Rate Distortion Optimization

- Consider the problem:
  Minimize distortion D subject to constraint R_c on the # of bits R.

$$\min_{D} \quad \text{subject to} \quad R_c < R \qquad \oplus$$

- Soln: Use Lagrangian optimization:

$$\min_{J} \quad \text{where} \quad J = D + \lambda R \qquad (*)$$

- Each soln to $(*)$ for a given λ corresponds to an optimal soln to $\oplus$ for a particular value of R_c

Budget Constraint Allocation

- Find the optimal quantizer, or operating point $x(i)$ for each coding unit i s.t.

$$\sum_{i=1}^{N} r_{i\,x(i)} \leq R_T \qquad \text{Eq. 1}$$

and some metric

$$f(d_{1\,x(1)}, d_{2\,x(2)}, \ldots d_{N,x(N)})$$ is

minimized.

- Example. $f(\cdot \text{----}) = \sum_{i=1}^{N} d_{i\,x(i)}$

__Thm__ If the mapping $x'(i)$ for $i=1,2,\dots N$ minimizes

$$\sum_{i=1}^{N} d_{ix(i)} + \lambda r_{ix(i)}$$

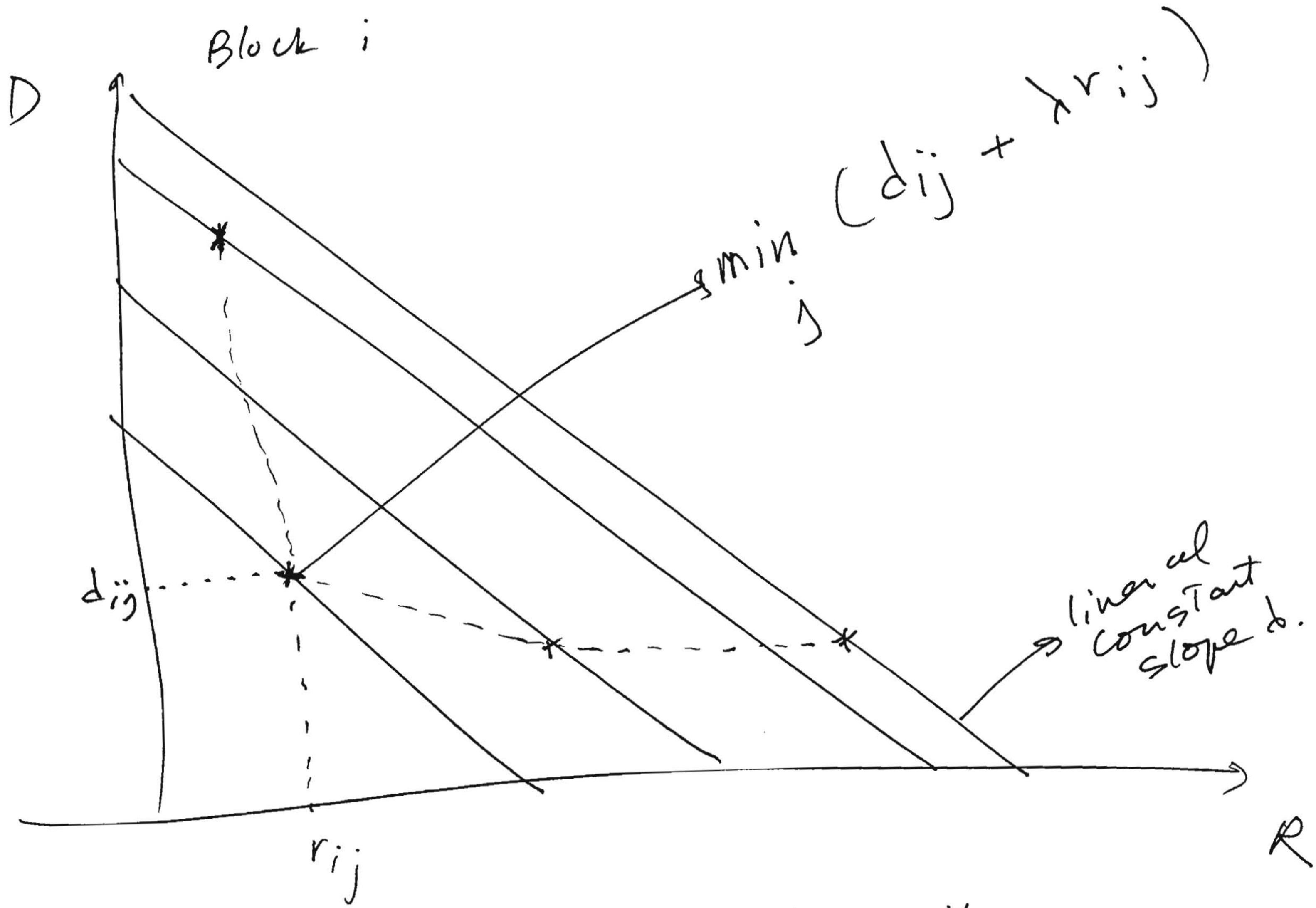
then, it is also the optimal soln to the budget constrained problem for the case where

$$R_T = R(\lambda) = \sum_{i=1}^{N} r_{ix'(i)}$$

- Note: $\min \left\{ \sum_{i=1}^{N} d_{ix(i)} + \lambda r_{ix(i)} \right\} = \sum_{i=1}^{N} \min \left\{ d_{ix(i)} + \lambda r_{ix(i)} \right\}$

$\Rightarrow$ minimum can be computed independently for each coding unit.

- For each coding unit i, the point on R_D characteristic that minimizes $d_{ix(i)} + \lambda r_{ix(i)}$ is that point at which the line of absolute slope λ is tangent to the convex hull of the R_D characteristic



λ is same for all coding units.