## EECS 210 Applied Electromagnetic Theory

Fall 2006
Tu, Th 12:30-2
Office Hours M, (W), 11AM
400 Cory

## Homework \#1: Due start of class Th Sep 7th

1.1) Divergence Theorem: Consider a vector $\vec{A}=x \vec{x}+y \vec{y}$ and a three dimensional rectangular box that goes from $(x=0, y=0, z=0)$ to $(x=a, y=b, z=c)$. Show that the integral of the divergence of this vector over the volume of this box is equal to the dot product of the vector with the local normal outward normal over the surface of the box.
1.2) Gauss Law Projected on Unit Sphere: Consider a surface define by $x^{2}+(y-0.5)^{2}+z^{2}=4$ and a charge at $(0,0,0)$. Evaluate Gauss's law over this surface as follows.
a) Outline how to evaluate the integral directly and carry out the steps that are not too algebraically intense.
b) Now carry out a more tractable approach by
a. showing that integrating the normal displacement over the original surface can be mapped into an integral over a unit sphere centered on the charge, and
b. carrying out this integral.
1.3) Integral over a singularity: Consider a charge at the origin and 4 other charges at arbitrary locations. Take a vanishingly small sphere about the origin of radius $\varepsilon$ and integrate both sides of Poisson's equation for the potential over this volume.
a) Outline an argument as to why the integral over the contributions from the 4 distant charges will be vanishingly small.
b) Use spherical coordinates and the formulas inside the front cover of Jackson to evaluate the Laplacian operating on the $1 / r$ behavior of the potential as a function of radius and note the dependence on radius
c) Show that this singular behavior can be integrated and gives the desired $4 \pi$ factor on the right hand side to multiply $\rho / \varepsilon$.
d)

For Problem set \#2
2.1) Surface charge singularity contribution: Consider the potential due a continuous distribution of charge on a smooth surface as an observation point $\bar{X}_{o}^{\prime}$, that is $\varepsilon$ away from the surface approaches an observation point $\bar{X}_{0}$ on the surface. Consider a small circle in the surface about $\bar{x}_{0}$ of radius a over which the charge density can be approximated as constant. Break up the integral for the potential at $\mathbf{x}_{0}$ into a part outside the circle and a part over the circle. Show that when $\mathrm{e} \ll$ a the integral for the potential (written out as a function of $\left|\vec{X}_{\text {surface }}-\vec{x}_{0}^{1}\right|$ )over the circular disk integrates to a factor of $2 \pi$ time $\sigma / \varepsilon$.

