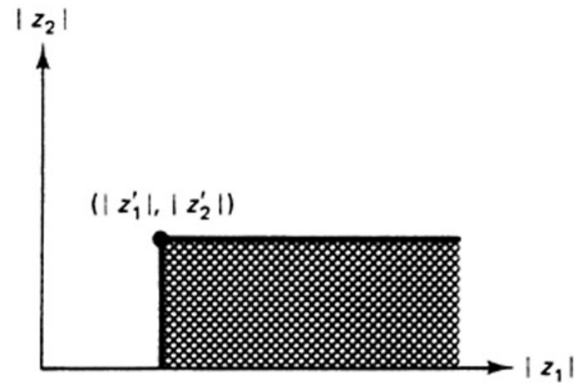

Figure P2.17

The constraint map shows that for any (z_1', z_2') in the ROC, all (z_1, z_2) such that $|z_1| \geq |z_1'|$ and $|z_2| \leq |z_2'|$ are also in the ROC.

(a) Determine the output mask and input mask for the system.

(b) Determine the impulse response of the above LSI system. Your answer should be a closed-form expression. (*Hint:* Consider doing an inverse z-transform.)

2.18. Consider the following difference equation:

$$y(n_1, n_2) + y(n_1 + 1, n_2) + y(n_1 + 1, n_2 + 1) + y(n_1, n_2 + 1) + y(n_1 - 1, n_2 + 1) + y(n_1 - 1, n_2) = x(n_1, n_2).$$

Assume that $x(n_1, n_2)$ is a finite-extent sequence.

(a) How many different recursively computable LSI systems can be obtained from the above difference equation? For each of the systems, sketch the output mask and input mask.

(b) For each of the systems in (a), sketch the region of support of $y(n_1, n_2)$ when $x(n_1, n_2) = \delta(n_1, n_2)$.

2.19. Consider a recursively computable LSI system with the output mask and input mask shown in the following figure. Assume that $x(n_1, n_2)$ is a first-quadrant support sequence.

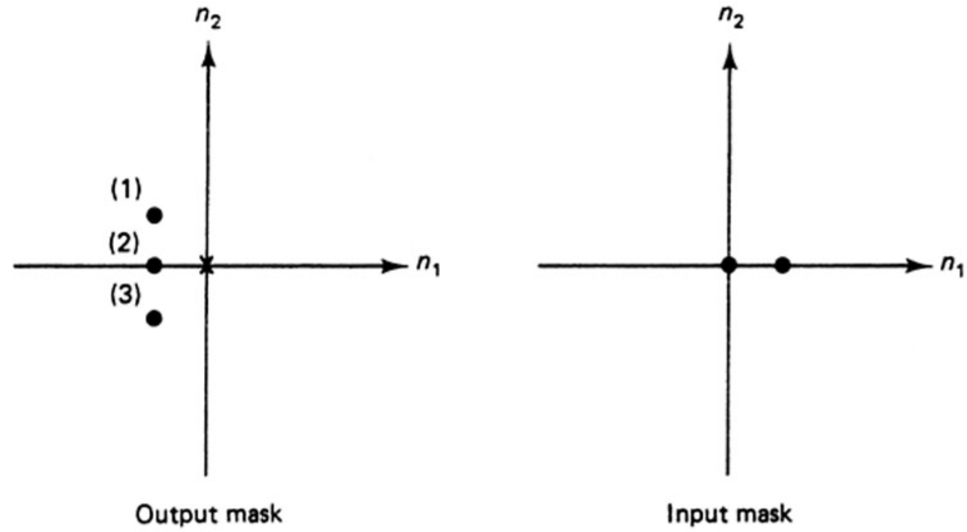

Figure P2.19

(a) Write the difference equation that has the above output and input masks.
(b) Find the boundary conditions that lead to a linear and shift-invariant system.
(c) Determine R_h, the region of support of the impulse response of the LSI system obtained in (b).
(d) Determine one recursion direction that can be used for the LSI system obtained in (b).
(e) Determine the system function.

2.20. Suppose we have designed an IIR filter whose system function is given by

$$H(z_1, z_2) = \frac{1 + z_1^{-1}}{1 + 2z_1^{-1} + 4z_2^{-1}}.$$

The IIR filter is an LSI system. Suppose the filter was designed by attempting to approximate a desired second-quadrant support sequence $h_d(n_1, n_2)$ with $h(n_1, n_2)$. We wish to filter the following 2×2-point input $x(n_1, n_2)$.

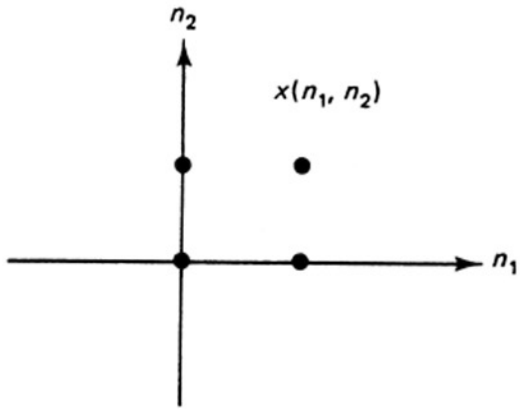

Figure P2.20

Determine the output $y(n_1, n_2)$.

2.21. Consider an LSI system with a third-quadrant support output mask and a third-quadrant support input mask. Show that the impulse response of the system has first-quadrant support. A digital filter with a first-quadrant support impulse response is sometimes referred to as a *causal* or *spatially causal* filter.

2.22. Consider a digital filter whose impulse response $h(n_1, n_2)$ has all its nonzero values in the shaded region in the following figure.

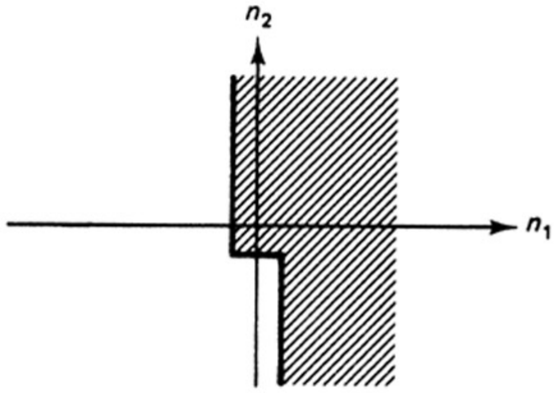

Figure P2.22a

A digital filter of this type is called a *nonsymmetric half plane* filter. Show that a filter whose output and input masks are shown in the following figure is a nonsymmetric half plane filter.

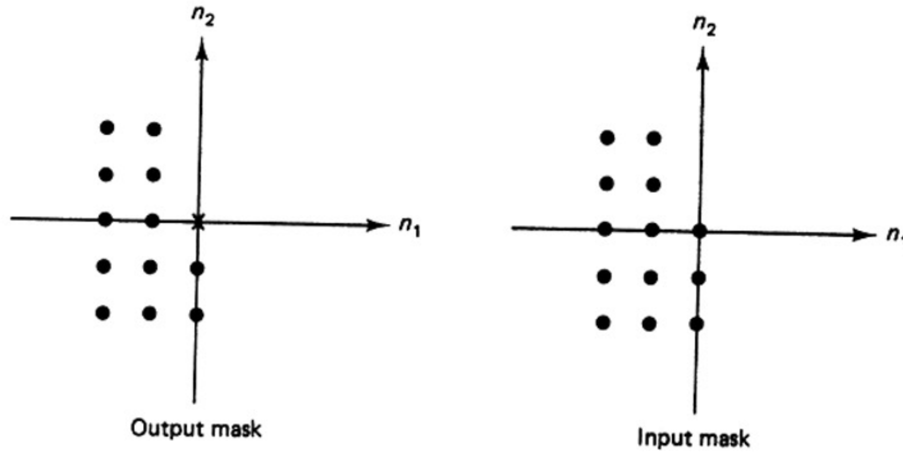

Figure P2.22b

2.23. Consider a 2-D LSI system with its system function denoted by $H(z_1, z_2)$. Consider the following four pieces of information.

(I1) $$H(z_1, z_2) = \frac{1}{1 + z_1^{-1} + z_2^{-1} + z_1^{-1}z_2 + z_1^{-1}z_2^{-1} + z_1z_2^{-1}}.$$

(I2) The system is recursively computable.

(I3) The ROC of $H(z_1, z_2)$ satisfies the conditions of the following constraint map, which states that for any (z_1', z_2') in the ROC, all (z_1, z_2) such that $|z_1| \leq |z_1'|$ and $|z_2| \leq |z_2'|$ is also in the ROC.

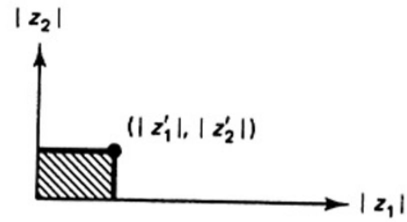

Figure P2.23a

(I4) The input to the system, $x(n_1, n_2)$, is a 3×3-point sequence given by

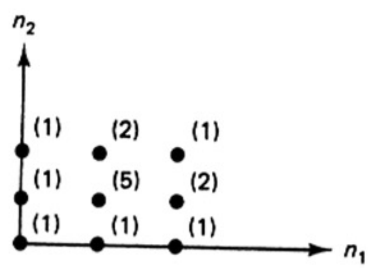

Figure P2.23b

(a) Suppose that only the information in (I1), (I2), and (I4) is available. How many different systems would satisfy the constraints imposed by the available information?

(b) Suppose all the above information in (I1), (I2), (I3), and (I4) is available. Sketch the output and input masks and determine $y(0, 0)$, the response of the system evaluated at $n_1 = n_2 = 0$.

2.24. When the input $x(n_1, n_2)$ is a finite-extent sequence, a necessary and sufficient condition for a computational procedure to be recursively computable is that the output mask have wedge support. When the input $x(n_1, n_2)$ is an infinite-extent sequence, the condition that the output mask have wedge support is necessary but not sufficient for the system to be recursively computable. To illustrate this, consider the following specific computational procedure:

$$y(n_1, n_2) \leftarrow y(n_1 - 1, n_2) + y(n_1, n_2 - 1) + y(n_1 - 1, n_2 - 1) + x(n_1, n_2).$$

Determine an infinite-extent quadrant support input $x(n_1, n_2)$ such that the boundary conditions obtained to force the computational procedure to be an LSI system do not make sense. In this case, the system is not recursively computable, even though the output mask has wedge support.

2.25. Determine whether or not the following filters are stable. Assume that they have first-quadrant support impulse responses.

(a) $$H(z_1, z_2) = \frac{1}{(1 - \frac{1}{2}z_1^{-1})(1 - \frac{1}{3}z_2^{-1})(1 - 2z_2^{-1})}$$

(b) $$H(z_1, z_2) = \frac{1}{1 + 5z_1^{-1} + 3z_2^{-1} + 2z_1^{-1}z_2^{-3} + 3z_2^{-6}}$$

(c) $$H(z_1, z_2) = \frac{1}{10 - 2z_1^{-1} - 2z_2^{-1} - 3z_1^{-1}z_2^{-1}}$$

(d) $$H(z_1, z_2) = \frac{1}{1 + 4z_1^{-1} + 3z_2^{-1} + z_1^{-1}z_2^{-2} + z_2^{-4}}$$

(e) $$H(z_1, z_2) = \frac{1}{1 - 0.6z_2^{-1} - 0.81z_1^{-1}z_2^{-1}}$$

(f) $$H(z_1, z_2) = \frac{1}{1 - 0.8z_1^{-1} - 0.7z_2^{-1} + 0.56z_1^{-1}z_2^{-1}}$$

2.26. Consider a first-quadrant support system with the system function $H(z_1, z_2)$ given by

$$H(z_1, z_2) = \frac{1}{1 - az_1^{-1} - bz_2^{-1}}.$$

Determine the conditions on a and b for which the system is stable.

2.27. Consider an LSI system whose output mask is sketched below.

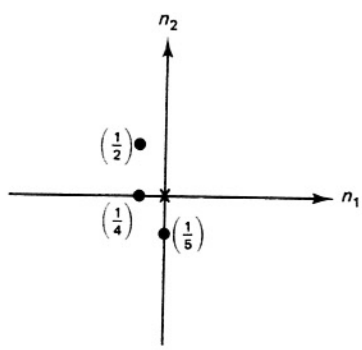

Figure P2.27

Determine whether or not the system is stable.