Problem 1. Consider the matrices:

\[
A_1 = \begin{bmatrix}
6 & 5 & -1 \\
-1 & 0 & 1 \\
3 & 3 & 2
\end{bmatrix} \quad A_2 = \begin{bmatrix}
2 & 0 & 1 \\
2 & 5 & -2 \\
3 & 4 & 0
\end{bmatrix}
\]  

(1)

By defining an appropriate similarity transform in each case, put each matrix into either diagonal or Jordan form.

Problem 2.

Suppose \( A \in \mathbb{C}^{n \times n} \) is such that \( \det(A) = 0 \). Is \( \det(e^A) = 0 \)? Explain why or why not.

Problem 3.

A matrix \( A \in \mathbb{R}^{6 \times 6} \) has minimal polynomial \( s^3 \). Give bounds on the rank of \( A \).

Problem 4.

A matrix \( A \) has minimal polynomial \( (s - \lambda_1)^2(s - \lambda_2)^3 \). Find \( \cos(e^A) \) as a polynomial in \( A \).

Problem 5.

In the preceding problem, assume that \( A \) has characteristic polynomial \( (s - \lambda_1)^5(s - \lambda_2)^3 \) and that it has four linearly independent eigenvectors. Write down the Jordan form \( J \) of this matrix and write down \( \cos(e^A) \) explicitly.

Problem 6.

Let \( A \in \mathbb{R}^{n \times n} \) be non-singular. True or false: the nullspace of \( \cos(\log(A)) \) is an \( A \)-invariant subspace.

Problem 7.

Consider \( A \in \mathbb{R}^{n \times n}, b \in \mathbb{R}^n \). Show that \( \text{span}\{b, Ab, \ldots, A^{n-1}b\} \) is an \( A \)-invariant subspace.