

Detour into Numerical Analysis

well Conditioned

Problem

ill Conditioned.

Well Condi

Algorithms

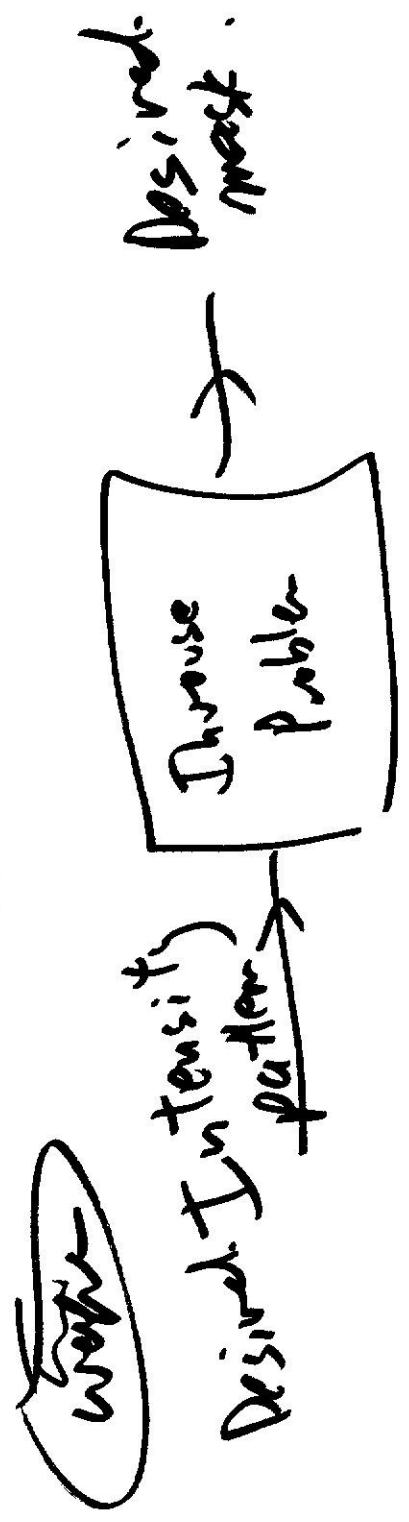
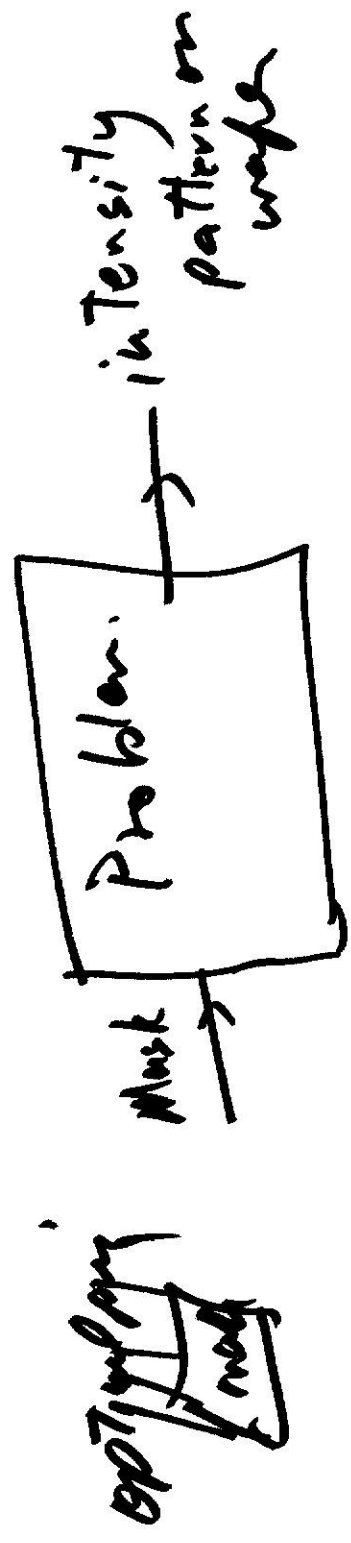
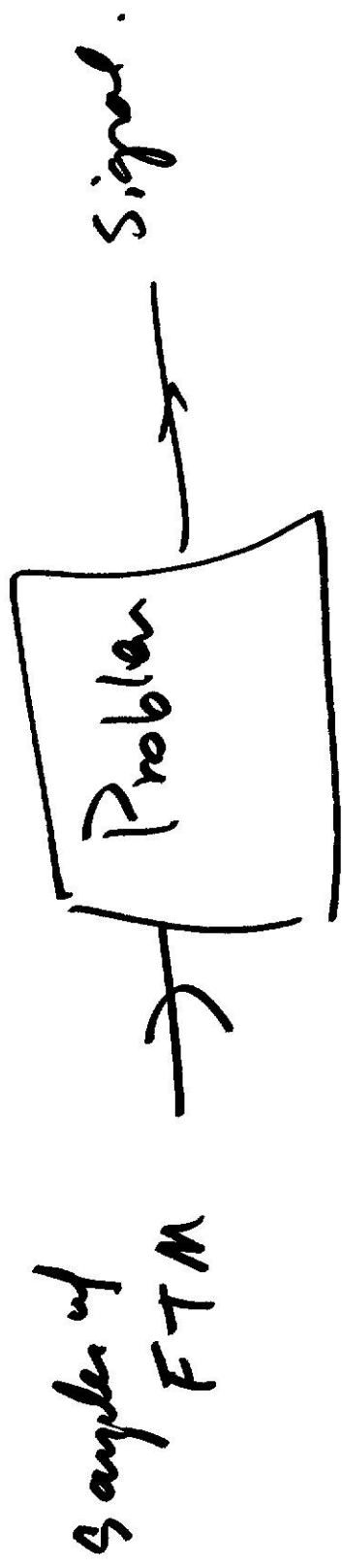
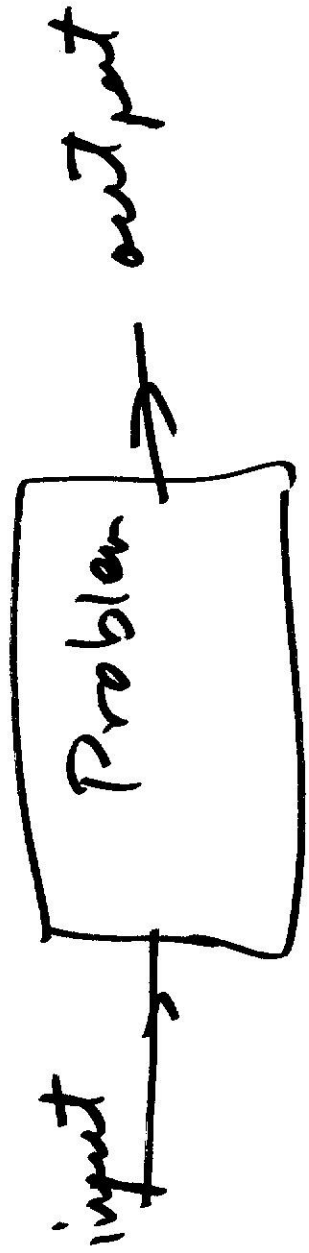
applied to problem

Ill Conditioned.

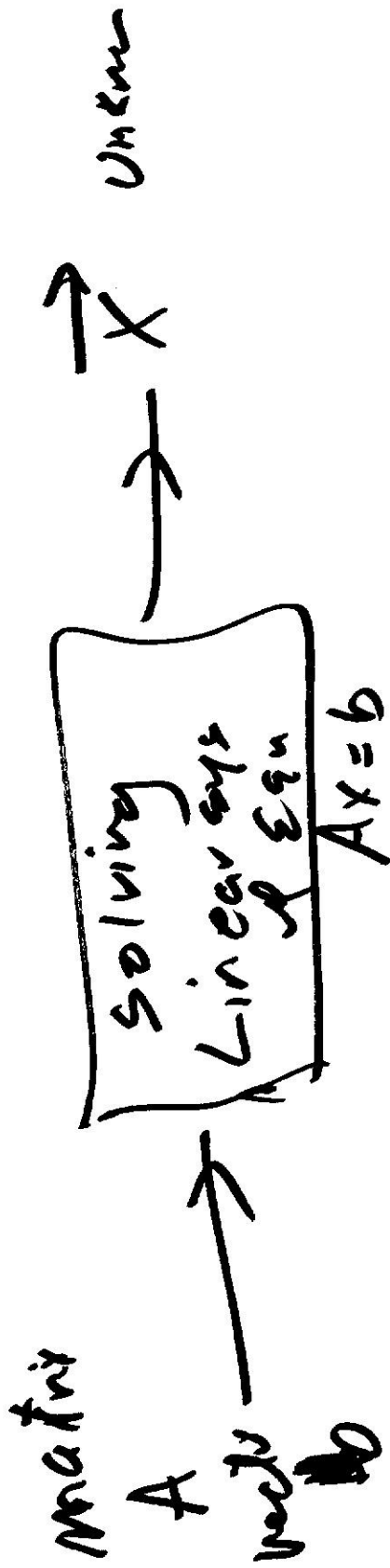
Condition # = measure of how

"well conditioned"

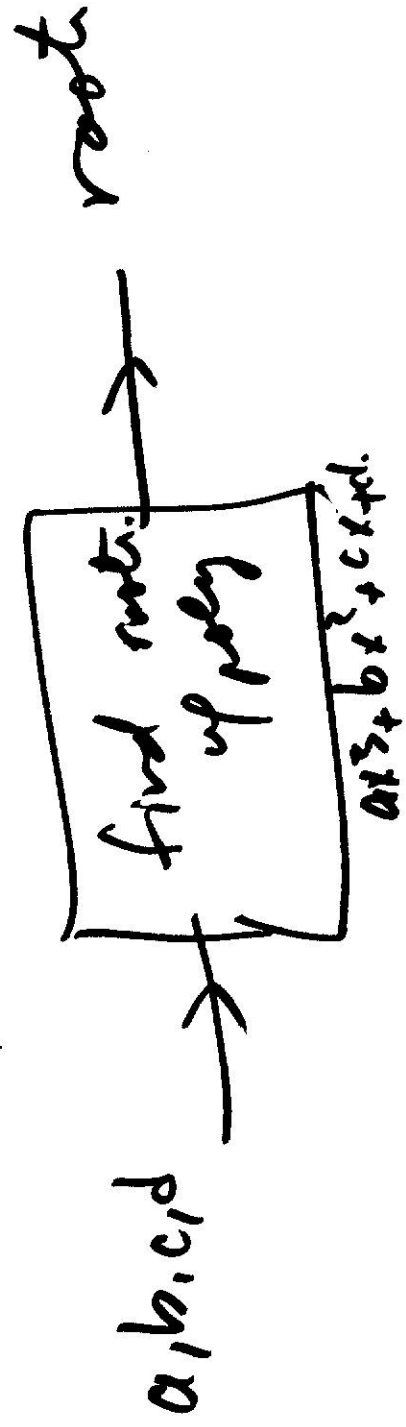
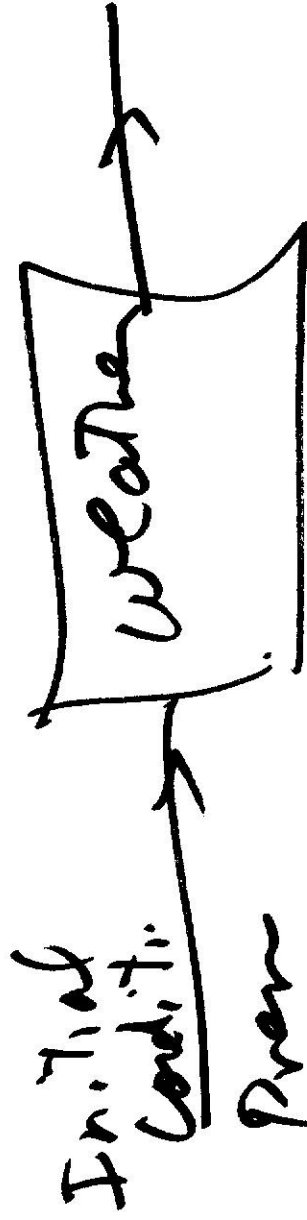
either alg or a problem is.



wafer



Tomorrow
 Temp
 Fair
 ...



Algorithm:

$$\underline{C} \vec{A} \vec{x} = \vec{b}$$

Method to solve a particular problem.

List of possible Algorithms

IT/1 Cond
alg.

① Gauss Elimination

② Compute Inverse

③ Gauss Elimination with Pivoting

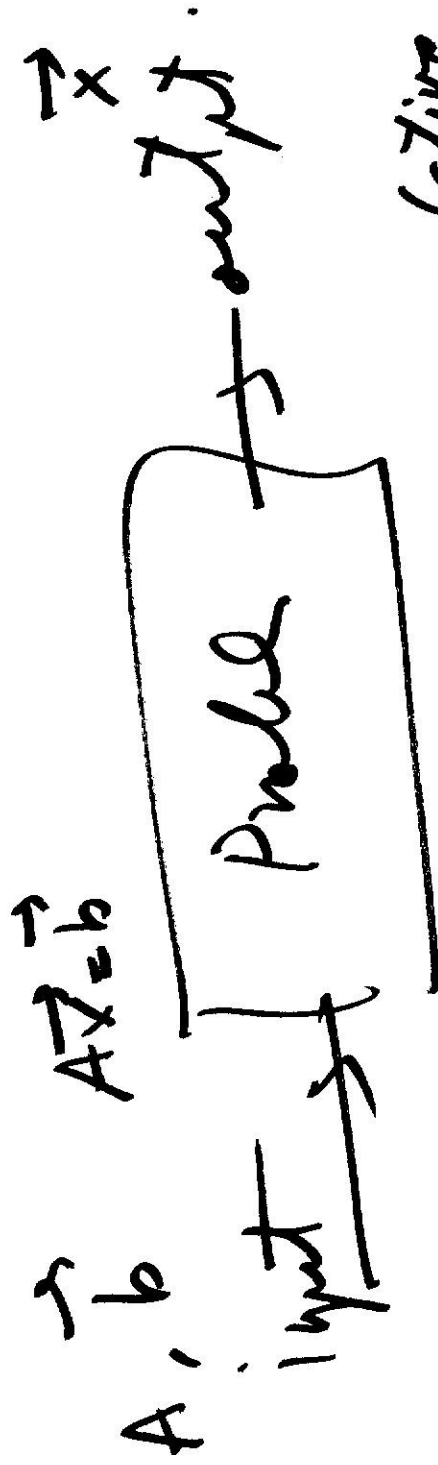
④ SVD excellent

⑤ QR Decomposition. excellent

⑥ Iterative alg. large problem

Condition of Problem

Condition # = Perturbation in output
perturbation in input.



relative change output
relative change input

Condition # = $10^6, 10^{10}, 10^{12}$ \rightarrow problem is ill-condition

Condition # is small \rightarrow problem is well-conditioned

$$Ax = b$$

$A \rightarrow$ Condition # = $\frac{\text{largest singular value}}{\text{smallest singular value}}$

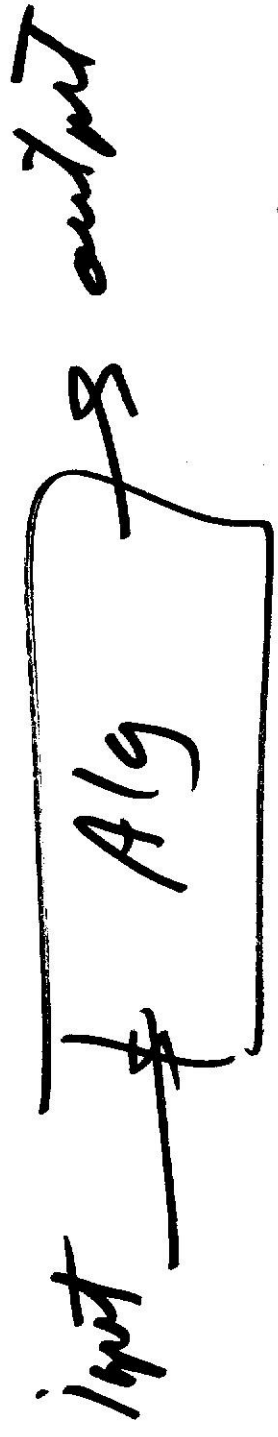
$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

Condition # of problem depends on the "Intrinsic" aspects of the problem itself. \rightarrow good answer



Problem \rightarrow Ill condition \rightarrow out of luck.

Condition # of Alg



$$\text{Condition \#} = \frac{\text{relative change in output}}{\text{relative change in input}}$$

A = very large condition

$$A \times x = b$$

Recn from FTM

- Even though theoretically Hayf/Mc Clellan showed uniqueness in practice \Rightarrow ill conditioned

- 2D polynomials can almost closely be approximated by few terms

- Close form \rightarrow Izraolovitz + Lim

- Iterative

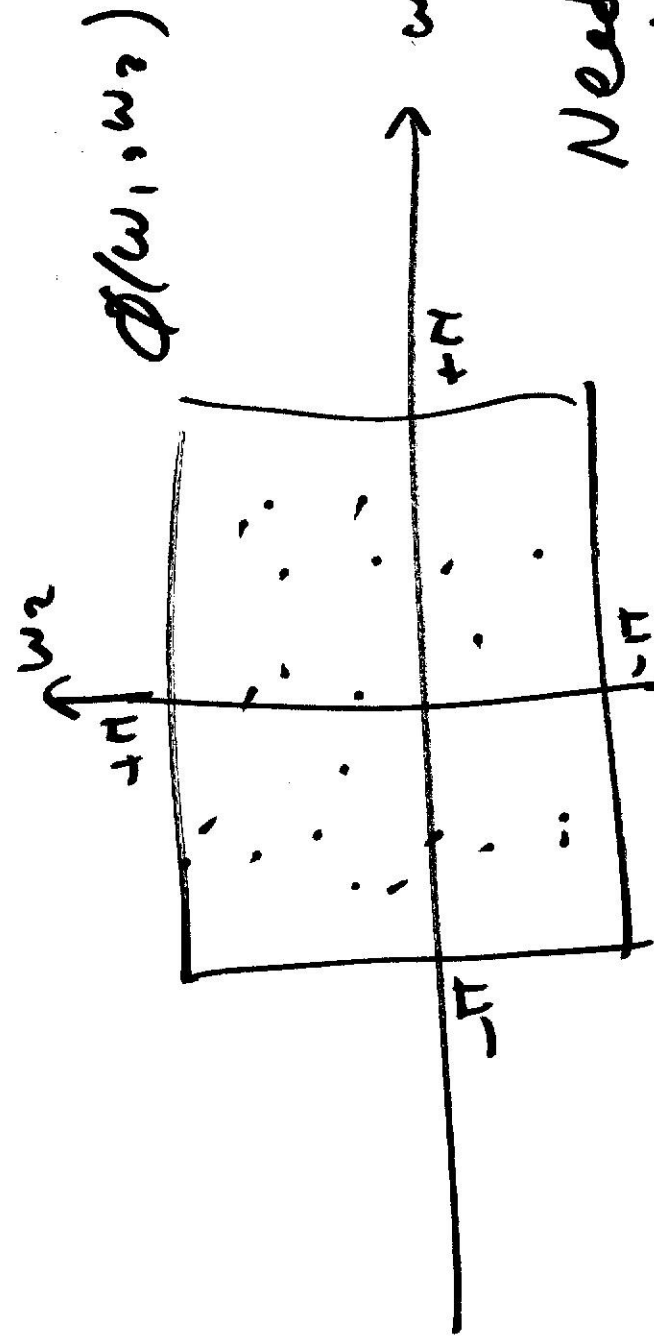
Oppenheim + Mersereau \rightarrow 1972
Proceedings of IEEE

P.S.T.

Recan from F.T. Phase

$\neq \phi$

$$X(n_1, n_2) \xrightarrow{\text{D.T.F.T}} X(\omega_1, \omega_2) = \sum_{n_1} \sum_{n_2} x(n_1, n_2) e^{-j\omega_1 n_1 - j\omega_2 n_2}$$



$\omega_1 \rightarrow$ continuous variables

Need to have samples of ϕ out more than N^2

Patrick Van Hove \approx 1982

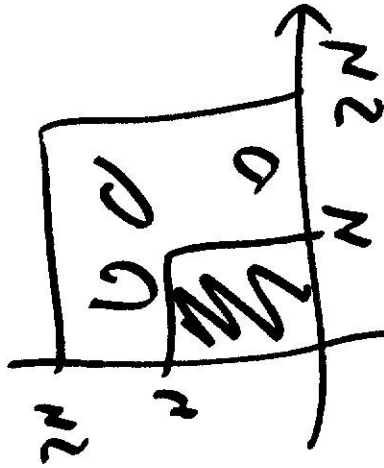
Two Alg. → iterative.

→ direct. • of F.T.

the 1982 → Even quantizing phase to
one bit & yet nearest.
signal successful.

Iteration

$N \times N$ signal.
 Know POS: $N \times N$.
 → Samples at $2N \times 2N$
 of phase of F.T. ϕ_k



~~Iteration~~

$x = \text{original signal}$

