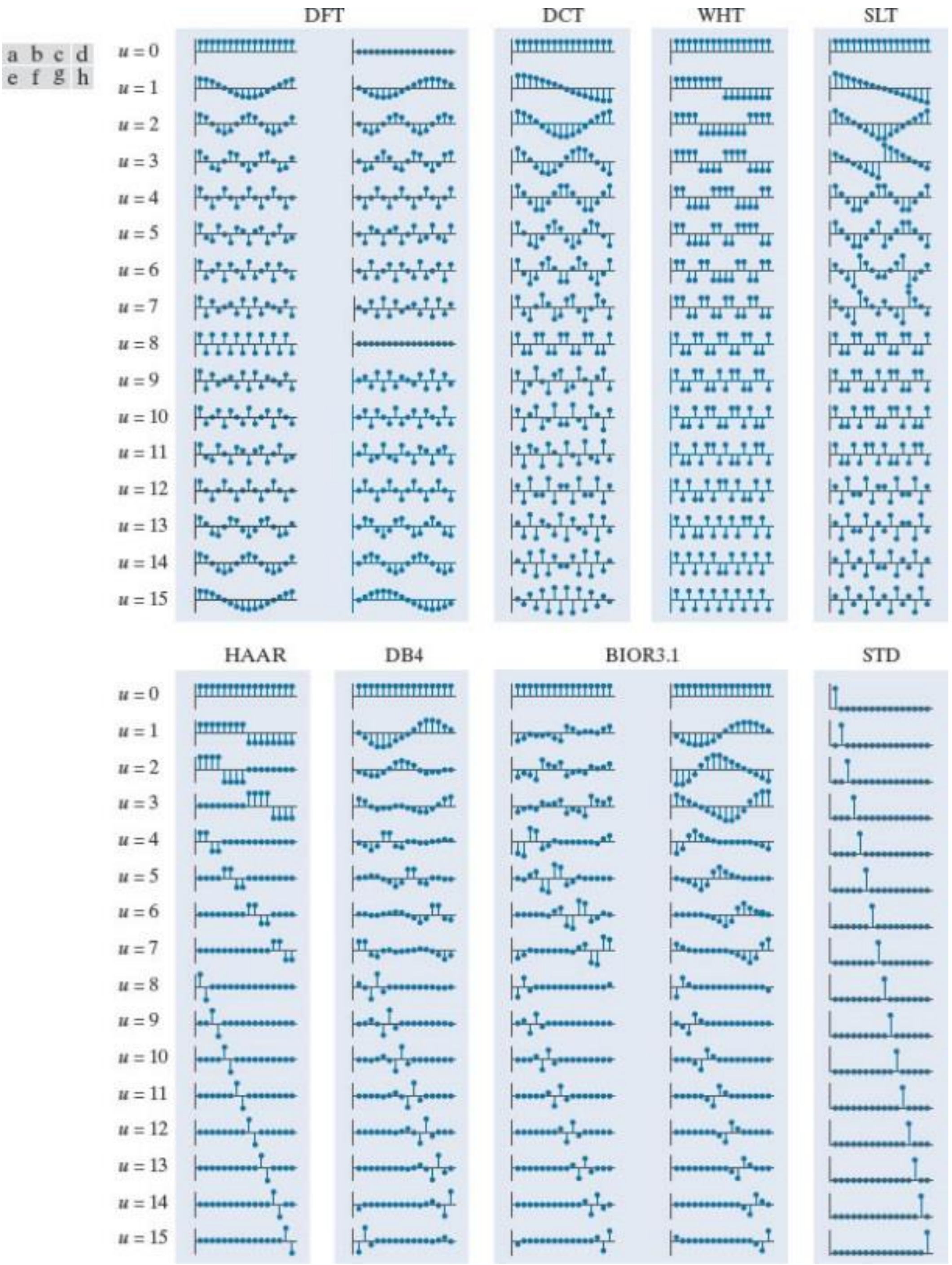
Tiling of the time frequency plane

Figure 6.3



Basis vectors (for N = 16) of some commonly encountered transforms: (a) Fourier basis (real and imaginary parts), (b) discrete Cosine basis, (c) Walsh-Hadamard basis, (d) Slant basis, (e) Haar basis, (f) Daubechies basis, (g) Biorthogonal B-spline basis and its dual, and (h) the standard basis, which is included for reference only (i.e., not used as the basis of a transform).

How to determine time & frequency extent of a function $h(t)$ in the time-frequency plane.

Let $$P_h(t) = \frac{|h(t)|^2}{\|h(t)\|^2}$$ probability density fn.

Mean : $$\mu_t = \frac{1}{\|h(t)\|^2} \int_{-\infty}^{+\infty} t\,|h(t)|^2\,dt.$$

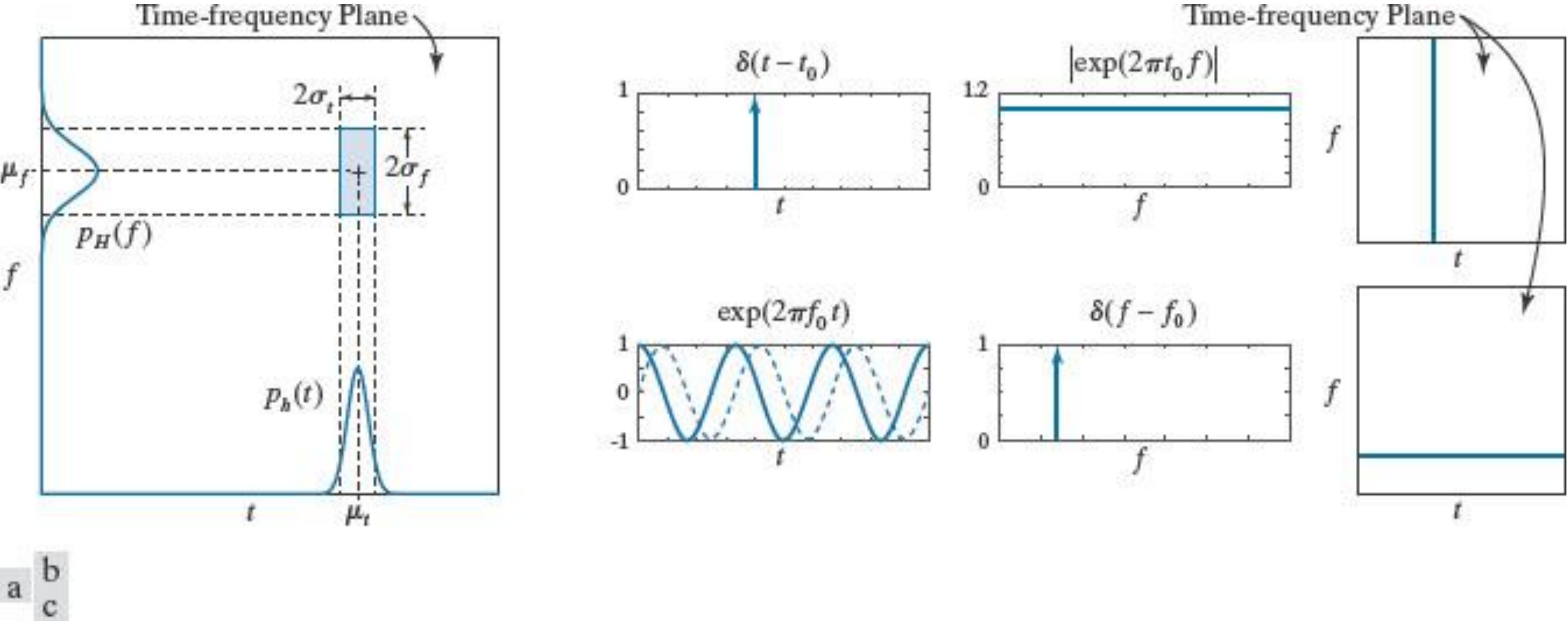
variance $$\sigma_t^2 = \frac{1}{\|h(t)\|^2} \int_{-\infty}^{+\infty} (t-\mu_t)^2\,|h(t)|^2\,dt.$$

$$F.T\{h(t)\} = H(f)$$

$$p_H(f) = \frac{|H(f)|^2}{\|H(f)\|^2}$$ - probability density fn for $H(f)$

mean $$\mu_f = \frac{1}{\|H(f)\|^2} \int_{-\infty}^{+\infty} f\,|H(f)|^2\,df$$

variance $$\sigma_f^2 = \frac{1}{\|H(f)\|^2} \int_{-\infty}^{+\infty} (f-\mu_f)^2\,|H(f)|^2\,df$$



(a) Basis function localization in the time-frequency plane. (b) A standard basis function, its spectrum, and location in the time-frequency plane. (c) A complex sinusoidal basis function (with its real and imaginary parts shown as solid and dashed lines, respectively), its spectrum, and location in the time-frequency plane.

Recall Haar basis fn:

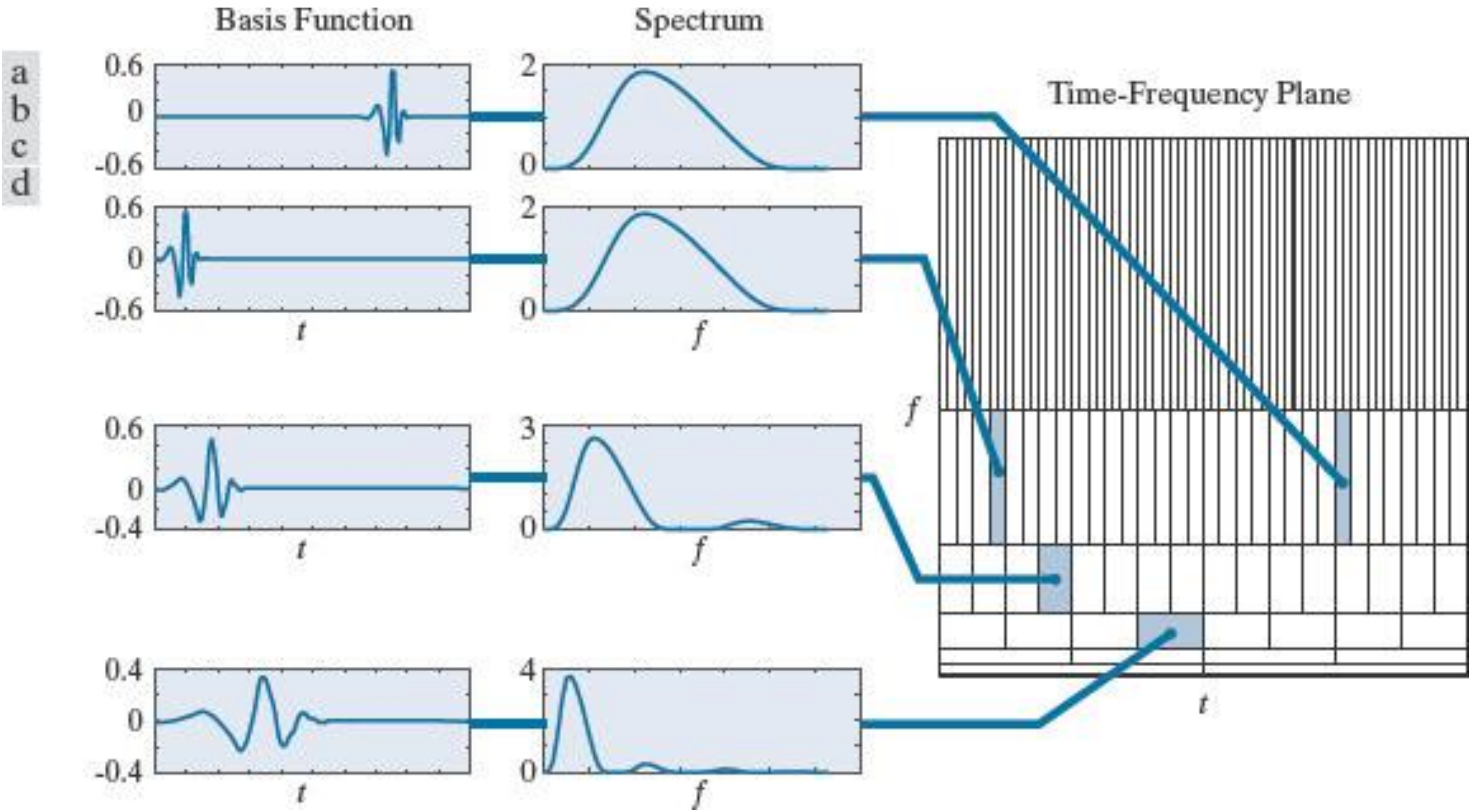
$$\psi_{s,\tau}(t) = 2^{s/2}\,\psi(2^s t - \tau) \qquad \tau, s \text{ integer}$$

$$\psi_{j,k}(x) = 2^{j/2}\,\psi(2^j x - k)$$

$$\text{F.T.}\left\{ \psi(2^s t) \right\} = \frac{1}{|2^s|}\,\Psi\left(\frac{f}{2^s}\right)$$

for $s > 0$ ⟶ spectrum is stretched

for $s < 0$ ⟶ spectrum is compressed



Time and frequency localization of 128-point Daubechies basis functions.