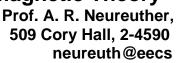
**EECS 210** Fall 2006 Tu, Th 12:30-2 400 Corv

## **Applied Electromagnetic Theory** Office Hours M, (W), 11AM





## Homework # 8: Due Start of Class Tuesday, Nov 21<sup>th</sup>

These problems are based on the solution of the coupled modes pp. 71-73 of the Kogelnik Chapter in Tamir. Assume a wavelength of 500nm for estimating distances.

## 8.1) Couple Modes: Physical Properties: Assume the co-directional case.

Tu, Th, (F) 10AM

- a) For the phase matched case when  $\delta = 0$ , determine the  $\kappa$  value required to couple 100% in a distance of 20  $\mu$ m. From this value suggest a rule of thumb for the value of  $\kappa L/\lambda$ .
- b) For optical image Lord Rayleigh pointed out that a phase error of 90 degrees from center to edge of the lens can be considered as a good measure of the defocus limit. Consider the coupler when the phase error in each wave is equal and opposite at  $\delta L = \pi/4$ . Assume the 100% coupling condition from a) holds, namely that  $\kappa L = \pi/2$ . Evaluate the complex quantities S and R. Hint: S comes out very similar to the Strehl ratio for the point spread function with one Rayleigh Unit defocus which is 0.78. Hint: Check your answer by conservation of energy as  $SS^* + RR^* = 1$ .

7.2) Coupled Modes: Phase-Matching for Periodic Case: Consider the Contra-directional coupler. The device structure consists of a film with refractive index 2.0 on a substrate with refractive index 1.5. Assume that a mode on this film has a k-vector 1.9 larger than  $k_0 = 2\pi/\lambda$ . The coupling of a plane wave into a mode in the film is produced a periodic height modulation with period P. The structure is uniform in the y-direction and the fields have no y-variation.

- a) Determine the period P that would be suitable for coupling a planewave making a  $30^{\circ}$ angle with the x-axis with its k-vector in the y = 0 plane in the backward direction. (The wave is propagating in the (-x,-z) direction and the mode is propagating in the +zdirection.)
- b) Determine the largest period that could be used in the coupler to not produce substrate radiation by the -2 spatial harmonic. Is the incident angle to couple via the -1 harmonic still backward? What is the minimum angle?

7.3) Coupled Mode Eigenvalues and Eigenvectors: Use the equations 2.6.28 and 2.6.29 and boundary conditions given just below with the eigenvalue-eigenfunction method to derive equations 2.6.30 and 2.6.31.

- a) Find the eigenvalues.
- b) Find the eigenvectors and normalize to unit energy.
- c) Match boundary conditions to find the amplitudes that multiply the eigenfunctions.
- d) Rearrange your results to obtain equations 2.6.30 and 2.6.31.
- e) Evaluate the two eigenfunctions at the three phase values of  $\delta = -2\kappa$ , 0,  $+2\kappa$ . Show that the relative weights of the first and second components of the eigenfunctions for  $\lambda_1$  and  $\lambda_2$  tend to switch identities as the phase mismatch goes from negative to positive. (Example: Define eigenvalue 1 as coming from the +sign in sqrt. This eigen vector for this eigenvalue will have its top component smaller than its lower component for negative values of  $\delta$ . At  $\delta = 0$  they will be equal and for  $\delta$  positive the top component will be larger than the bottom component.)